Bayesian Inference on Numerical Injections

Ilya Mandel
Northwestern University / MIT

on behalf of the NINJA Bayesian parameter estimation group

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Participants

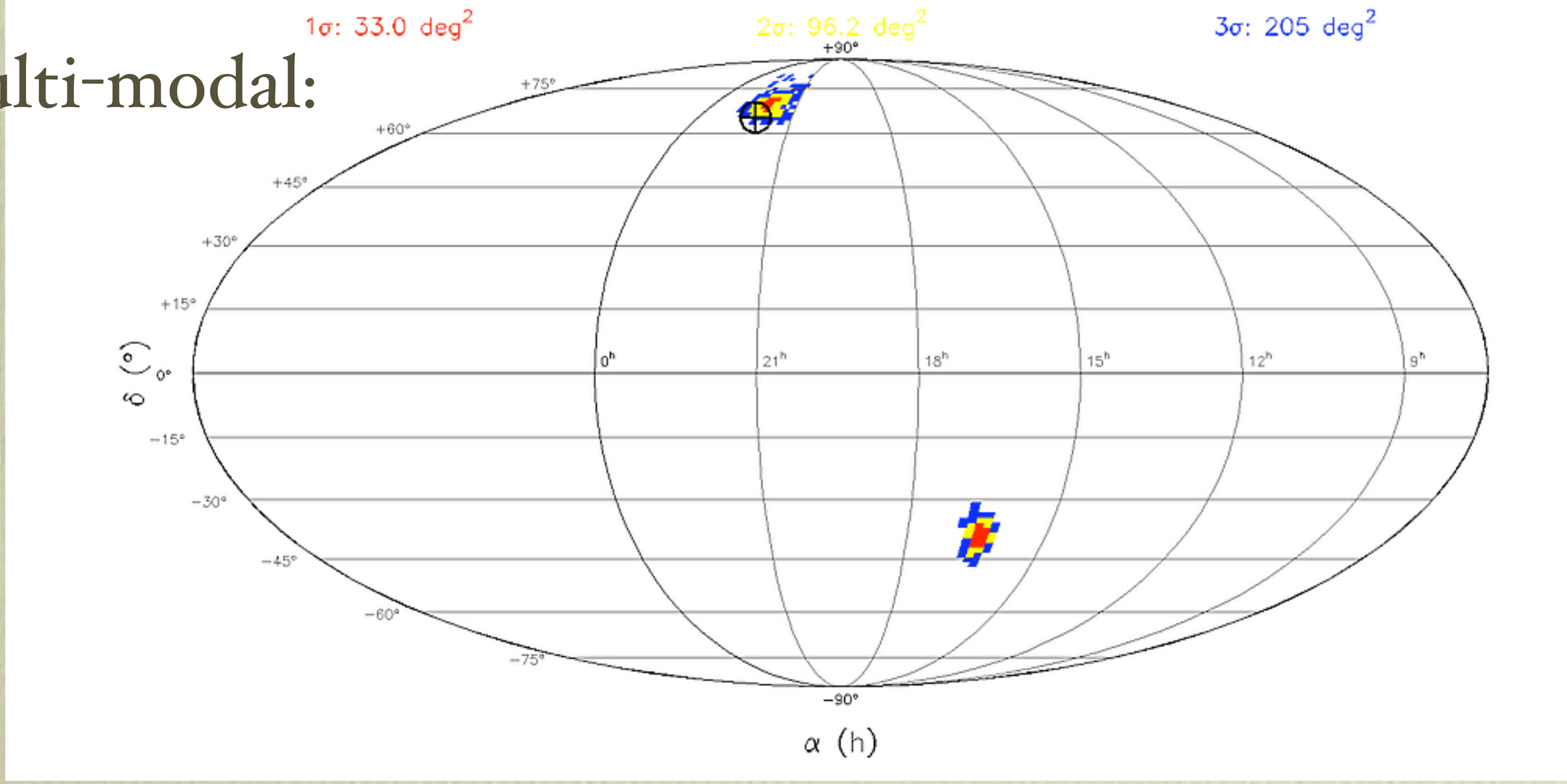
- Several data analysis groups are involved in parameters estimation on NINJA data:
  - FollowupMCMC - C. Röver (AEI Hannover)
  - SPINspiral MCMC - B. Farr, V. Raymond, I. Mandel, V. Kalogera (Northwestern), M. v. d. Sluys (U Alberta)
  - S. Nissanke, P. Ajith, M. Vallisneri (JPL/Caltech), J. Sievers, D. Holz (LANL), S. Hughes (MIT), N. Dalal (CITA)
  - Nested Sampling - B. Aylott, R. Smith, A. Vecchio (Birmingham), J. Veitch (Cardiff)
  - MultiNest - F. Feroz, J. Gair, P. Graff, M. Hobson (Cambridge)

NINJA Parameter Estimation

- Show that we can still accurately estimate parameters on more realistic injections using approximate waveforms
- Study systematic errors
- Determine appropriate waveform family to use “on the fly” via model selection
- Develop/test pipeline of passing triggers from detection searches
- Compare and validate parameter-estimation codes

Challenges

- Explore a large physical parameter space: 9 to 15 dimensions
- Analyze data streams coherently
- Make use of a priori information
- Infer posterior distribution on signal parameters
- Could be multi-modal:



Solution: Bayesian Inference

Compute the full posterior probability density function on the parameter space θ of the signal model H, given data $\{d\}$.

Prior Likelihood

Posterior

$$p(\vec{\theta}|\{d\}, H) = \frac{p(\theta|H)p(\{d\}|\theta, H)}{p(\{d\}|H)}$$

Evidence

Likelihood: $p(\{d\}|\theta, H) \propto e^{-<d-h(\theta)|d-h(\theta)>/2}$

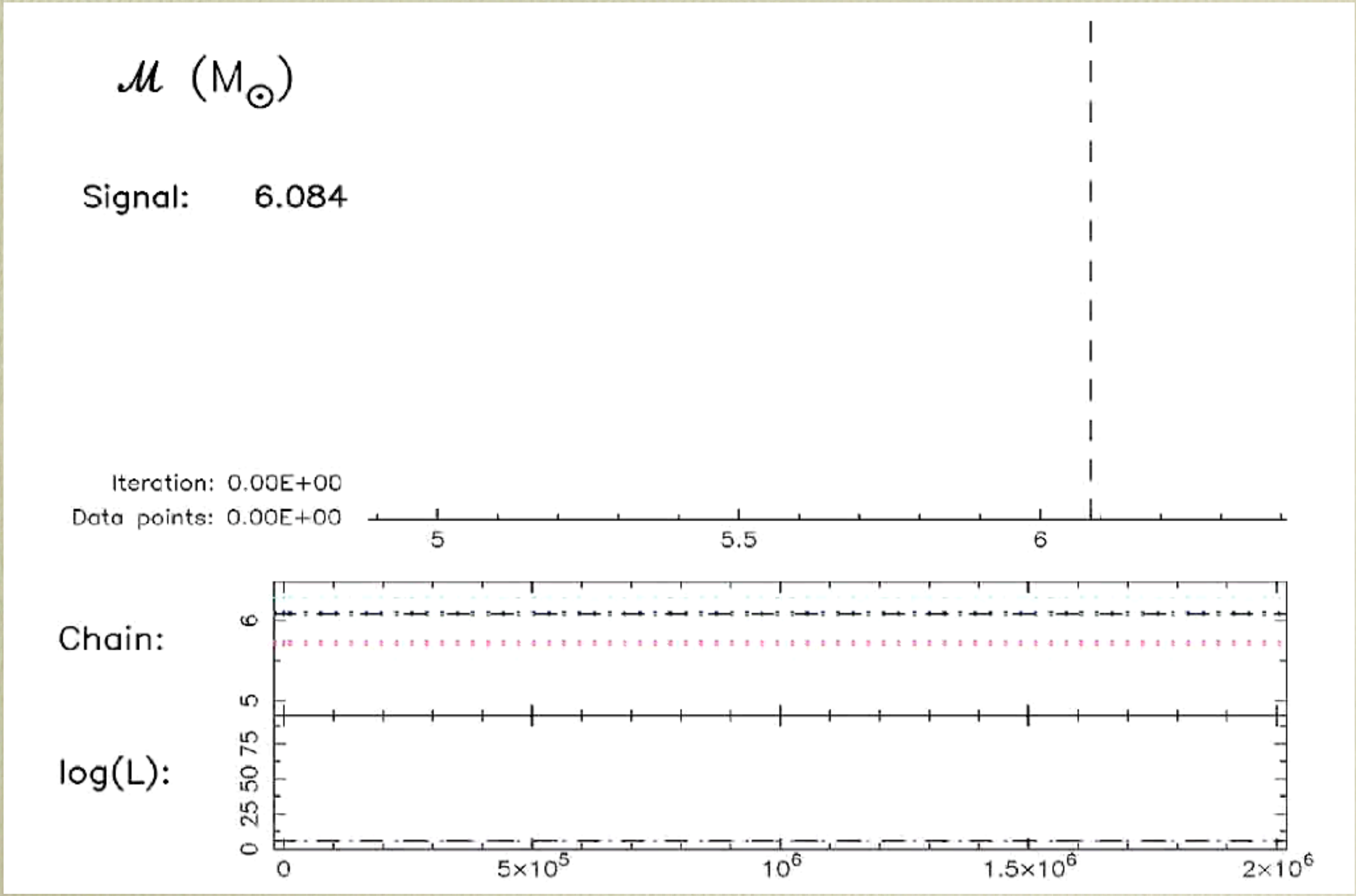
Evidence: $p(\{d\}|H) = \int p(\theta|H)p(\{d\}|\theta, H)d\theta$

Evidence comparison can be used for model selection

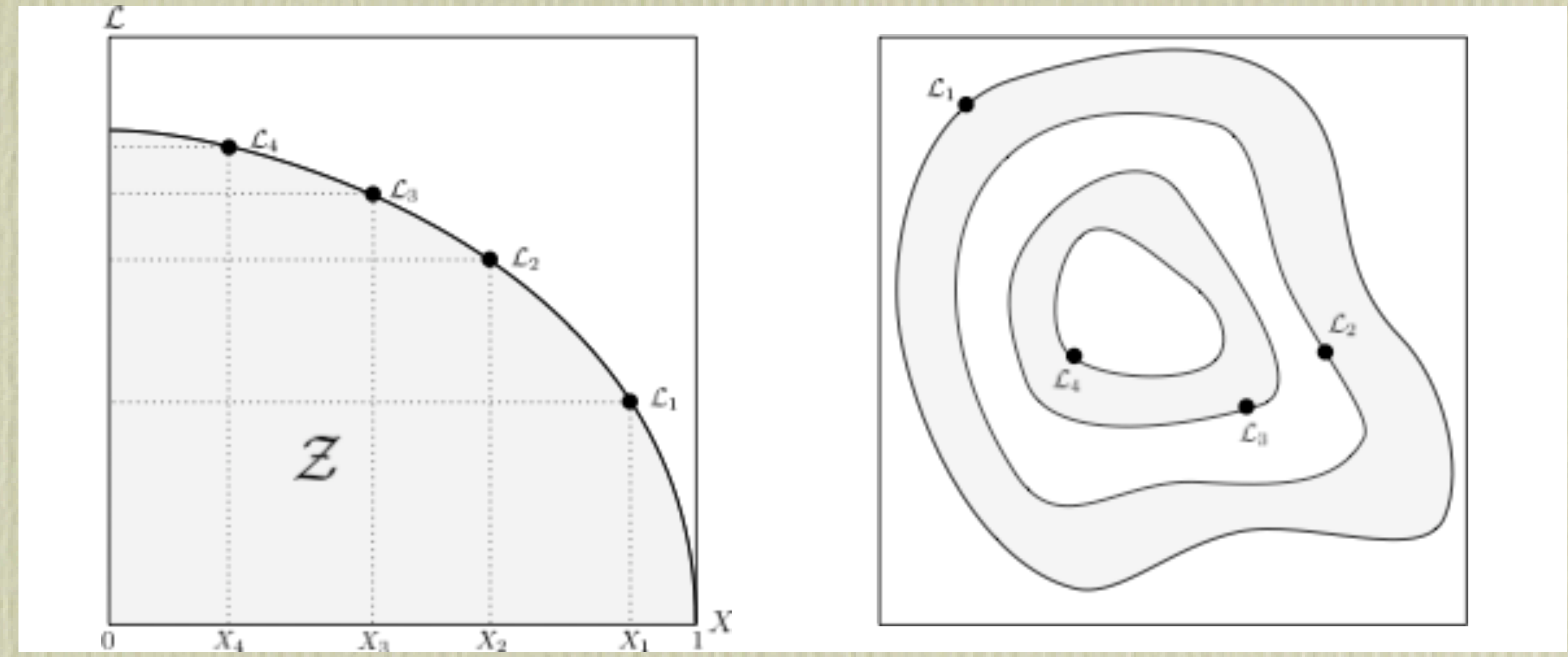
Approaches

Markov Chain Monte Carlo	Nested Sampling
•Stochastic sampling in multi-dimensional parameter space •Designed for evaluating posterior PDFs •Can also be used to compute evidence for model selection •Sample from the posterior accepting/rejecting new samples according to Metropolis-Hastings ratio {new vs. current sample} •Variations to improve sampling: jump proposal distributions; parallel tempering	•Stochastic sampling in multi-dimensional parameter space •Designed compute evidence for model selection •Can also be used for evaluating PDFs •Samples iteratively drawn from shrinking prior volume with new sample replacing lowest-likelihood sample {N live samples} •Variations in selection of next live point (e.g., InspNest uses MCMC; MultiNest - ellipsoidal clustering)

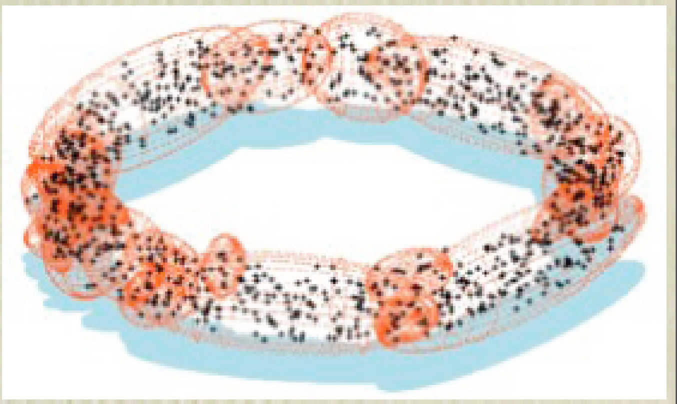
Markov Chain Monte Carlo



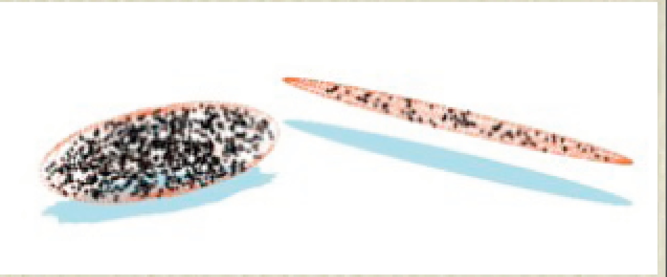
Nested Sampling



- Use a collection of N_{Live} samples from prior. At each iteration replace outermost sample with one drawn from within the contour.



- At each iteration the volume enclosed shrinks by factor $\sim e^{1/N_{Live}}$.
- Computes marginal likelihood: fit of data to a model
- Re-sample to get samples from posterior PDF
- MultiNest: Mode separation achieved via ellipsoidal decomposition of live point set. Live points updated by rejection-sampling.

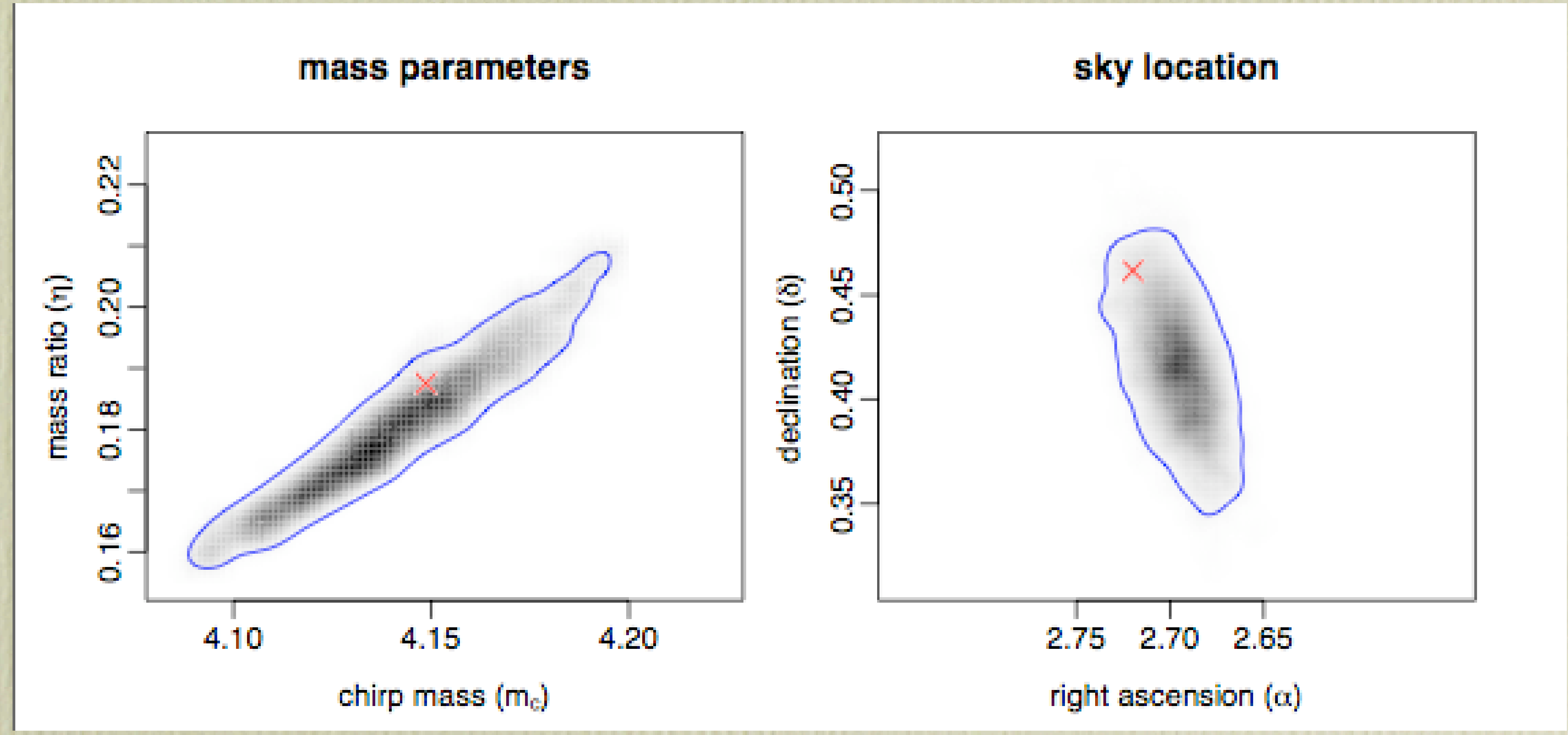


Preliminary NINJA Results

FollowupMCMC

non-spinning binary parameter estimation via MCMC (AEI)
[Röver et al, P.R.D. 75 62004 (2007)]

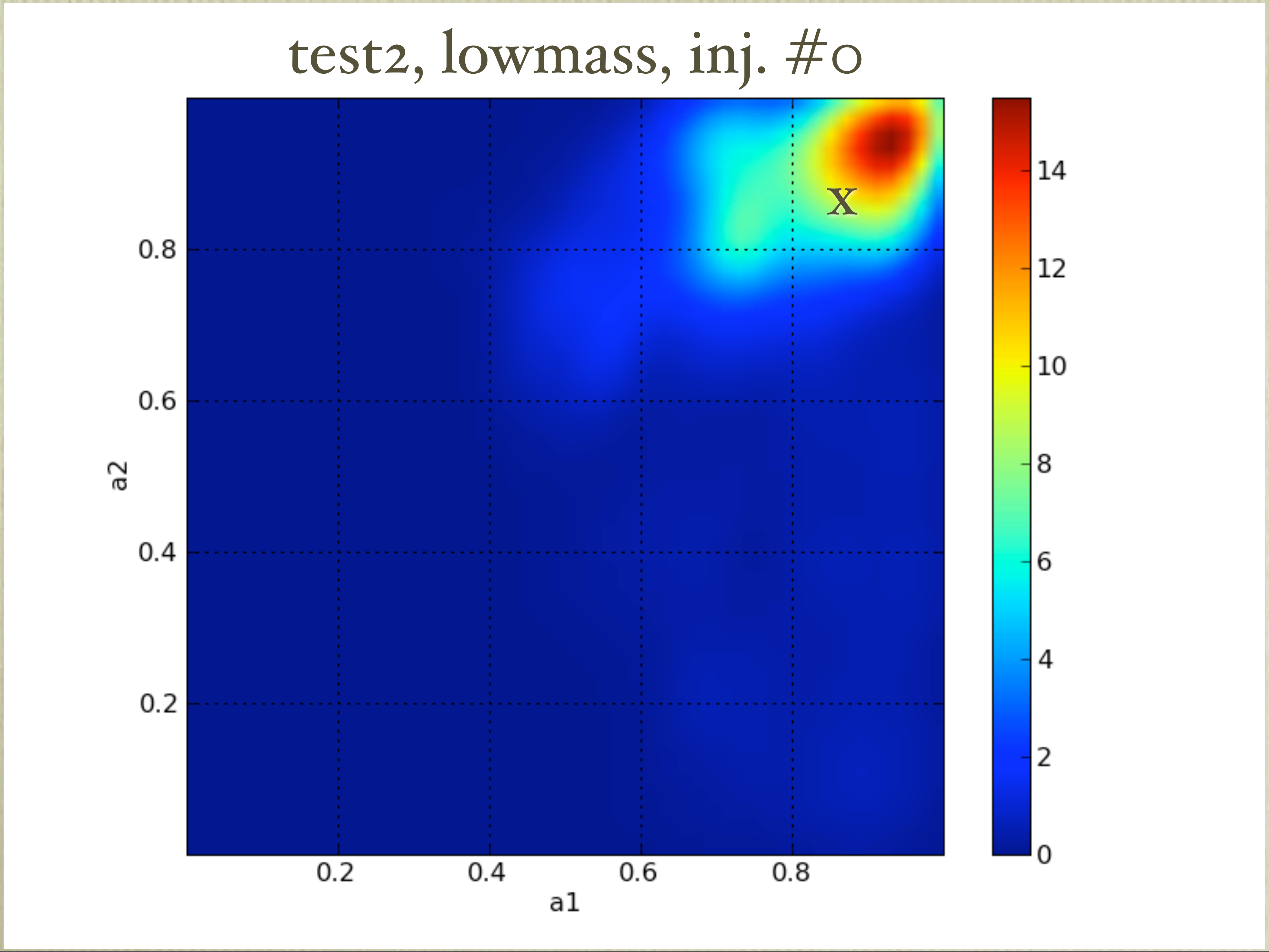
test2, lowmass, inj. #40



- Mc=4.148, eta=0.1785, SNR~14.2
- used IMRPhenomA waveforms

SPINspiral

(spinning binary parameter estimation via MCMC (Northwestern)
[van der Sluys et al., CQG 25 184011 (2008)]



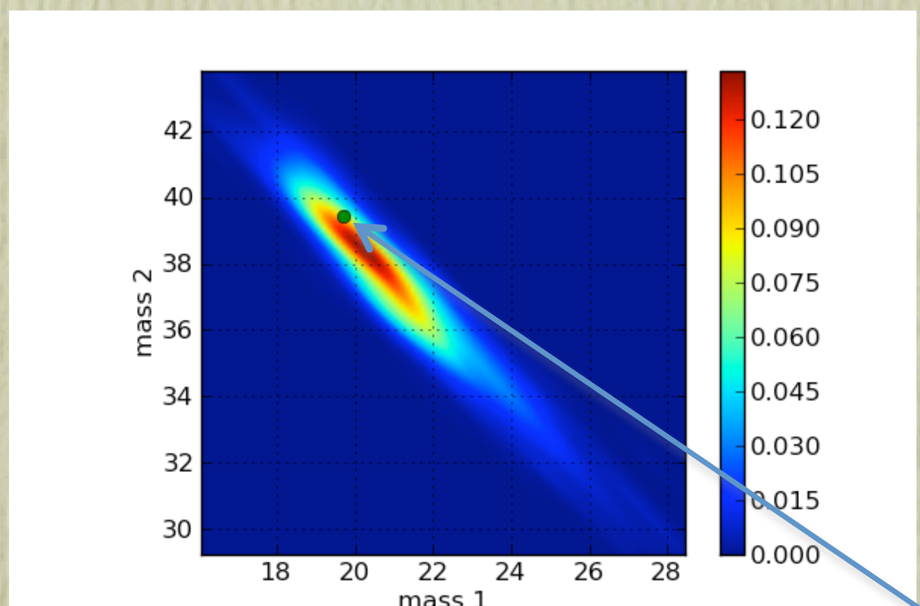
- SNR~17
- eta=0.25
- M_c=15.4
- used SpinTaylor waveforms
- based on 1 MCMC chain

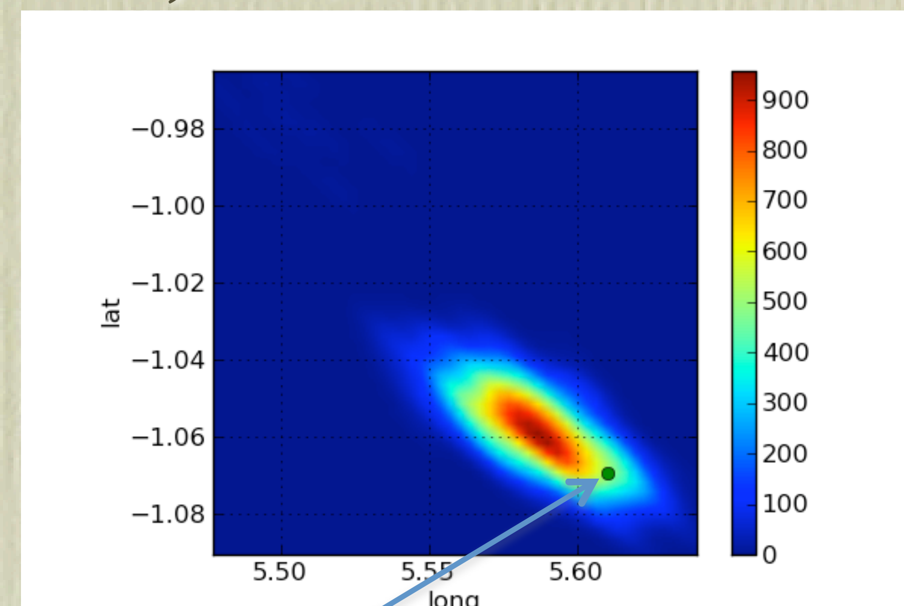
InspNest

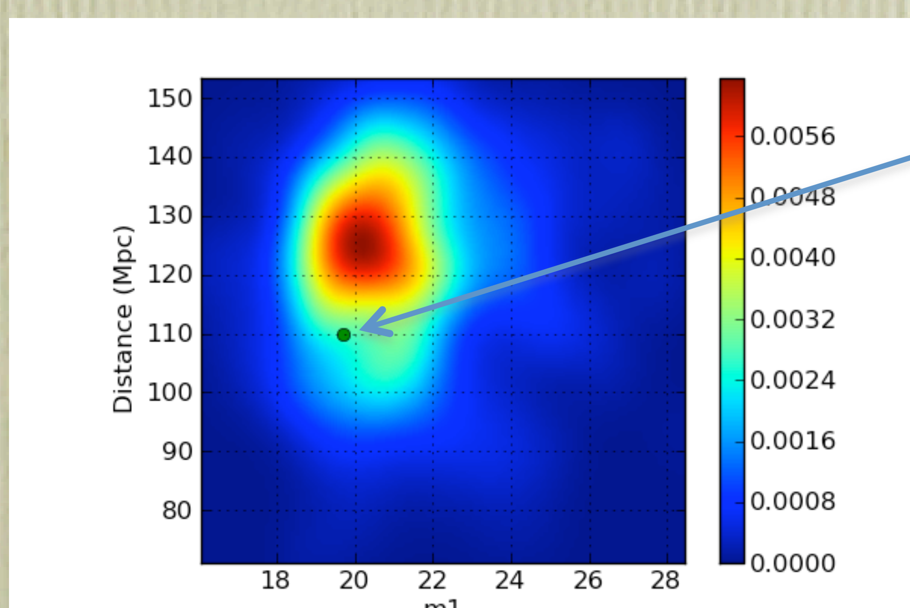
(non-spinning binary nested sampling code (Birmingham/Cardiff)

[Veitch & Vecchio, P.R.D. 81 62003 (2010)]

test2, highmass, inj. #98



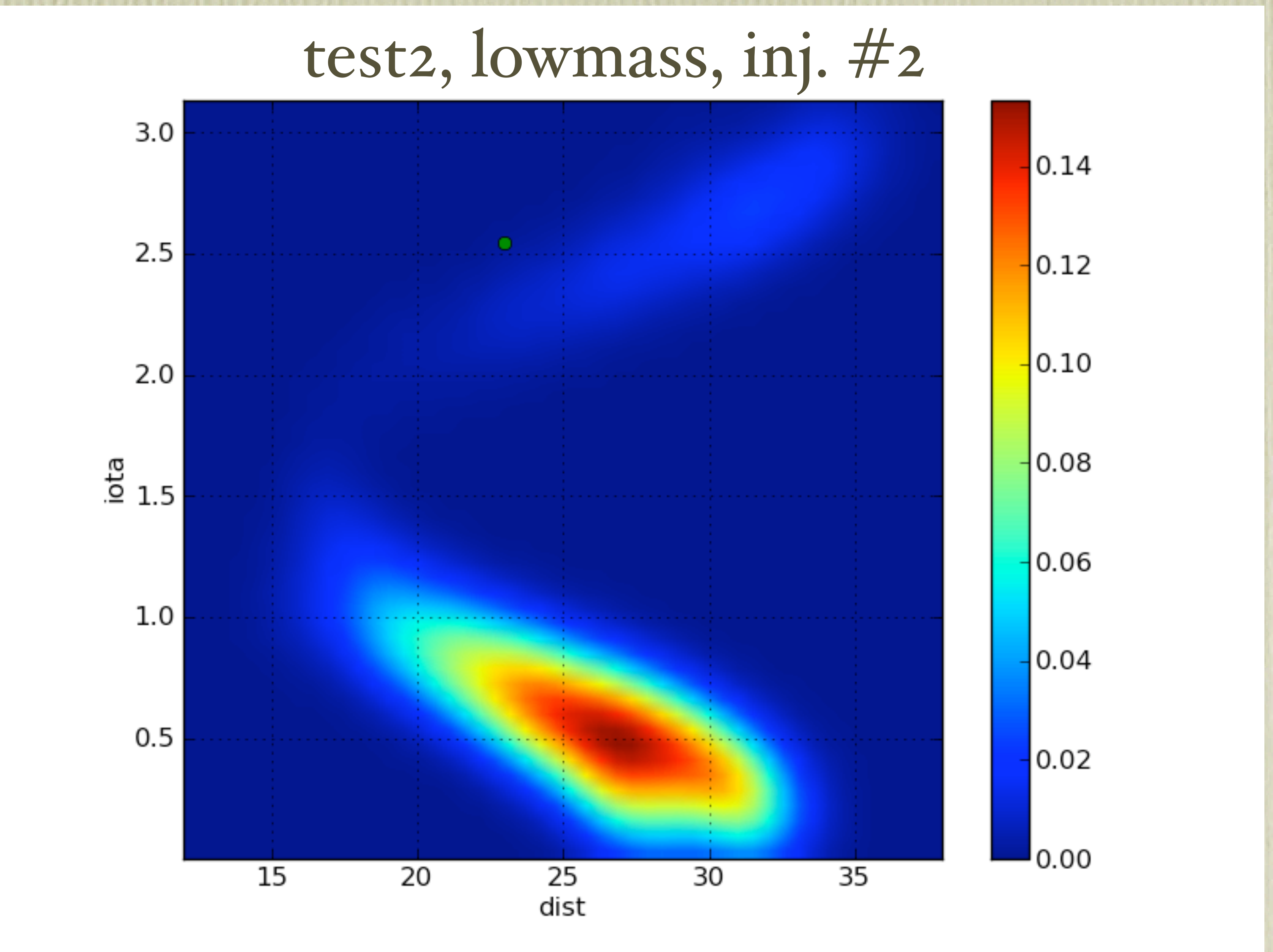




- Injection parameters, SNR~19
- used IMRPhenomA waveforms
- $B_{signal/noise}$=227

MultiNest

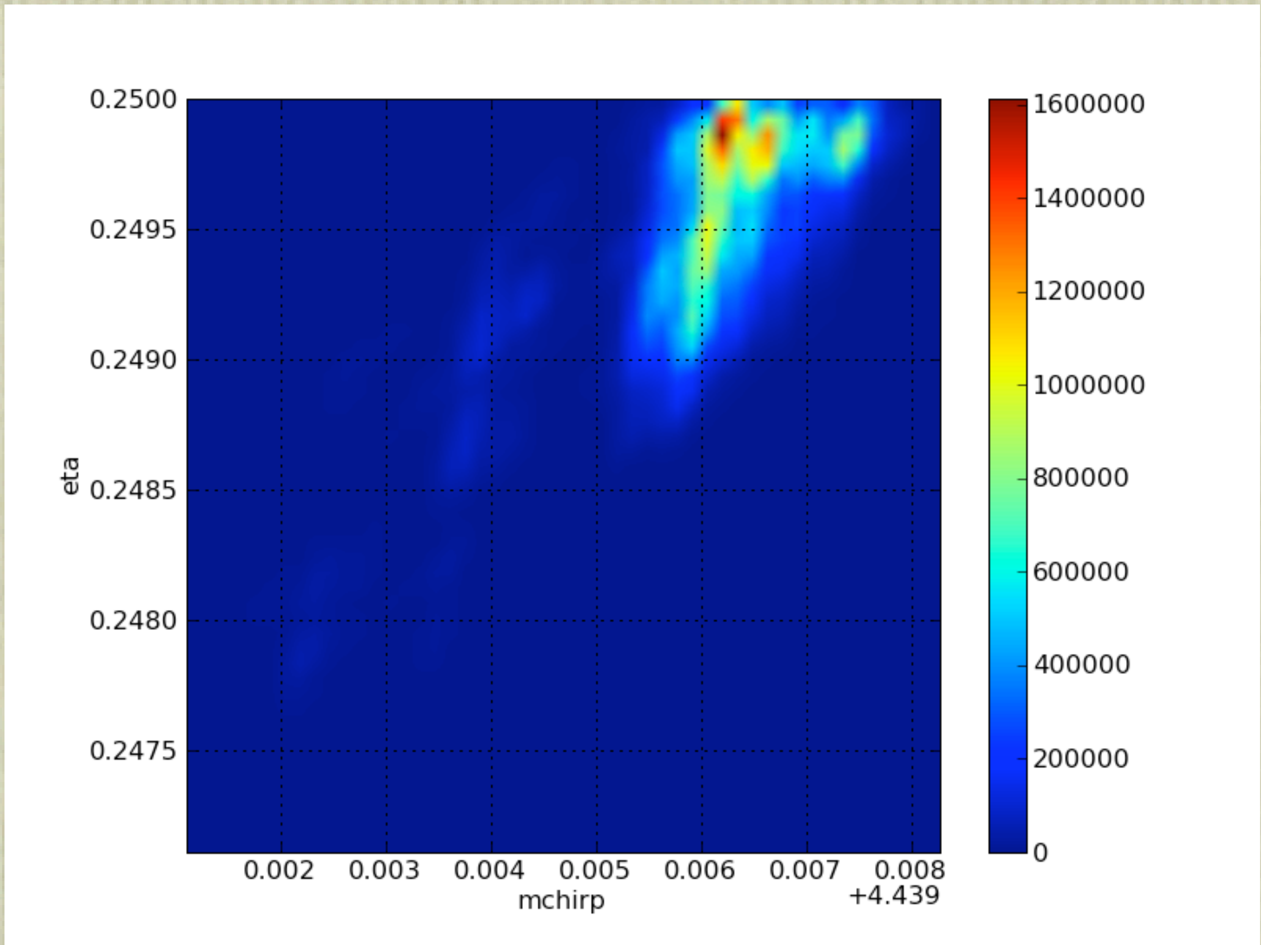
(non-spinning binary nested sampling via MultiNest (Cambridge)
[Feroz et al., MNRAS 398 4 1601 (2009)]

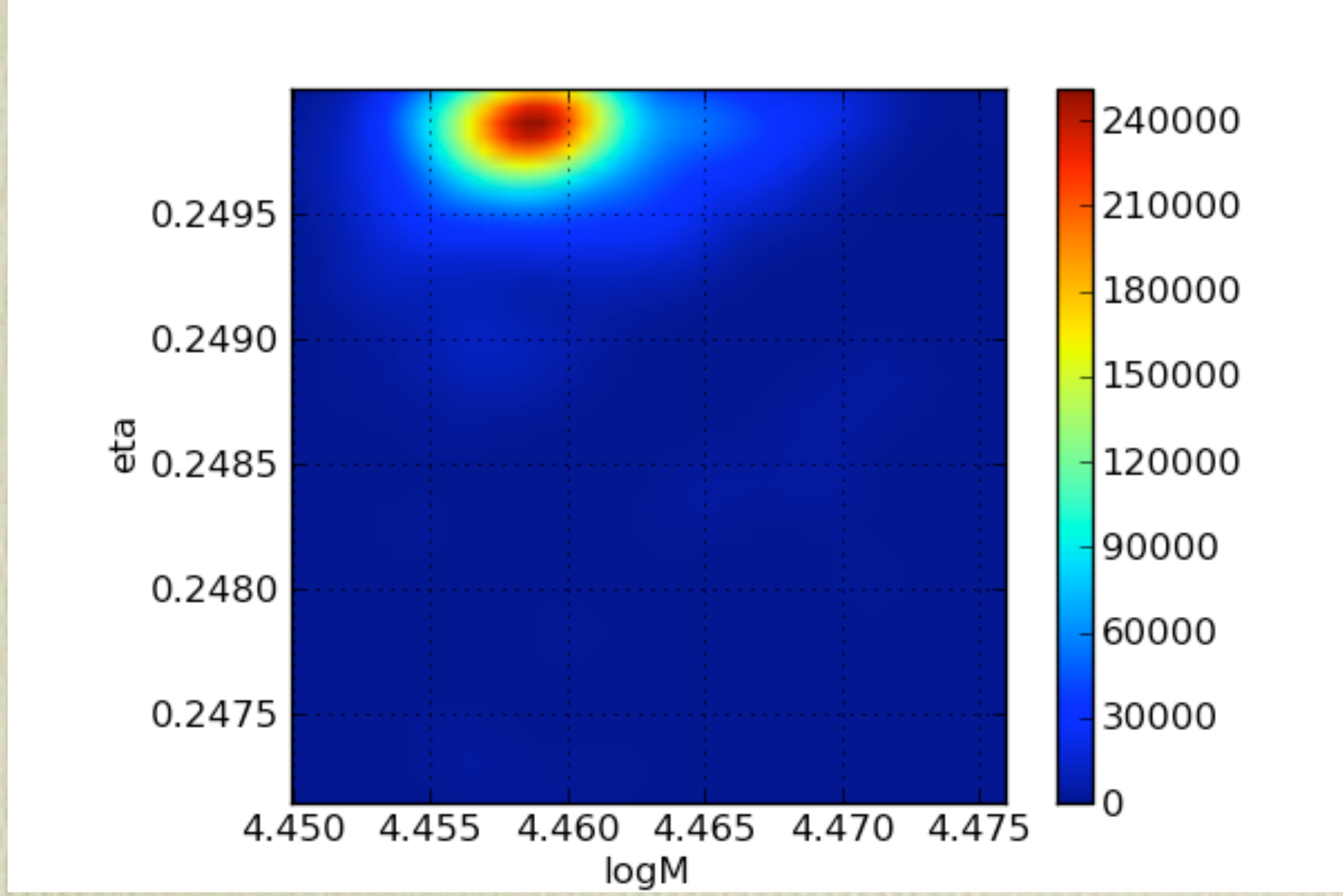


- SNR~33
- eta=0.25
- M_c=9.54
- used IMRPhenomA waveforms
- Note the inclination/ distance degeneracy

Systematic Errors

test2, lowmass, inj. #23





SPINspiral
~20 MCMC chains converged

InspNest

- used TaylorT2 & Taylor F2 waveforms
- SNR~70, eta=0.25, M_c=4.66

Summary

- We have several good tools in hand to perform parameter estimation on GW signals.
- We know that systematic errors can be extremely important, so we will need accurate waveforms.
- Model selection through evidence comparison may boost confidence in detection and allow us to pick the best waveform template family for a given signal.
- NINJA-2 will provide a good testbed for these studies.