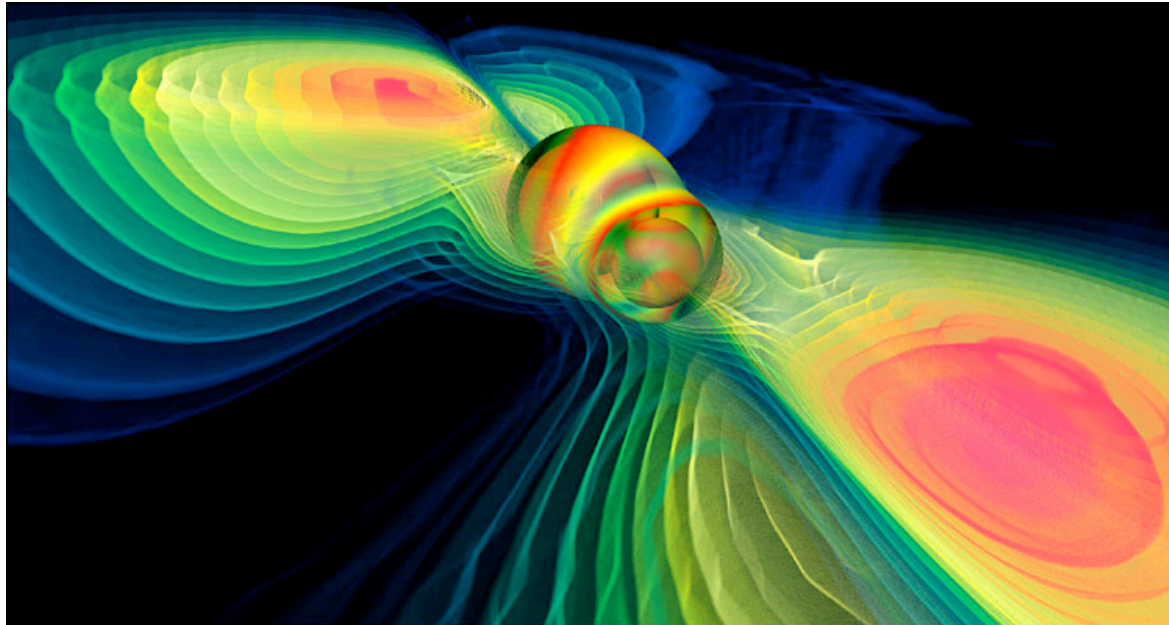


# Extracting the Distribution of Black-hole Parameters from Gravitational-wave Observations



(Image: MPI for Gravitational Physics / W.Benger-ZIB)

Ilya Mandel  
(NSF AAPF, @Northwestern University)

February 19, 2010

Formation and Evolution of Black Holes, Aspen

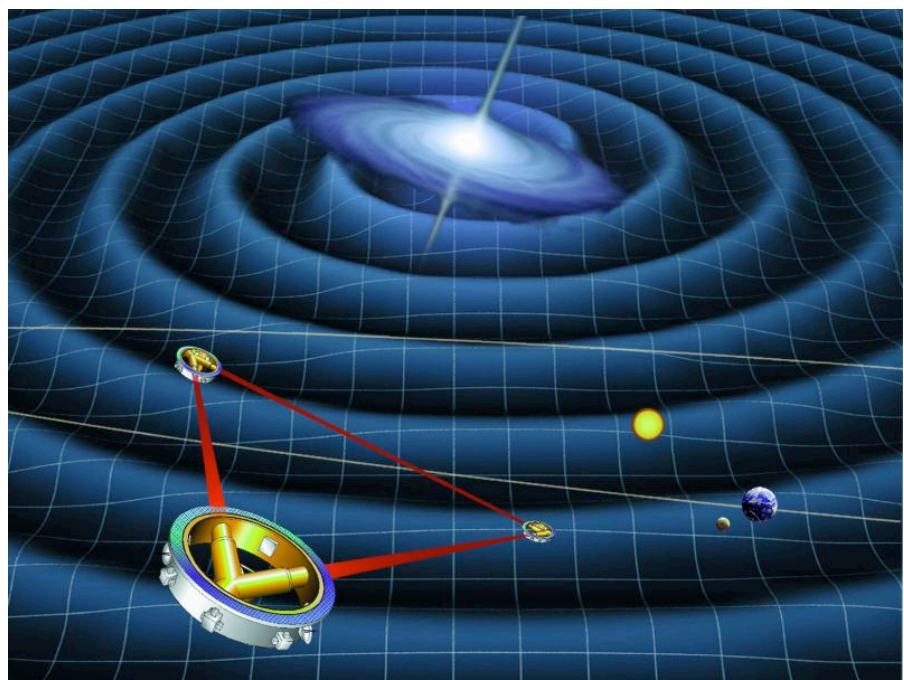
# Jantar Mantar, Delhi



# Gravitational-wave observatories



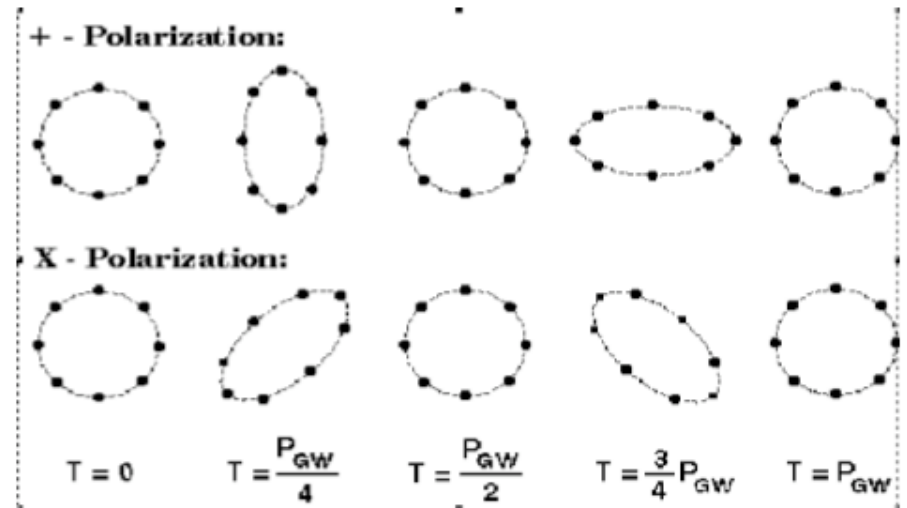
LIGO:  
Laser  
Interferometer  
Gravitational-wave  
Observatory



LISA:  
Laser  
Interferometer  
Space  
Antenna

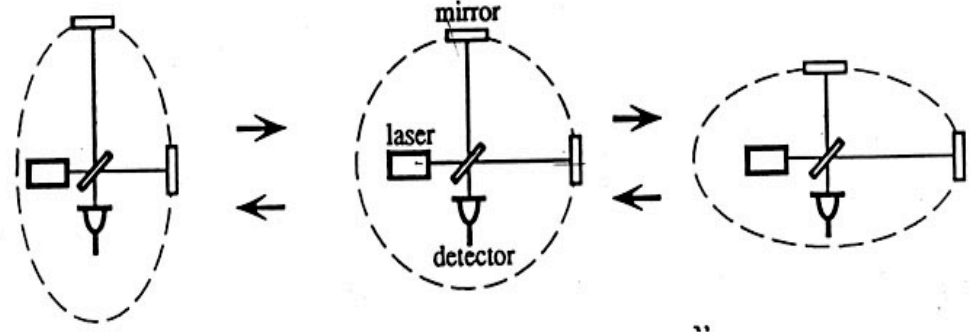
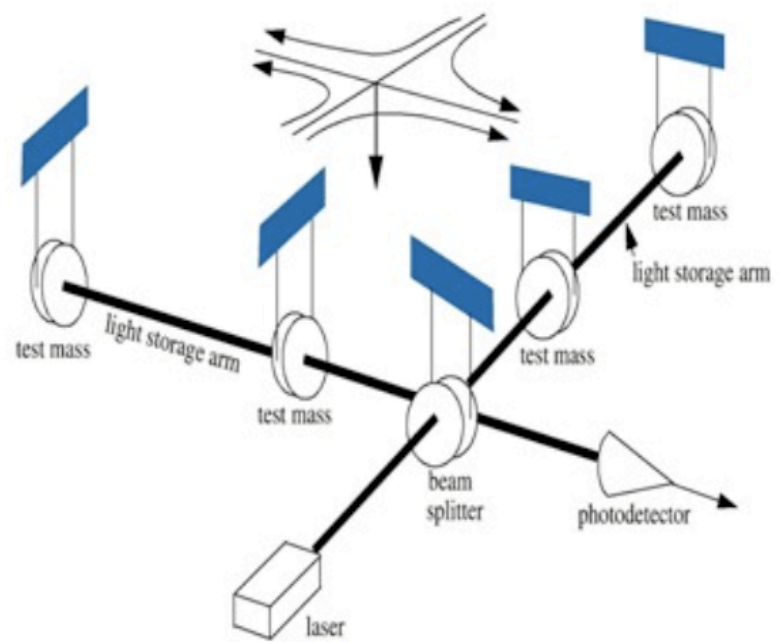
# Gravitational Waves

- Ripples in spacetime
- Caused by time-varying mass quadrupole moment; GW frequency is twice the orbital frequency for a circular, non-spinning binary



- Indirectly detected by Hulse & Taylor [binary pulsar]
- Huge amounts of energy released: 5% of mass-energy of a supermassive black hole binary is comparable to the electromagnetic radiation emitted from an entire galaxy over the age of the universe!
- GWs carry a lot of energy, but interact weakly: can pass through everything, **including** detectors!

# Opportunity and Challenge

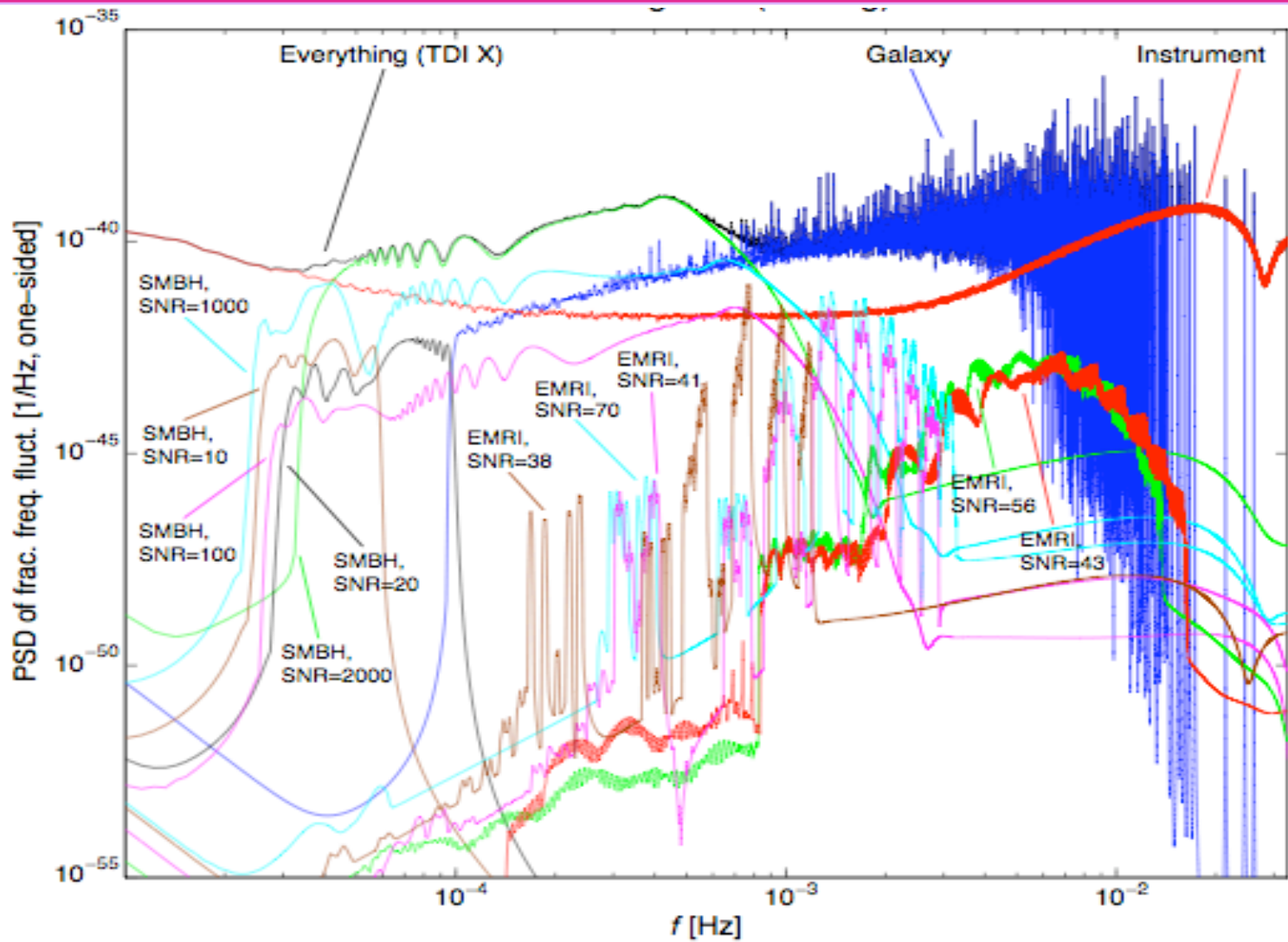


## Michelson-type interferometers

# LISA Binary Sources

- LIGO sensitive @ a few hundred Hz
  - » NS-NS, NS-BH, BH-BH binaries
- LISA sensitive @ a few mHz
  - » massive black-hole binaries
    - merger tree models to describe history of Galactic mergers
    - could be detected anywhere in Universe, SNR up to thousands
    - a few to tens of detections [e.g., Sesana et al., 2005]
  - » extreme-mass-ratio inspirals of WDs/NSs/BHs into SMBHs
    - complicated modeling of dynamics in Galactic centers: loss cone problem, resonant scattering, etc.
    - could detect tens to hundreds to  $z \sim 1$  [e.g., Gair et al., 2004]
  - » galactic white dwarf (and compact object) binaries
    - 30 million in Galaxy, create noise foreground [Farmer & Phinney, 2003]
    - 20,000 resolvable

# Embarrassment of riches



# LISA Data Analysis

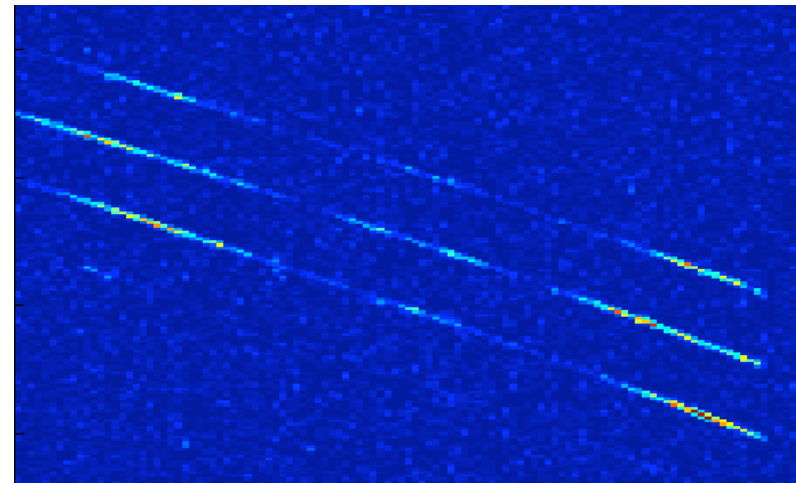
$$h(t) = h(M_1, M_2, \vec{S}_1, \vec{S}_2, \theta, \phi, D_L, e, \dots; t) \quad 17 \text{ parameters}$$

## What has already been accomplished?

	MLDC 1	MLDC 2	MLCD 1B	MLDC 3
GB	<ul style="list-style-type: none"> <li>• Verification ✓</li> <li>• Unknown, isolated ✓</li> <li>• Unknown, interfering ✓</li> </ul>	<ul style="list-style-type: none"> <li>• Galaxy of <math>3 \times 10^6</math> ✓</li> </ul>	<ul style="list-style-type: none"> <li>• Verification ✓</li> <li>• Unknown, isolated ✓</li> <li>• Unknown, confused ✓</li> </ul>	<ul style="list-style-type: none"> <li>• Galaxy of <math>6 \times 10^7</math> chirping ✓</li> </ul>
MBH	<ul style="list-style-type: none"> <li>• Isolated ✓</li> </ul>	<ul style="list-style-type: none"> <li>• 4–6x, over Galaxy and EMRIs ✓</li> </ul>	<ul style="list-style-type: none"> <li>• Isolated ✓</li> </ul>	<ul style="list-style-type: none"> <li>• Over Galaxy spinning, precessing ✓</li> </ul>
EMRI		<ul style="list-style-type: none"> <li>• Isolated ✓</li> <li>• 4–6x, over Galaxy and SMBHs</li> </ul>	<ul style="list-style-type: none"> <li>• Isolated ✓</li> </ul>	<ul style="list-style-type: none"> <li>• 5 together, weaker ✓</li> </ul>
New				<ul style="list-style-type: none"> <li>• Cosmic string cusp bursts ✓</li> <li>• Cosmological background ✓</li> </ul>

Table by M. Vallisneri

Need innovative search techniques to separate many overlapping signals: Markov-Chain Monte Carlo, MultiNest, genetic algorithms

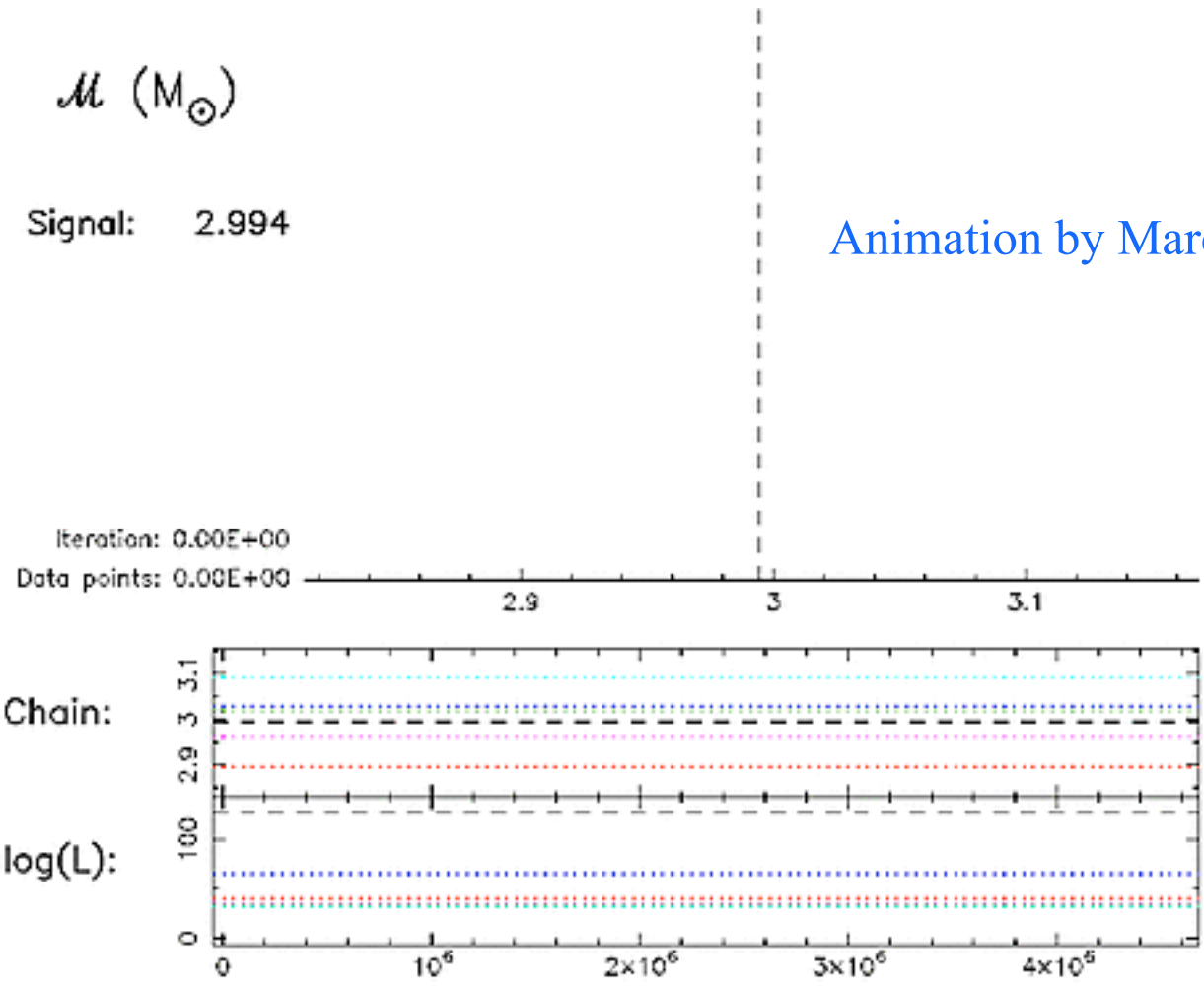


[Gair, IM, Wen, 2008, CQG 25 184031]

## Mock LISA Data Challenges

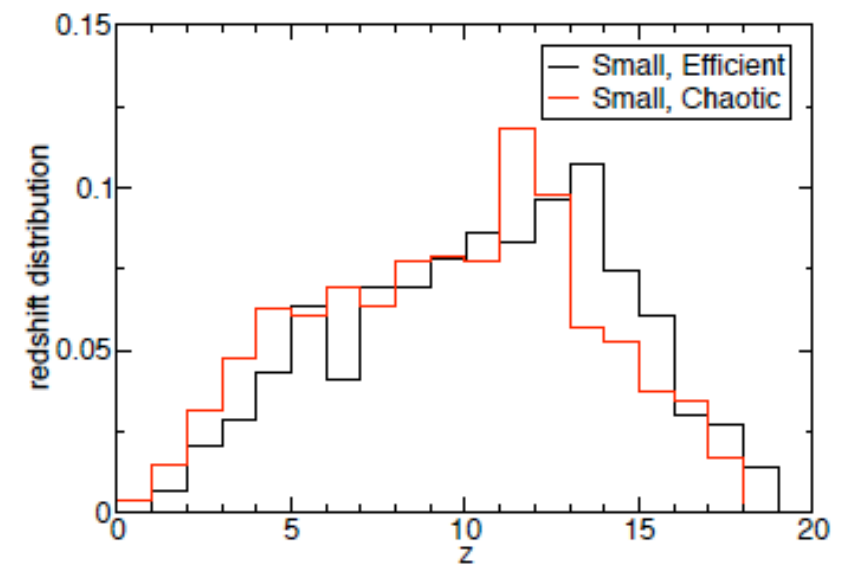
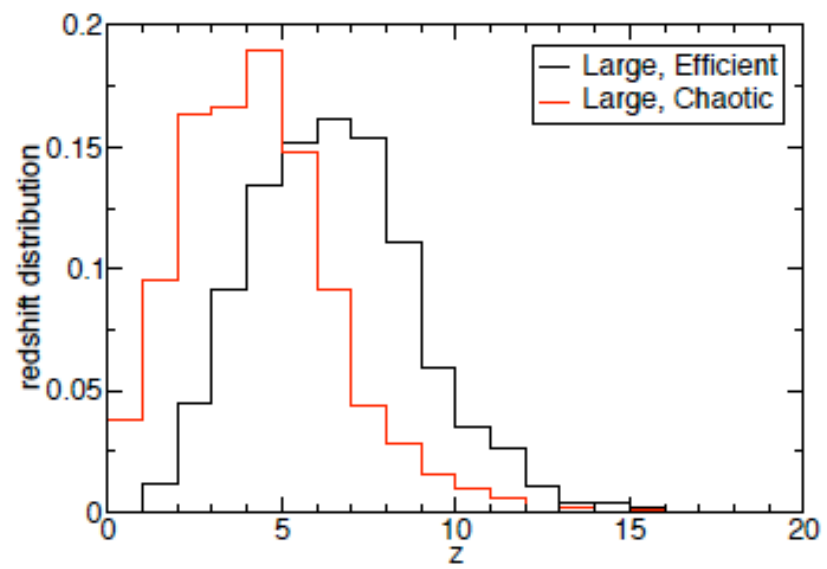


# Markov Chain Monte Carlo



Animation by Marc van der Sluys

# SMBH binaries

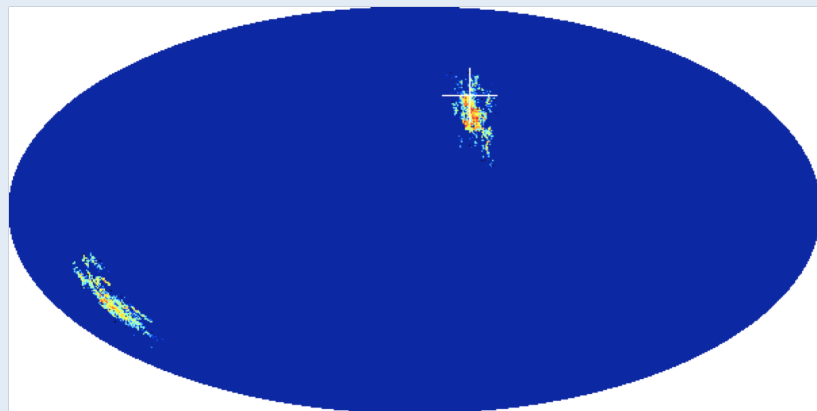


Model	$N$	$N_{\text{det}}$	$N_{10\%D_L}$	$N_{10 \text{ deg}^2}$	$N_{10 \text{ deg}^2, 10\%D_L}$	$N_{1 \text{ deg}^2}$	$N_{1 \text{ deg}^2, 1\%D_L}$
SE	80	33 (25)	21 (8.0)	8.2 (1.5)	7.9 (1.1)	2.2 (0.6)	1.7 (0.1)
SC	75	34 (27)	17 (4.4)	6.1 (0.4)	5.5 (0.4)	1.3 (0.1)	1.3 (0.1)
LE	24	23 (22)	21 (7.7)	10 (0.8)	10 (0.7)	2.2 (0.1)	1.2 (0.05)
LC	22	21 (19)	14 (4.3)	6.5 (0.5)	5.4 (0.5)	1.8 (0.04)	1.0 (0.1)

from [Arun et al. (LISA Parameter Estimation Taskforce), 2008, CQG 26, 094027]

# Mock LISA Data Challenge Results

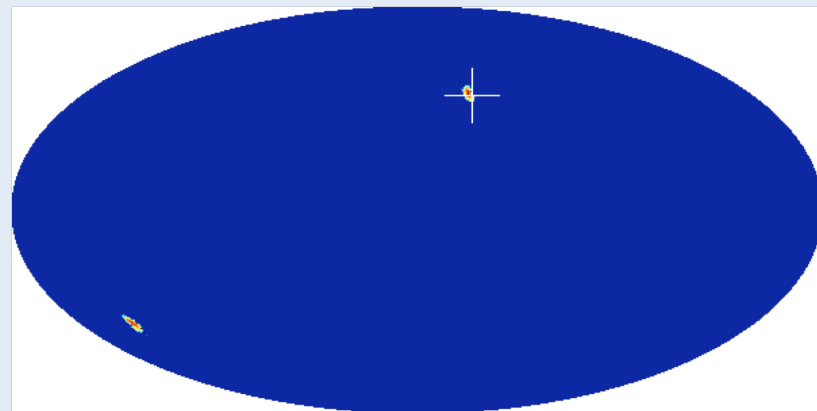
## Challenge 3.2 : Massive Black Holes



30 days before merger

Monte Carlo by Neil Cornish

## Challenge 3.2 : Massive Black Holes

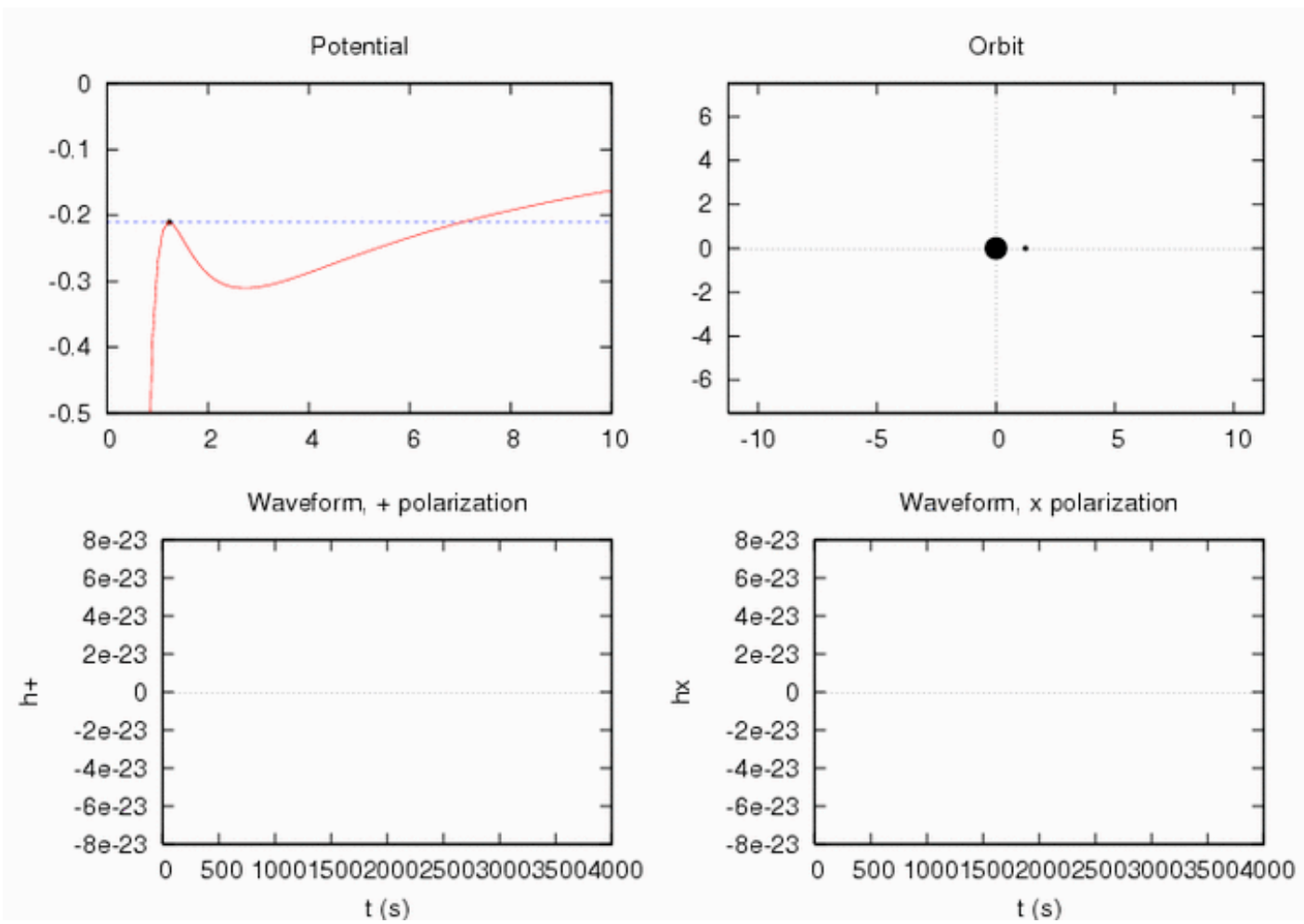


1 day before merger

Monte Carlo by Neil Cornish

source (SNR <sub>true</sub> )	group	$\Delta M_c/M_c$ $\times 10^{-5}$	$\Delta \eta/\eta$ $\times 10^{-4}$	$\Delta t_c$ (sec)	$\Delta \text{sky}$ (deg)	$\Delta a_1$ $\times 10^{-3}$	$\Delta a_2$ $\times 10^{-3}$	$\Delta D/D$ $\times 10^{-2}$	SNR	FF <sub>A</sub>	FF <sub>E</sub>
MBH-1 (1670.58)	AEI	2.4	6.1	62.9	11.6	7.6	47.4	8.0	1657.71	0.9936	0.9914
	CambAEI	3.4	40.7	24.8	2.0	8.5	79.6	0.7	1657.19	0.9925	0.9917
	MTAPC	24.8	41.2	619.2	171.0	13.3	28.7	4.0	1669.97	0.9996	0.9997
	JPL	40.5	186.6	23.0	26.9	39.4	66.1	6.9	1664.87	0.9972	0.9981
	GSFC	1904.0	593.2	183.9	82.5	5.7	124.3	94.9	267.04	0.1827	0.1426

# Extreme Mass Ratio Inspirals



Sound from  
Scott Hughes

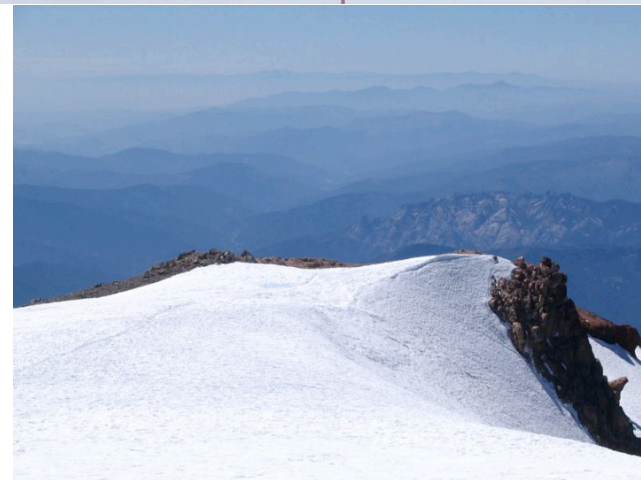
Animation  
from Jon Gair

# Exploring the spacetime...



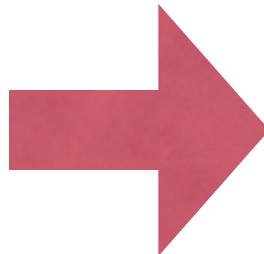
BHs @ Aspen. February 19, 2010

# ... taking lots of pictures

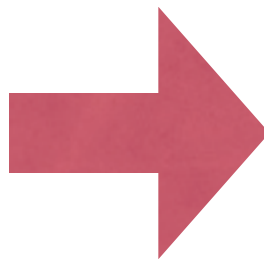


BHs @ Aspen. February 19, 2010

# Testing the no-hair theorem



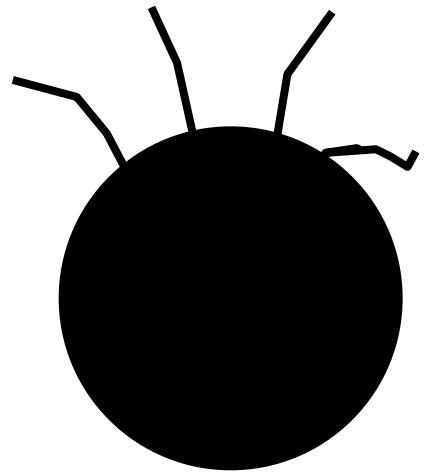
# Testing the no-hair theorem?



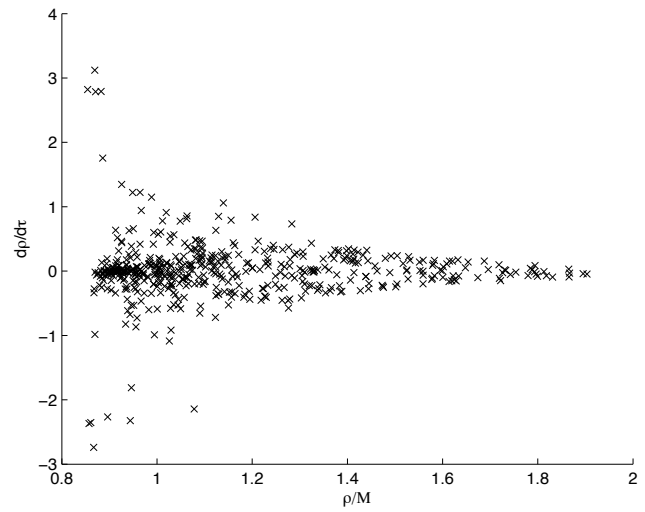
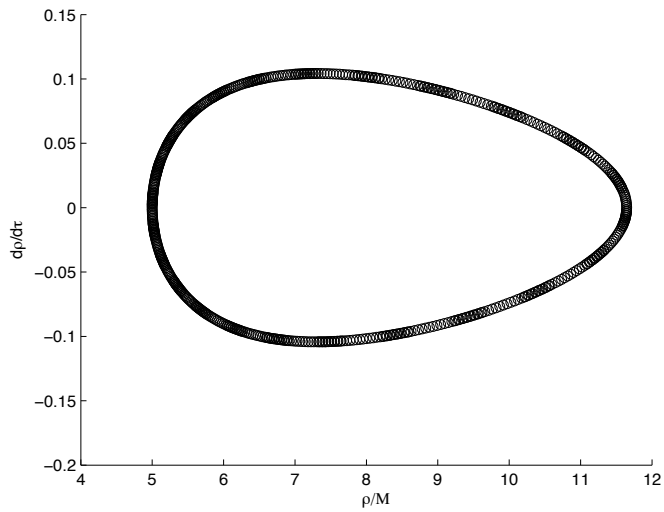
Stationary, vacuum, asymptotically flat spacetimes in which the singularity is fully enclosed by a horizon with no closed timelike curves outside the horizon are described by the Kerr metric



# Do black holes have hair?

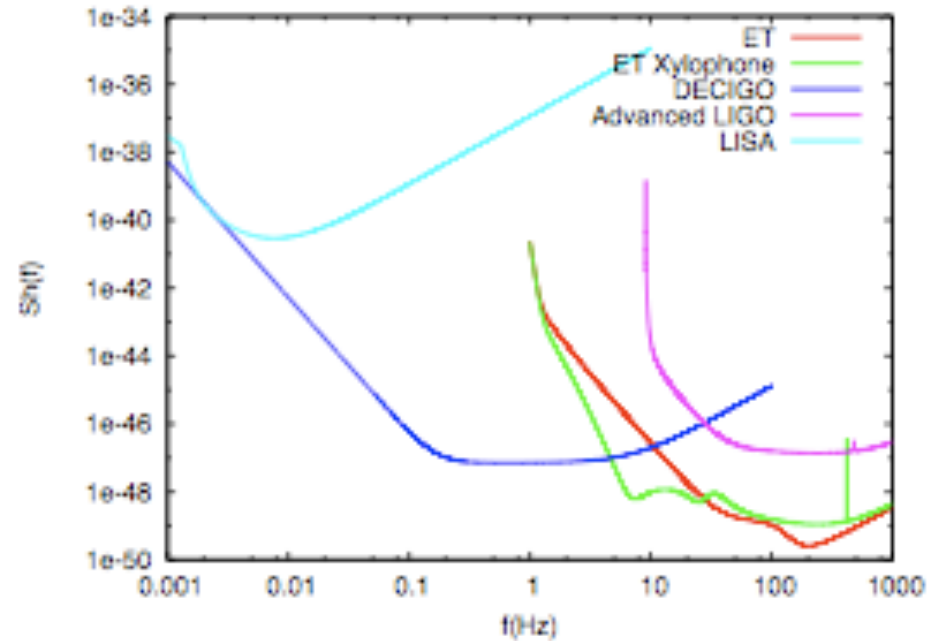


$$M_n + iS_n \neq M(ia)^n$$



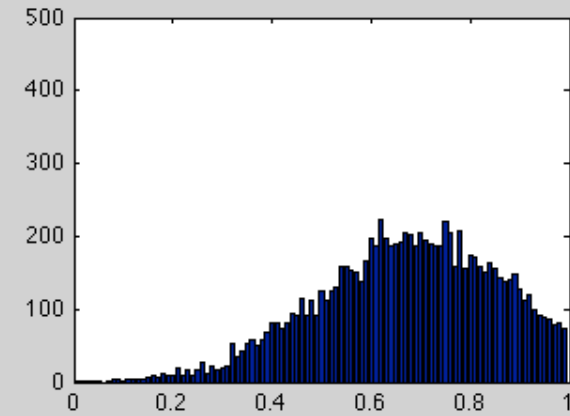
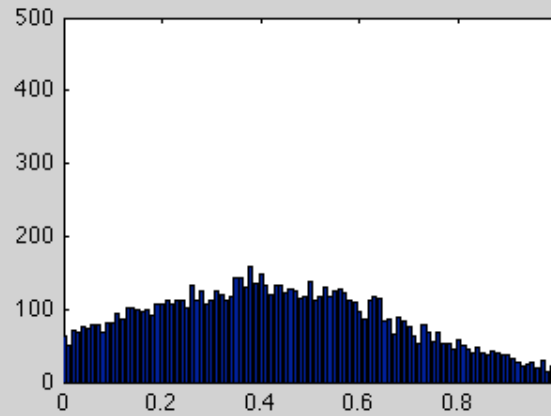
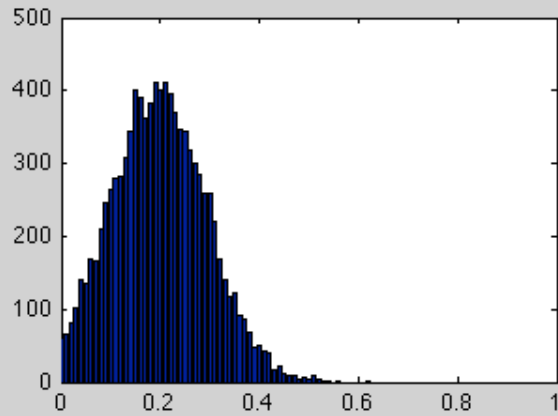
# Third-generation detectors

- The Einstein Telescope:
  - » Underground, sensitive to 1 Hz
  - » Exciting science example: mergers of light seeds of massive black holes at high redshifts [Sesana, Gair, IM, Vecchio, 2009]
- ALIA/DECIGO/BBO
  - » Space-based LISAs on steroids
  - » Exciting science example: using 300,000 merging binaries as standard candles for precision cosmology: Hubble constant to 0.1%,  $w$  to 0.01 [Cutler & Holz, 2009]
- Pulsar timing [see next talk!]
  - » Sensitive to SMBHBs @  $10^{-8}$  Hz

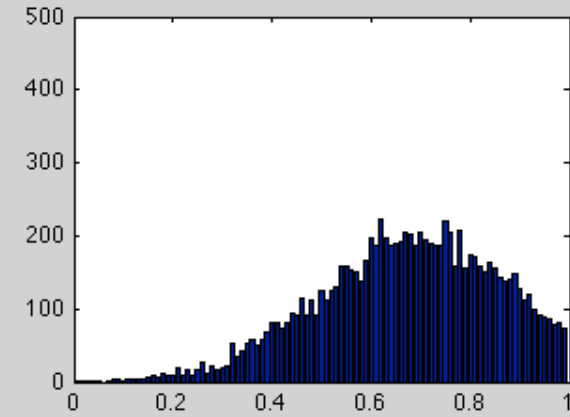
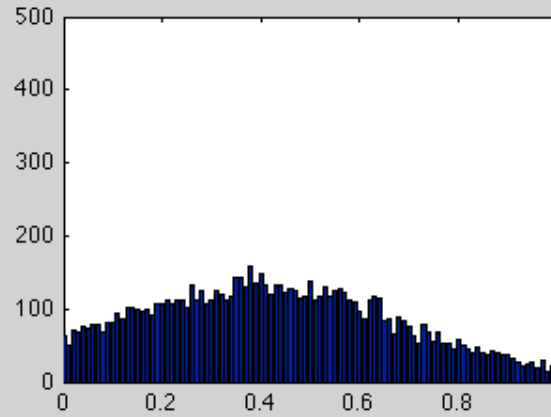
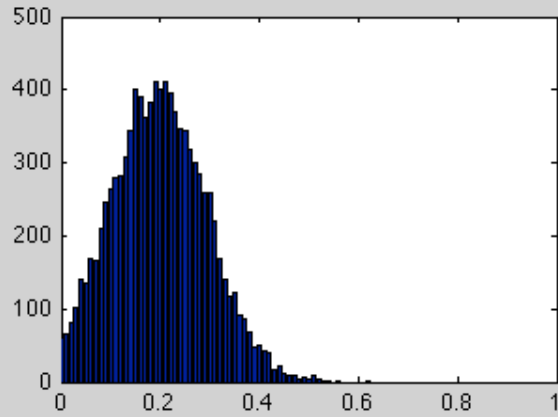


from [Gair, IM, Sesana, Vecchio, 2009]

# Parameter estimation on multiple GW detections: yield a set of individual marginalized posterior probability density functions

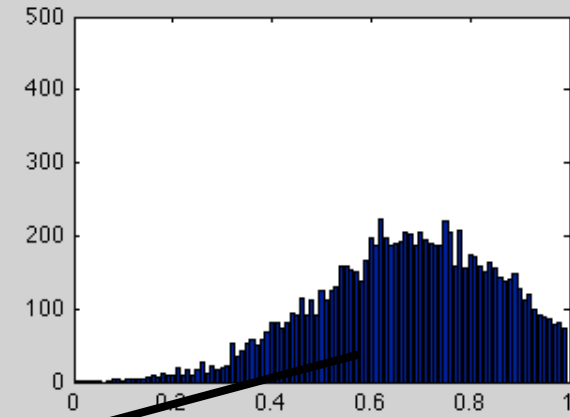
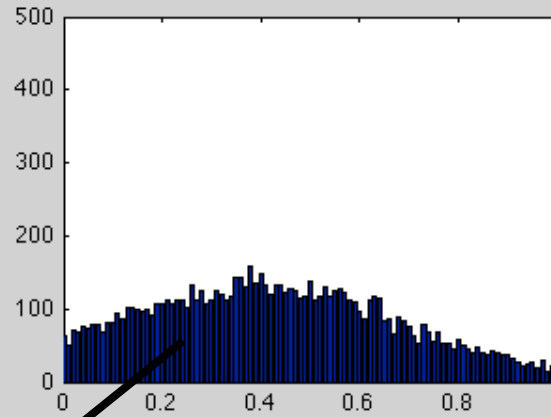
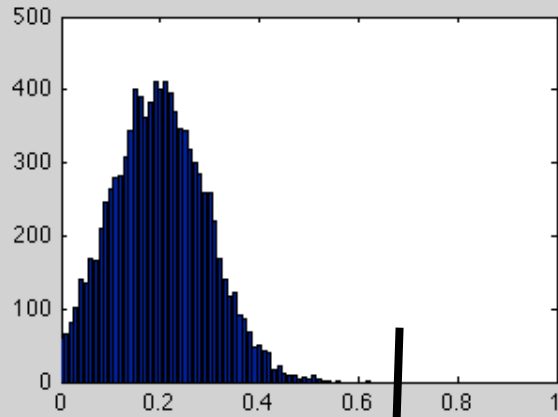


Parameter estimation on multiple GW detections:  
yield a set of individual marginalized posterior probability density functions



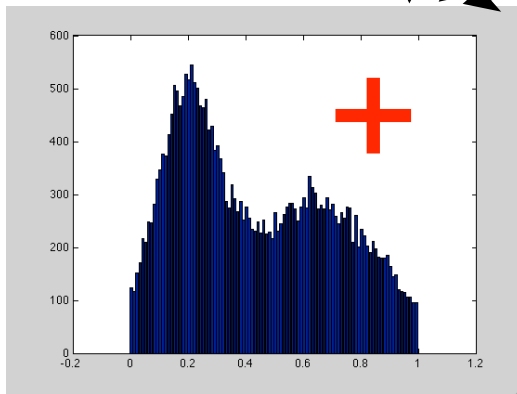
How do we combine these to make a statement about parameter distribution  
of the population being sampled?

Parameter estimation on multiple GW detections:  
yield a set of individual marginalized posterior probability density functions

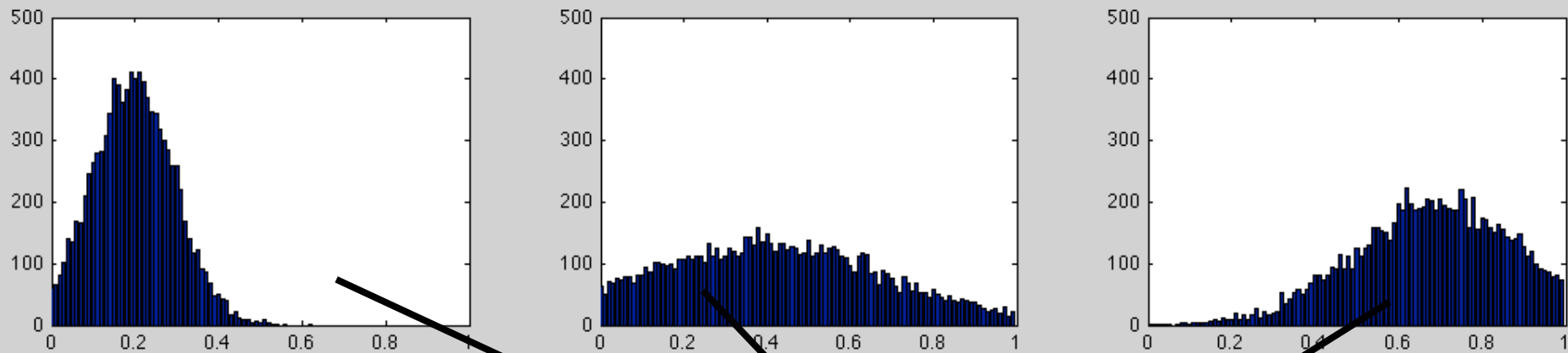


POLL: How do we combine these to make a statement about parameter distribution of the population being sampled?

A: Add them?



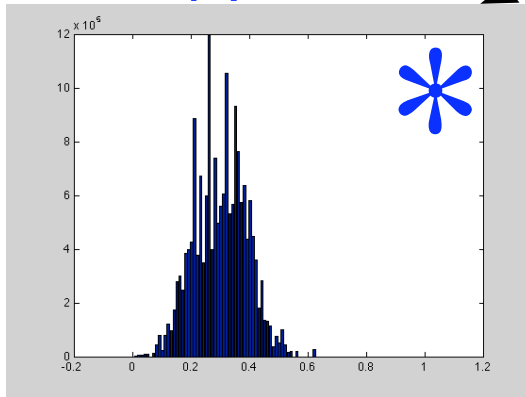
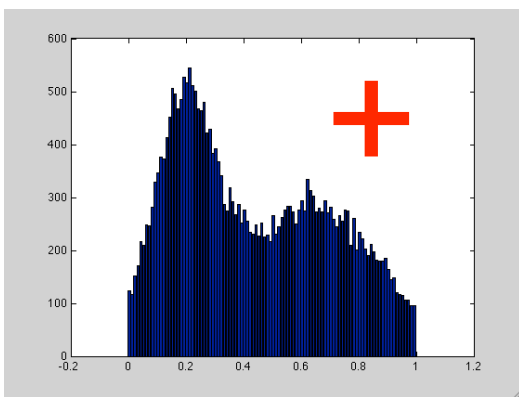
# Parameter estimation on multiple GW detections: yield a set of individual marginalized posterior probability density functions



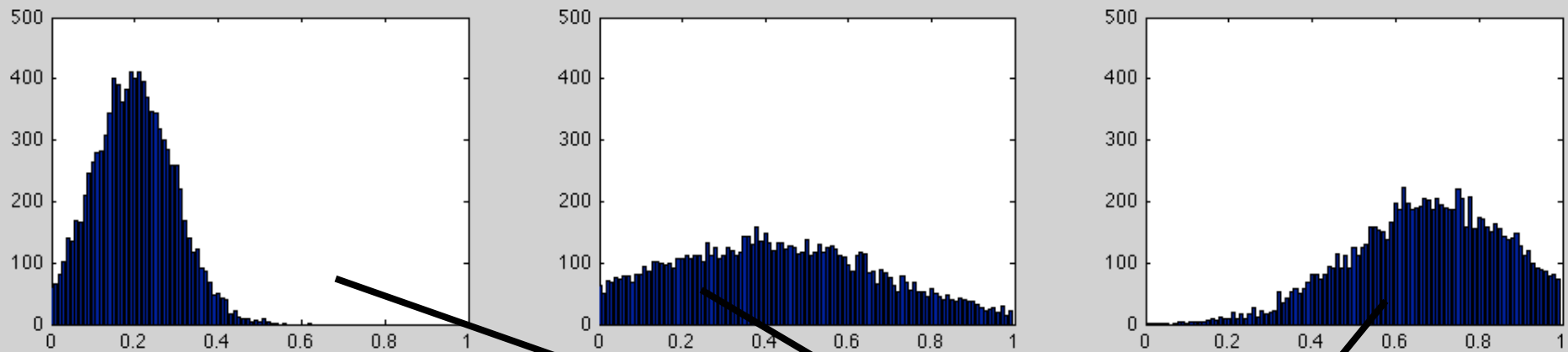
POLL: How do we combine these to make a statement about parameter distribution of the population being sampled?

A: Add them?

B: Multiply them?



# Parameter estimation on multiple GW detections: yield a set of individual marginalized posterior probability density functions

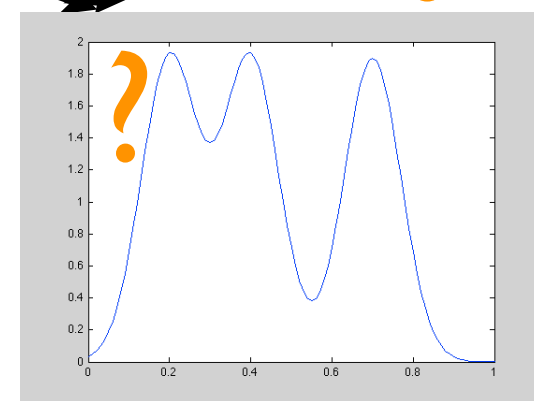
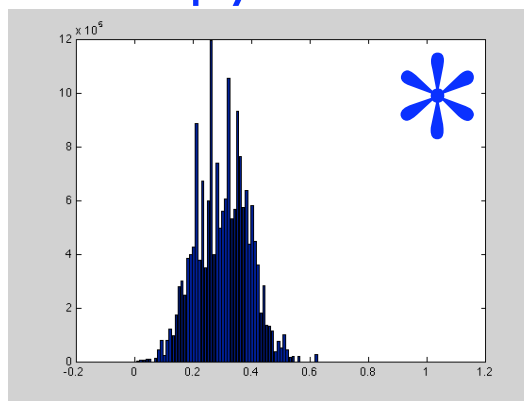
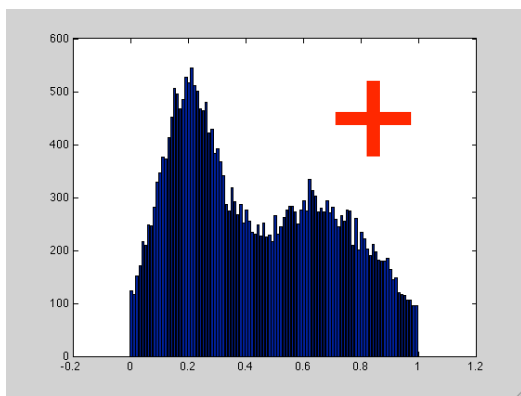


POLL: How do we combine these to make a statement about parameter distribution of the population being sampled?

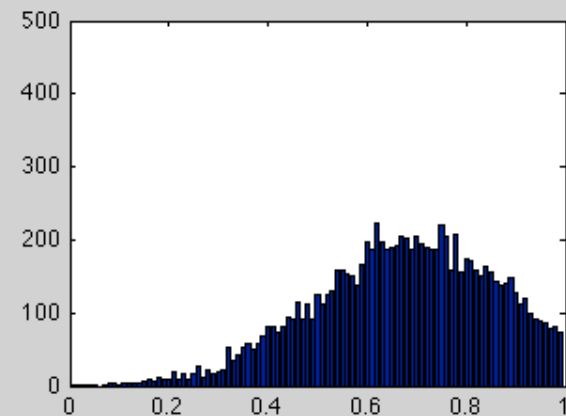
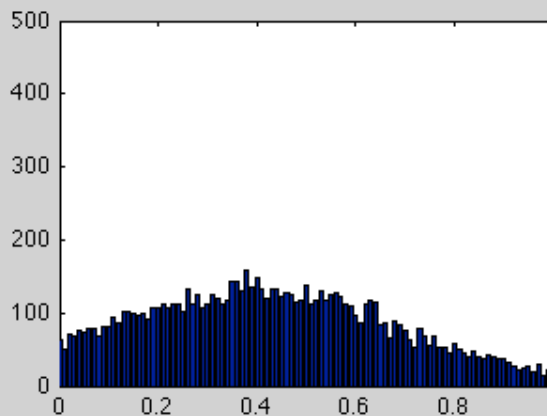
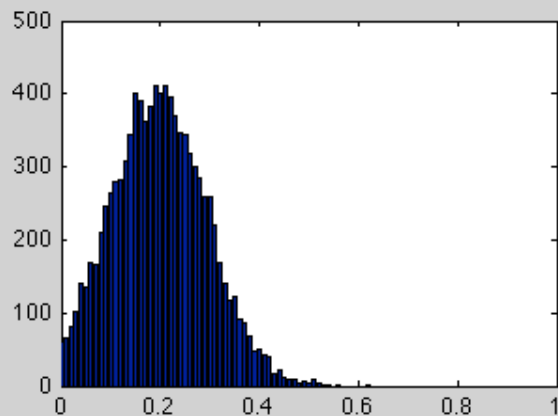
A: Add them?

B: Multiply them?

C: Something else?

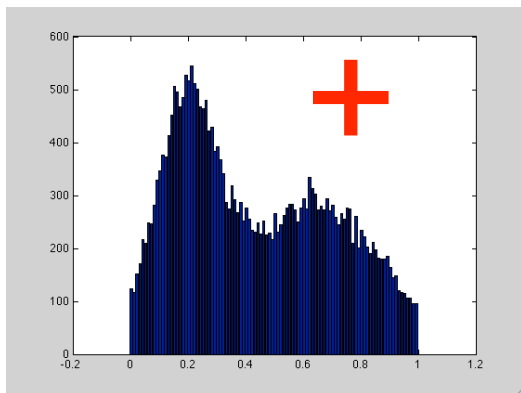


# Parameter estimation on multiple GW detections: yield a set of individual marginalized posterior probability density functions

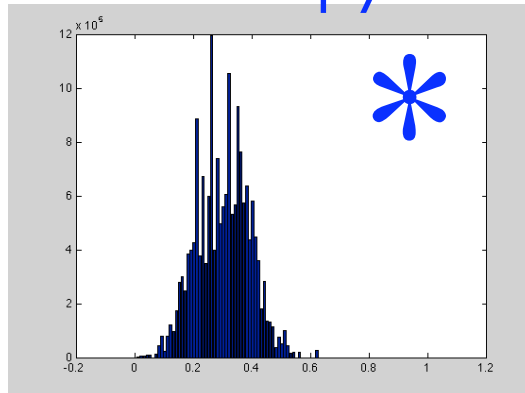


POLL: How do we combine these to make a statement about parameter distribution of the population being sampled?

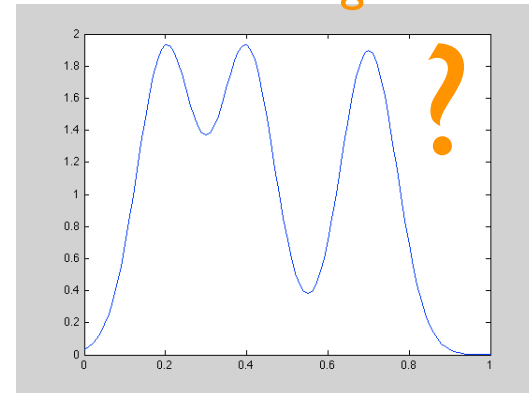
A: Add?



B: Multiply?

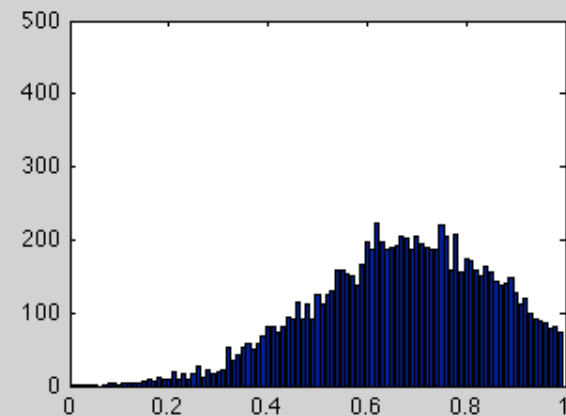
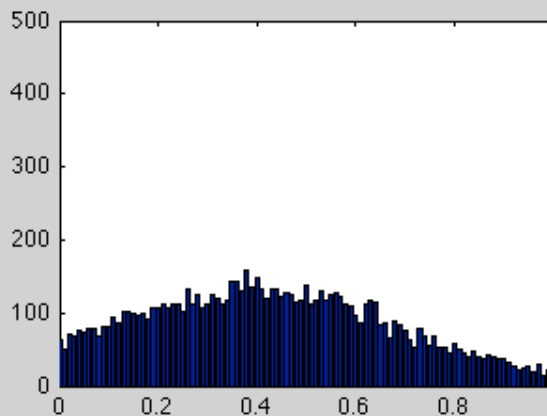
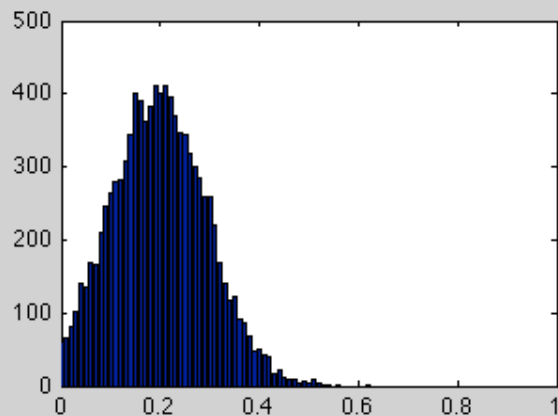


C: Something else?



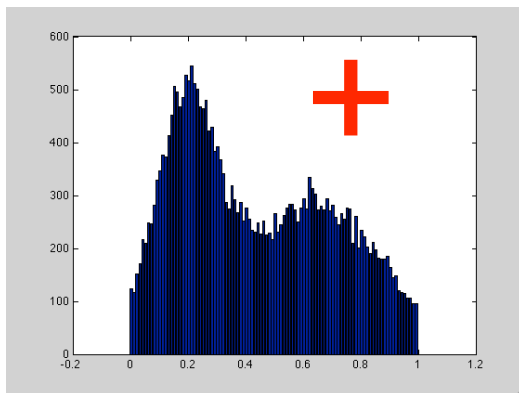


# Parameter estimation on multiple GW detections: yield a set of individual marginalized posterior probability density functions

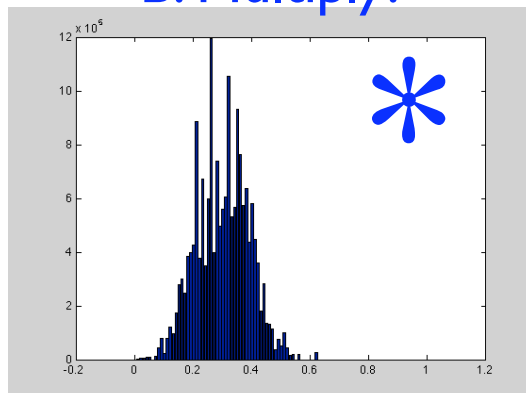


POLL: How do we combine these to make a statement about parameter distribution of the population being sampled?

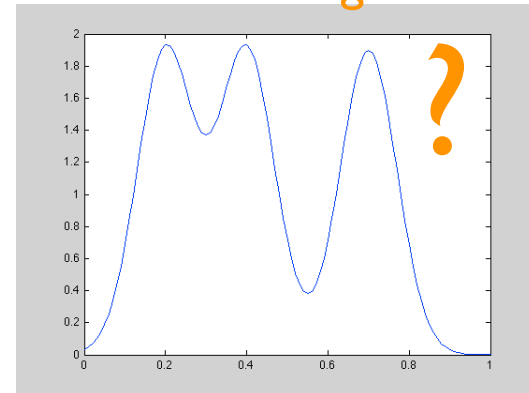
A: Add?



B: Multiply?



C: Something else?



See IM, 2010, arXiv:0912.5531 for the answer. :-)