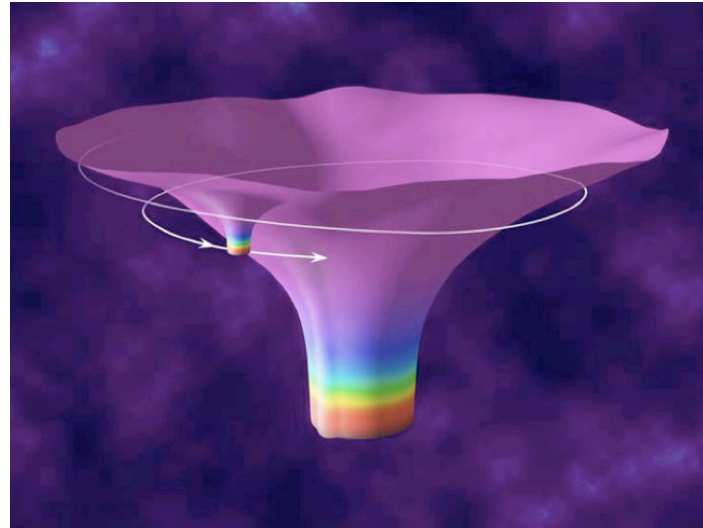


Testing General Relativity with Gravitational Waves from Extreme Mass Ratio Inspirals

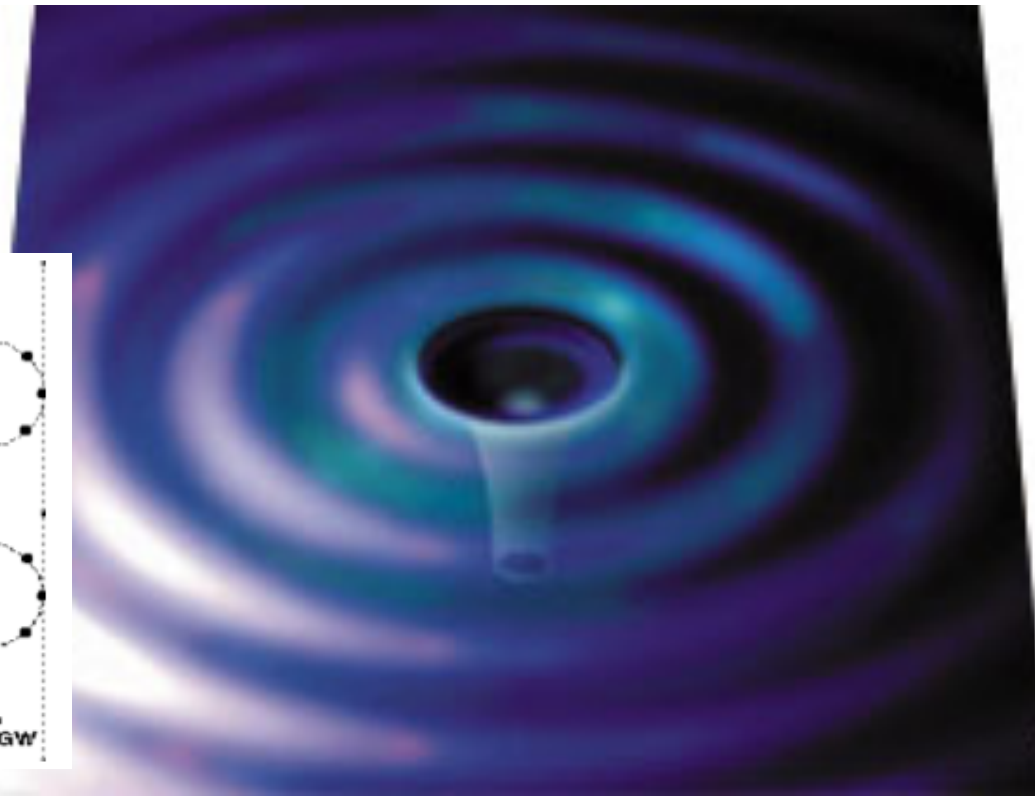
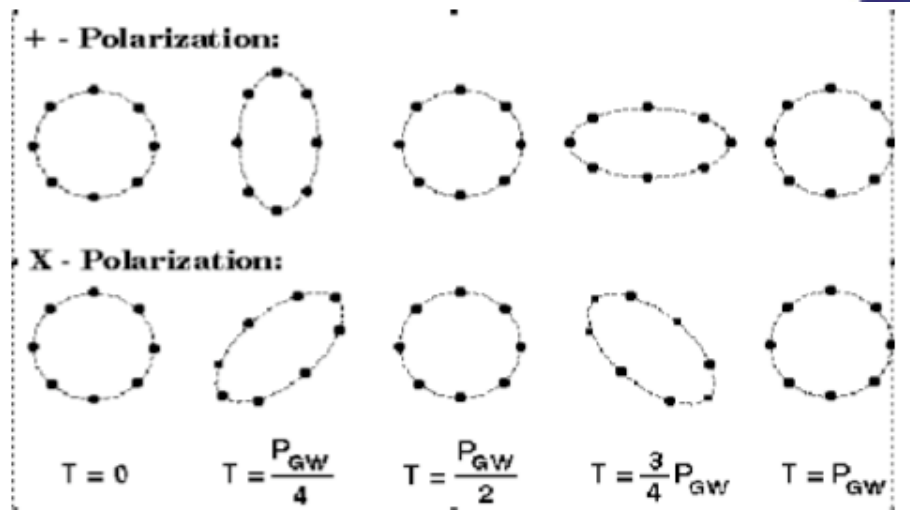


Ilya Mandel
(Northwestern University / MIT)

July 20, 2010
COSPAR-10, Bremen

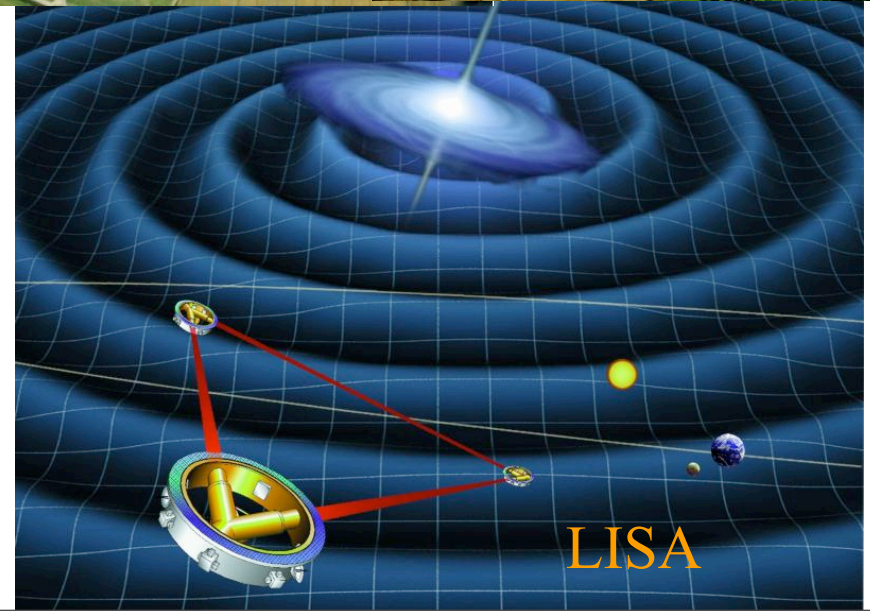
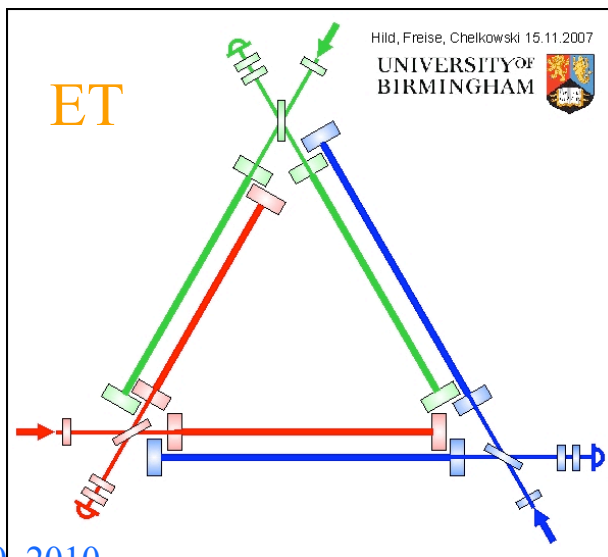
Gravitational Waves

- Ripples in spacetime:



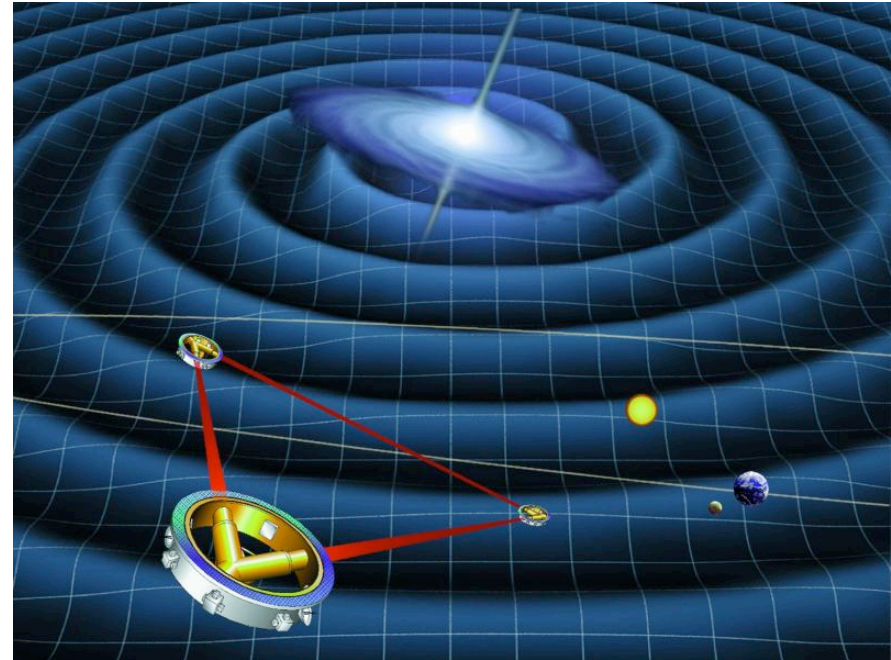
- Caused by time-varying mass quadrupole moment; GW frequency is twice the orbital frequency for a circular, non-spinning binary
- Indirectly detected by Hulse & Taylor: binary pulsar

Gravitational-wave observatories



LISA Binary Sources: EMRIs

- LIGO sensitive @ a few hundred Hz
 - » NS-NS, NS-BH, BH-BH binaries
- LISA sensitive @ a few mHz
 - » massive black-hole binaries
 - » galactic white dwarf (and compact object) binaries
 - » **extreme-mass-ratio inspirals of WDs/NSs/BHs into SMBHs**
 - could see tens to hundreds to $z \sim 1$ [e.g., Gair et al. 2004; Amaro-Seoane et al., 2007]

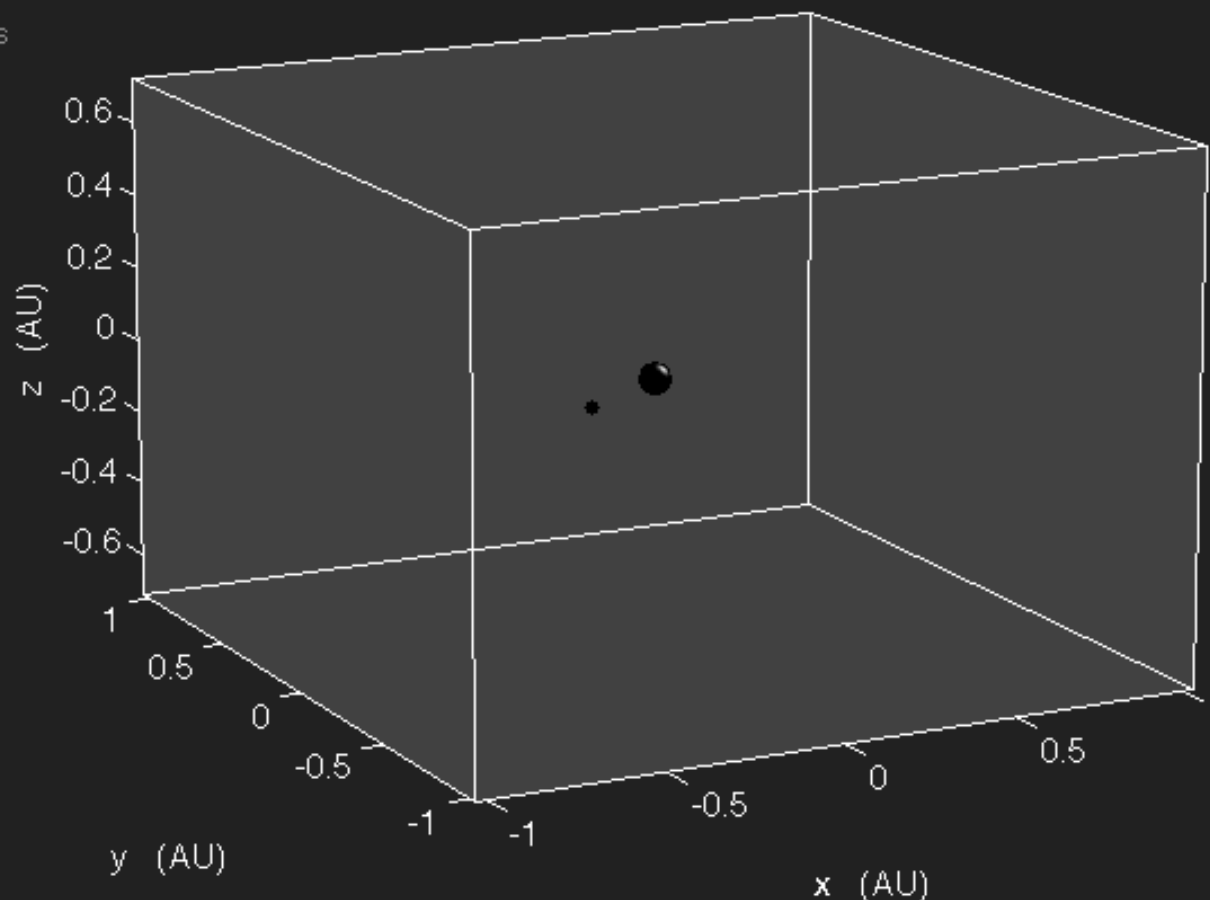


Extreme Mass Ratio Inspirals

Large black hole:
shown to scale
3,000,000 solar masses
90% maximal spin

Small black hole:
shown enlarged
540 solar masses
negligible spin

Trace duration:
1 day

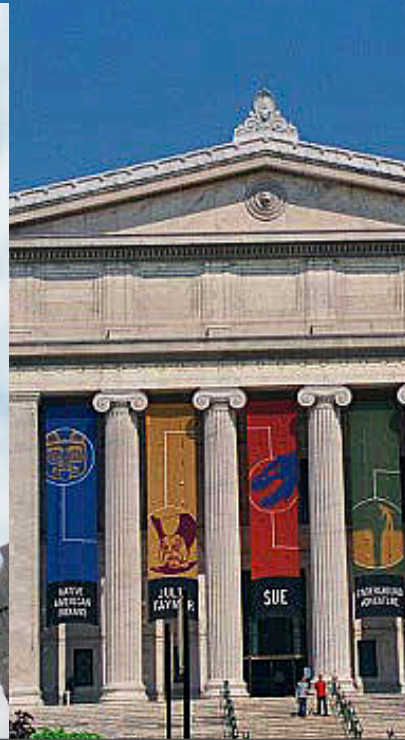


Steve Drasco
Max Planck Institute
for Gravitational Physics
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Exploring the spacetime...



... taking lots of pictures

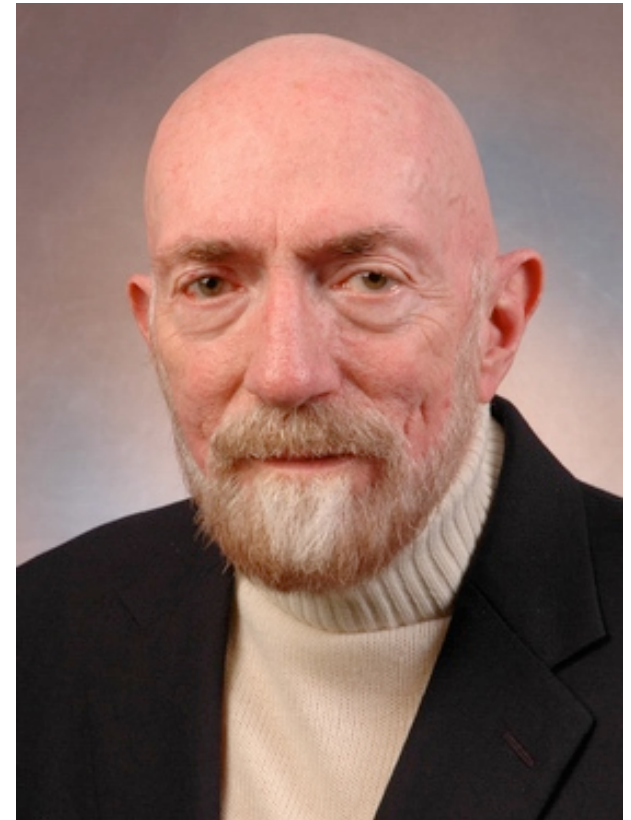
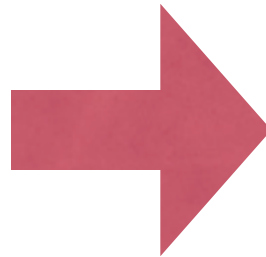


Deviations from Expectations

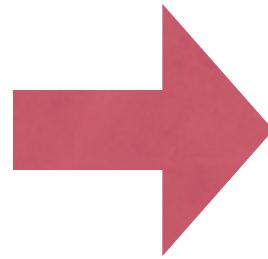
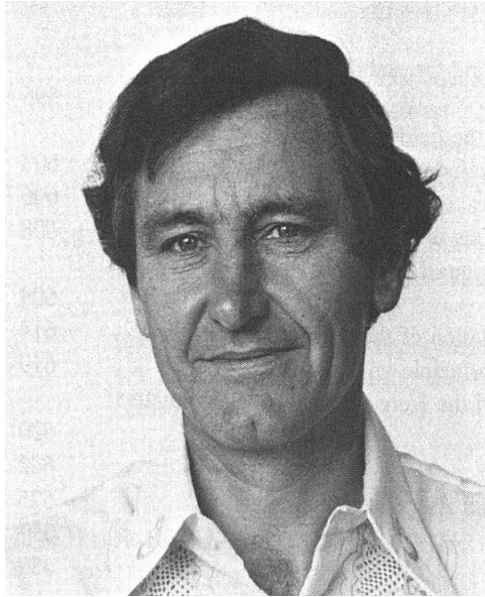
- Dirty EMRIs: Matter (disks, other compact objects) as an astrophysical perturber
 - » Gravitational influence of matter [Barausse et al., 2007]
 - » Hydrodynamic drag [Barausse & Rezzolla, 2008]
- GR breaks down
 - » Test specific alternative theories of gravity [e.g., Sopuerta and Yunes, 2009]
 - » Parametrized or phenomenological deviations [e.g., Yunes and Pretorius, 2009; IM et al., in prep.]
 - » Massive gravitons [e.g., Will 1997, Babak & Grishchuk 2003, Berti et al. 2005]
- GR is correct, but the central objects are not black holes
 - » Non-vacuum solutions of Einstein's equations (boson stars, gravistars)
 - » Vacuum solutions that violate some cherished assumptions (e.g., naked singularities)

Testing the “no-hair” theorem

Testing the no-hair theorem

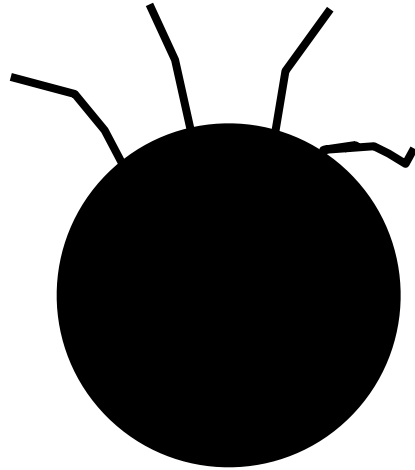


Testing the no-hair theorem?



Stationary, vacuum, asymptotically flat spacetimes in which the singularity is fully enclosed by a horizon with no closed timelike curves outside the horizon are described by the Kerr metric

Do black holes have hair?



$$M_n + iS_n \neq M(ia)^n$$

Ryan's theorem [1995]: GWs from nearly circular, nearly equatorial orbits in stationary, axisymmetric spacetimes encode all of the spacetime multipole moments... *in principle*

Bumpy black holes

- Perturbed black holes

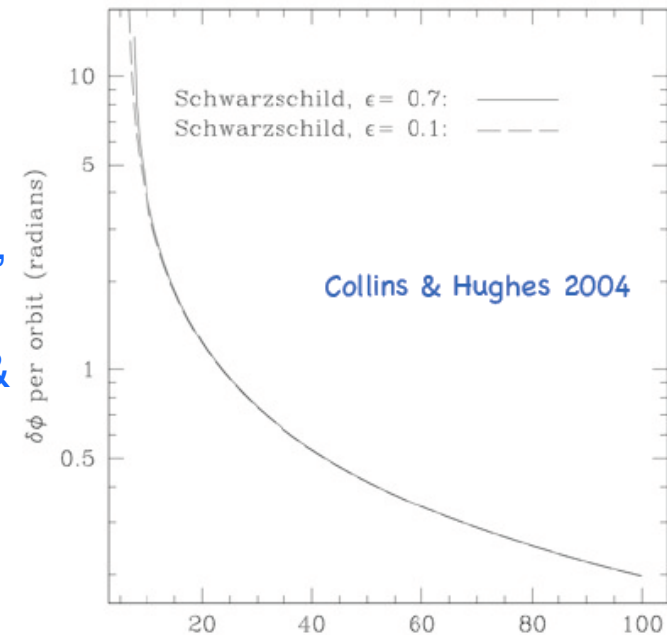
- » Perturbed Schwarzschild [Collins & Hughes, 2004]
- » Perturbed Kerr [Vigeland & Hughes, 2009]
- » Quasi-Kerr with slow spins [Glampedakis & Babak, 2006]
- » Kludge pN term due to mass quadrupole [Barack & Cutler, 2007]
- » Ringdown modes [Berti, Cardoso, Will, 2006]

- Exact solution of Einstein's equations

Manko-Novikov spacetime, an exact solution of Einstein's equations:

$$ds^2 = -f(\rho, z) (dt - \omega(\rho, z) d\phi)^2 + \frac{1}{f(\rho, z)} \left[e^{2\gamma(\rho, z)} (d\rho^2 + dz^2) + \rho^2 d\phi^2 \right]$$

Search for observable imprints of a “bumpy” spacetime, such as deviations from the full set of isolating integrals (energy, angular momentum, Carter constant) in Kerr [Gair, Li, IM, 2009, PRD 77:024035]

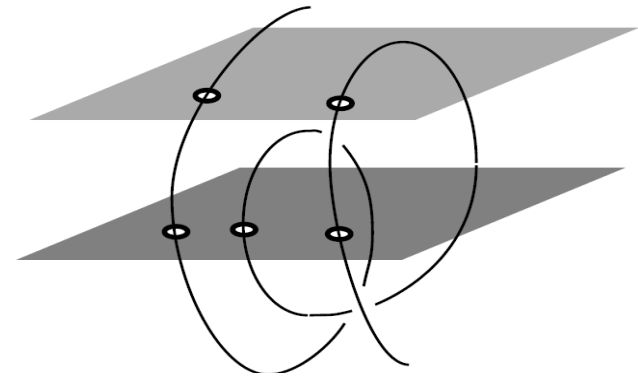


The emergence of chaos

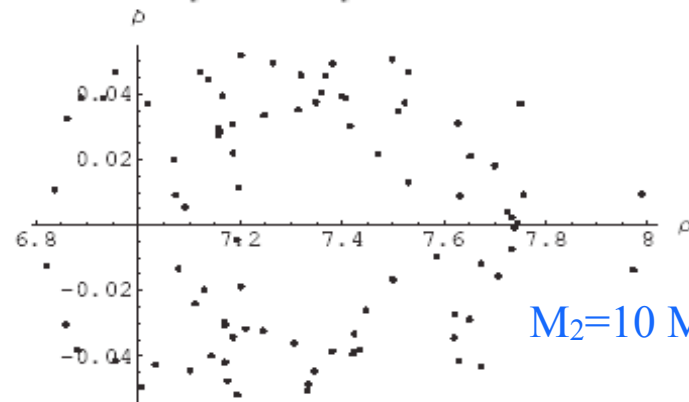
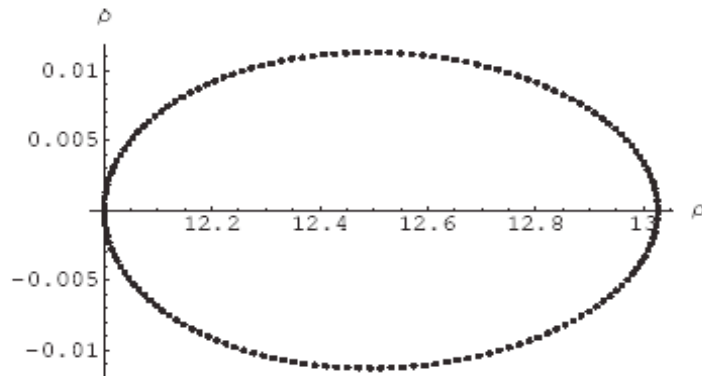
Solve the geodesic equation
and study Poincare maps:

- Plot dp/dt vs. ρ for $z=z_0$ crossings
- Phase space plots should be closed curves for all z_0 iff there is a third isolating integral [Carter constant]

$$\frac{\partial^2 x^\alpha}{\partial \tau^2} = -\Gamma_{\beta\gamma}^\alpha \frac{\partial x^\beta}{\partial \tau} \frac{\partial x^\gamma}{\partial \tau}$$

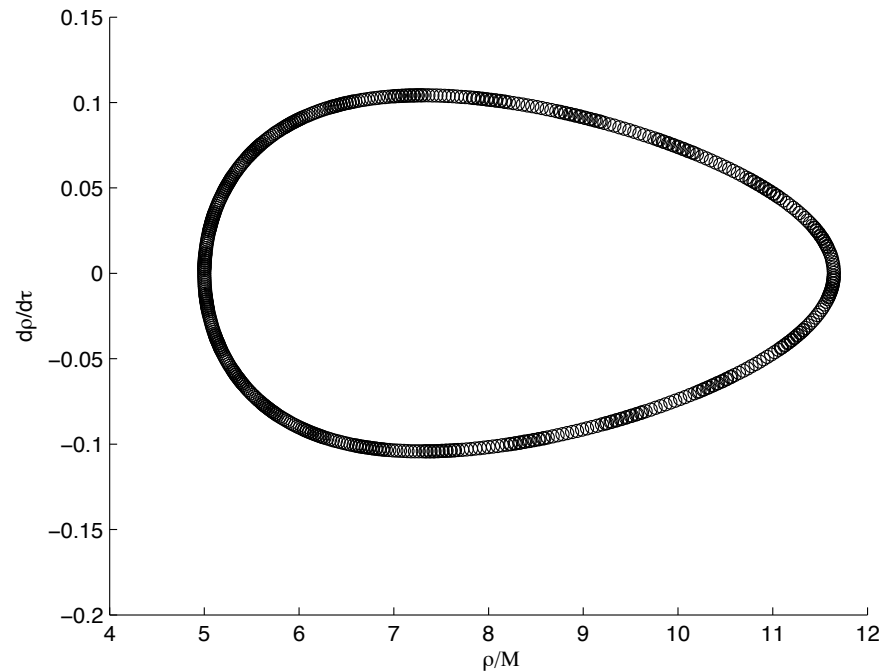


Newtonian+hexadecapole: $V(r, \theta) = -\frac{M_0}{r} + \frac{M_2}{r^3} P_2(\cos \theta) + \frac{M_4}{r^5} P_4(\cos \theta)$



$M_2=10 M_0$; $M_4=400 M_0$

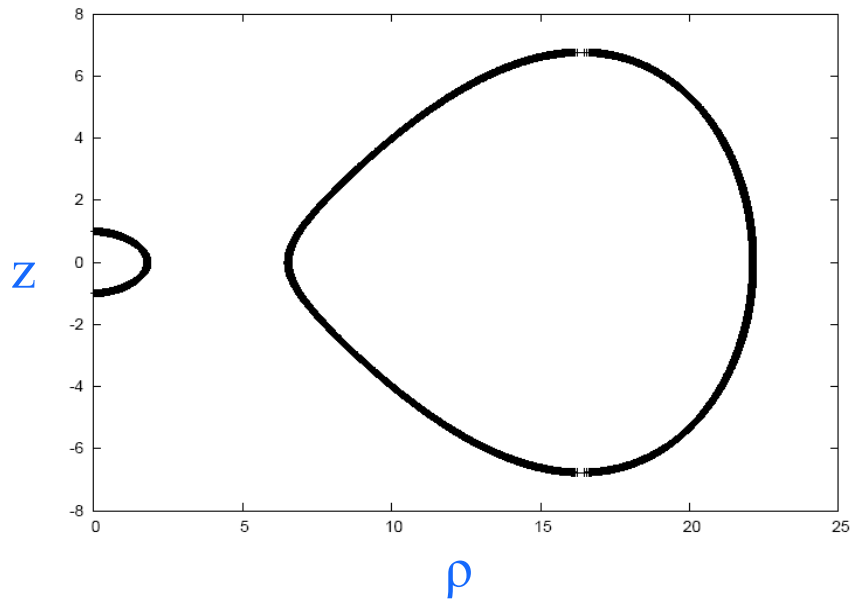
All is regular in “bumpy” spacetimes



Poincare map for $E=0.95$, $L_z=-3$, $a/M=0.9$, $q=0.95$

Or is it?...

Effective potential $(\dot{\rho}^2 + \dot{z}^2) = V(E, L_z, \rho, z)$ defines allowed bound orbits



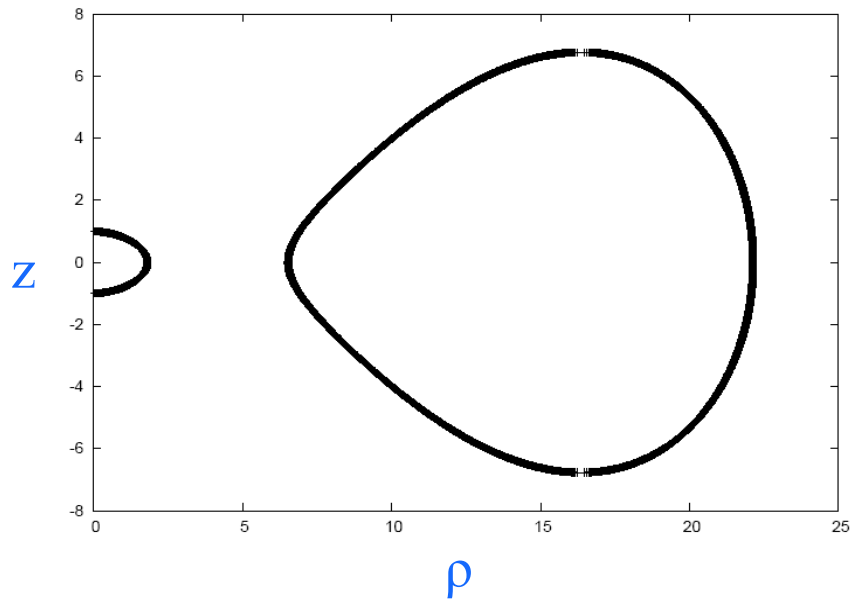
$E=0.95, L_z=-3, a/M=0.9, q=0$



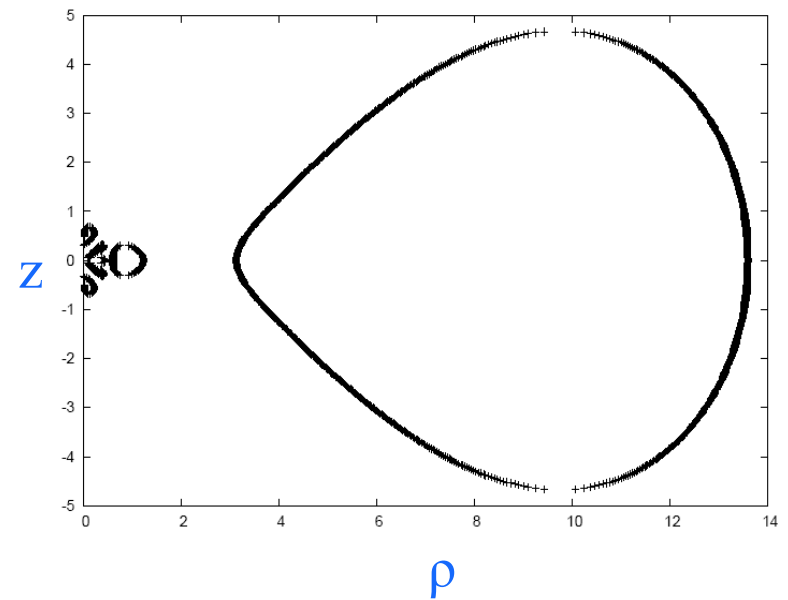
$E=0.95, L_z=-3, a/M=0.9, q=0.95$

Or is it?...

Effective potential $(\dot{\rho}^2 + \dot{z}^2) = V(E, L_z, \rho, z)$ defines allowed bound orbits

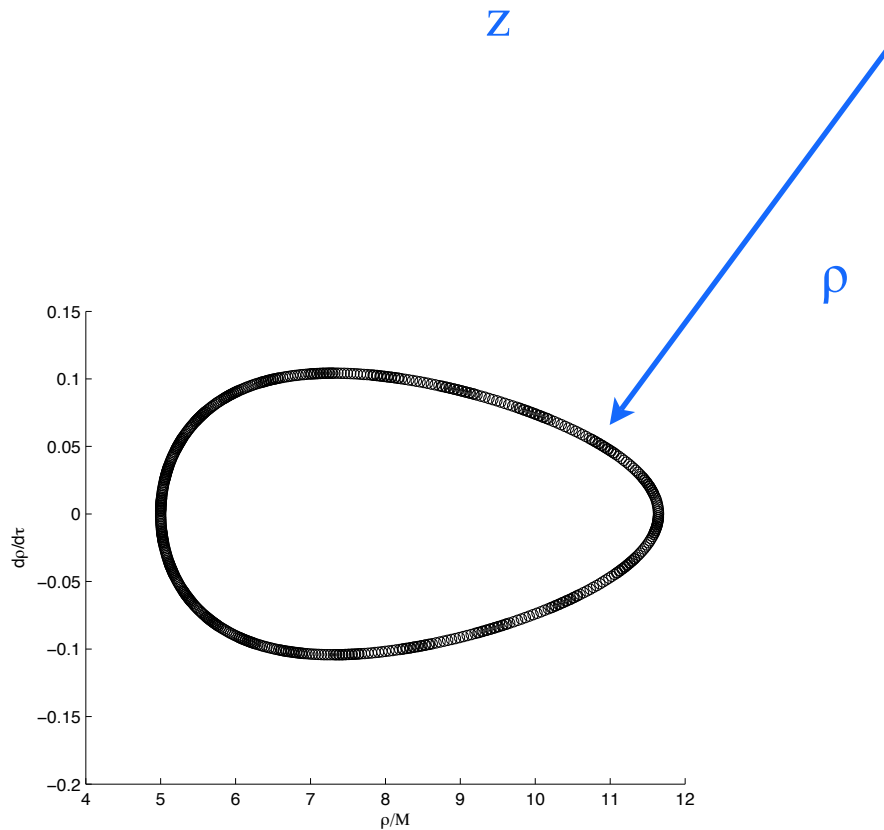


$E=0.95, L_z=-3, a/M=0.9, q=0$



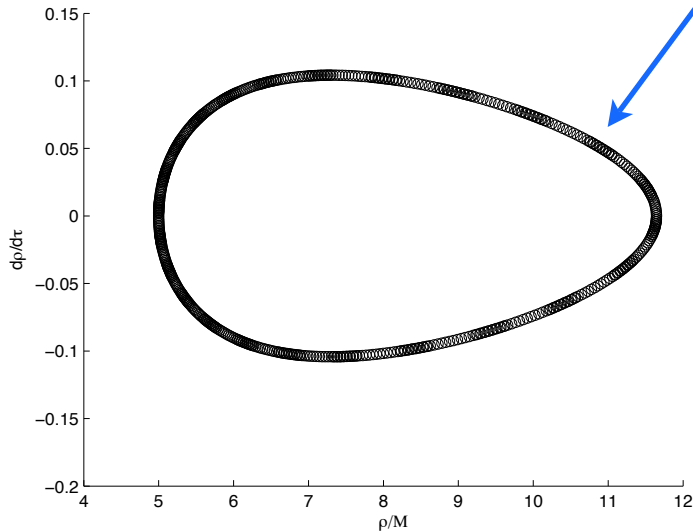
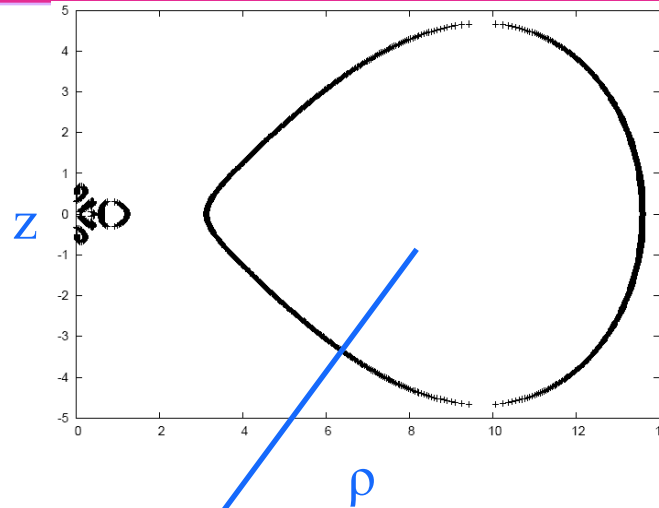
$E=0.95, L_z=-3, a/M=0.9, q=0.95$

It's a mad, mad, mad, mad geodesic



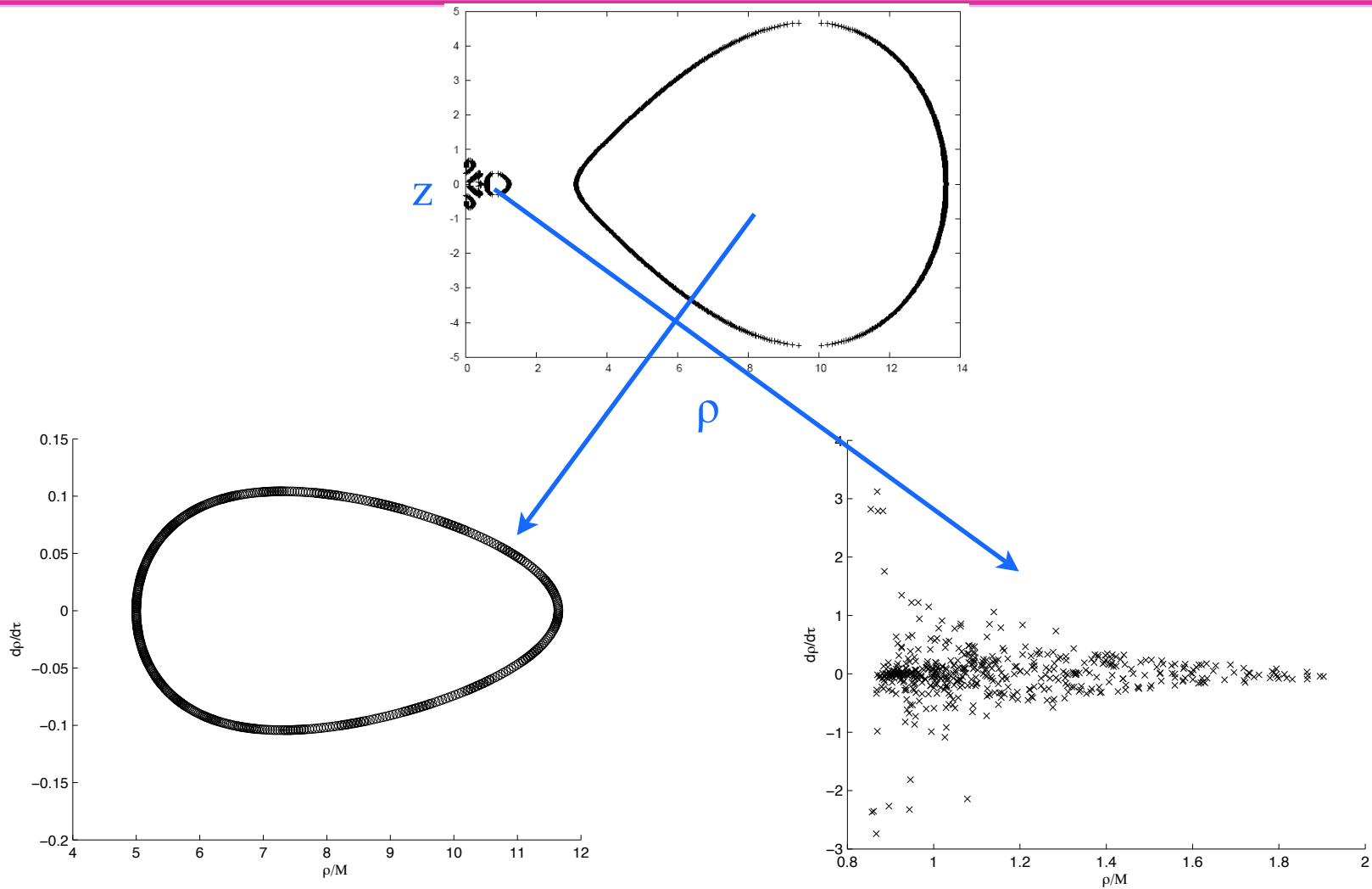
$$E=0.95, L_z=-3, a/M=0.9, q=0.95$$

It's a mad, mad, mad, mad geodesic



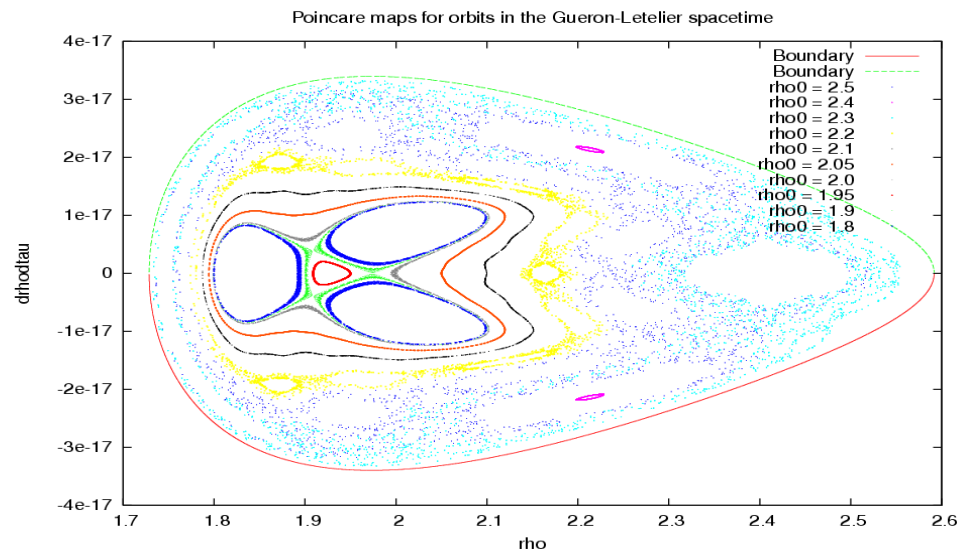
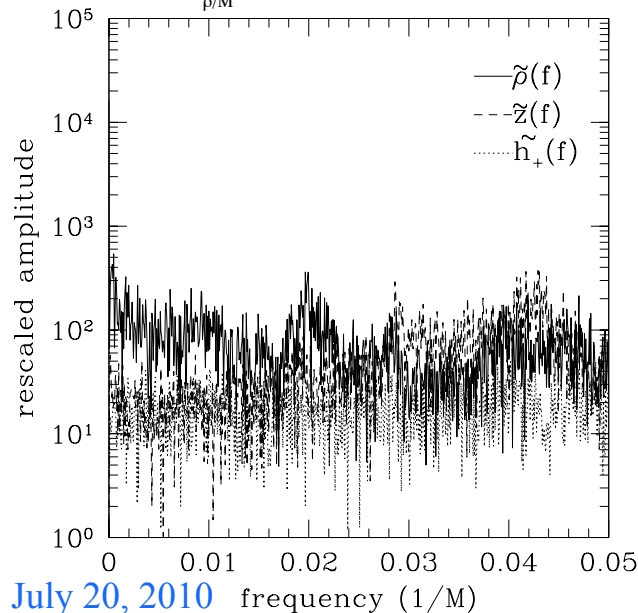
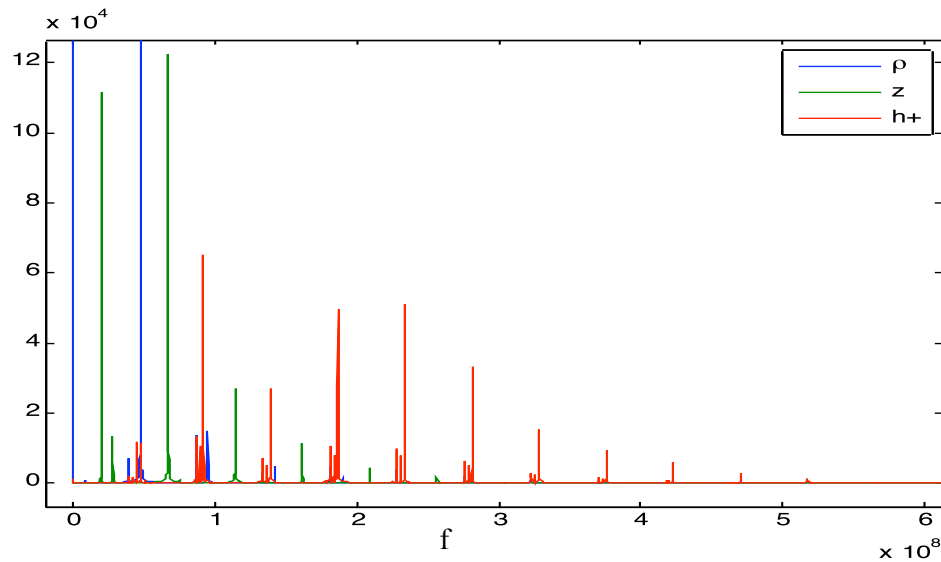
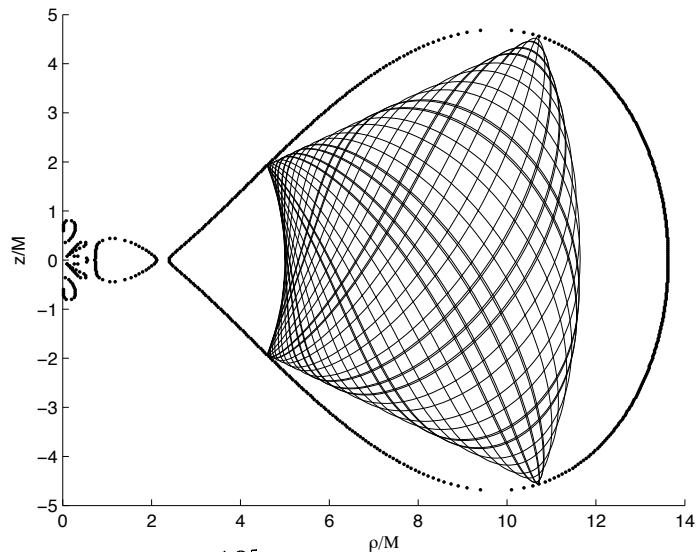
$$E=0.95, L_z=-3, a/M=0.9, q=0.95$$

It's a mad, mad, mad, mad geodesic



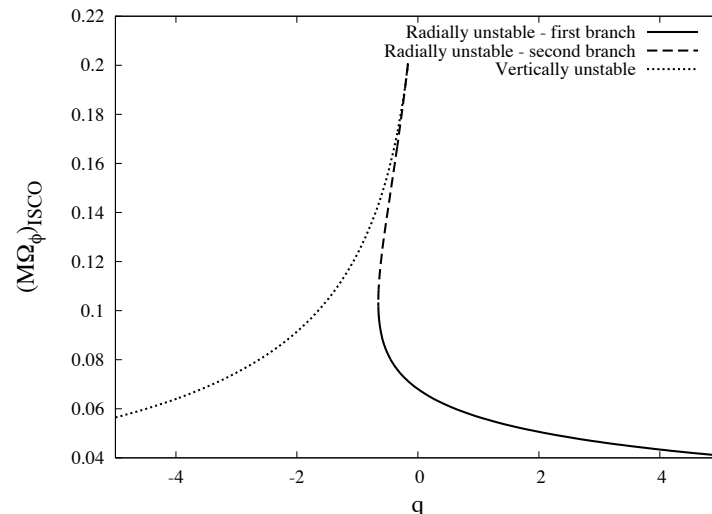
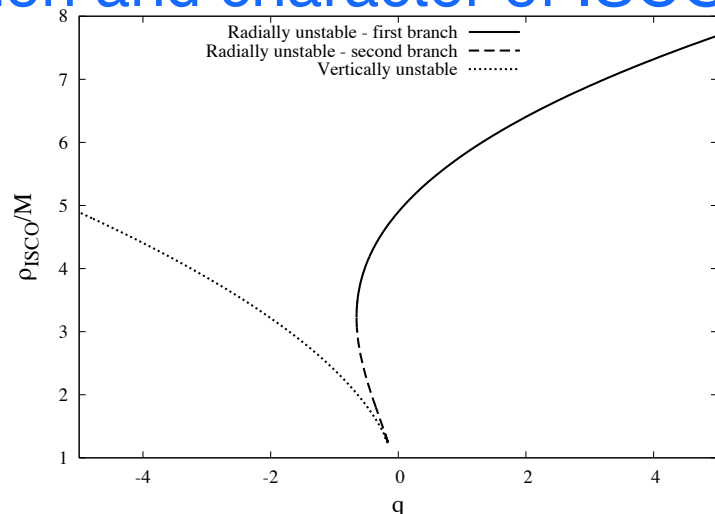
$E=0.95, L_z=-3, a/M=0.9, q=0.95$

Order and Chaos, side by side

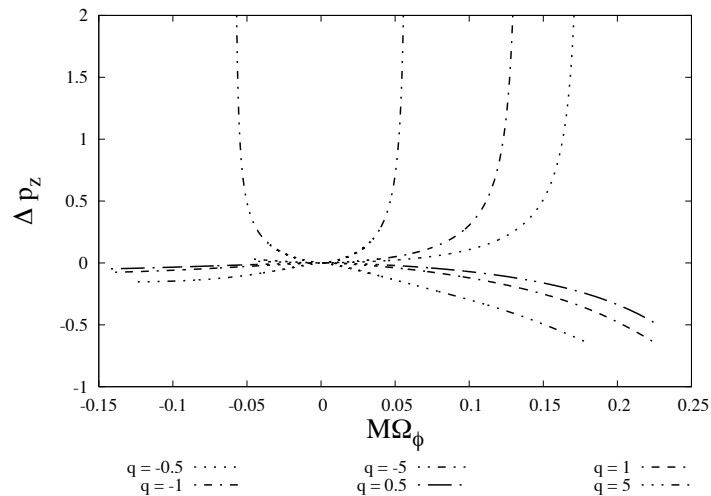
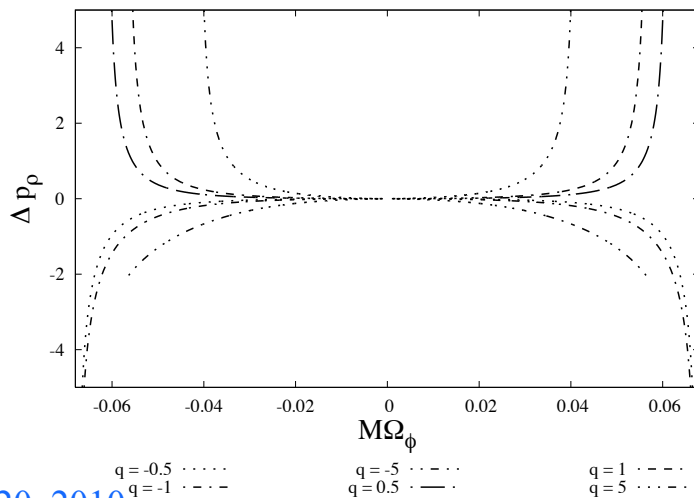


Other signs of non-Kerr spacetimes

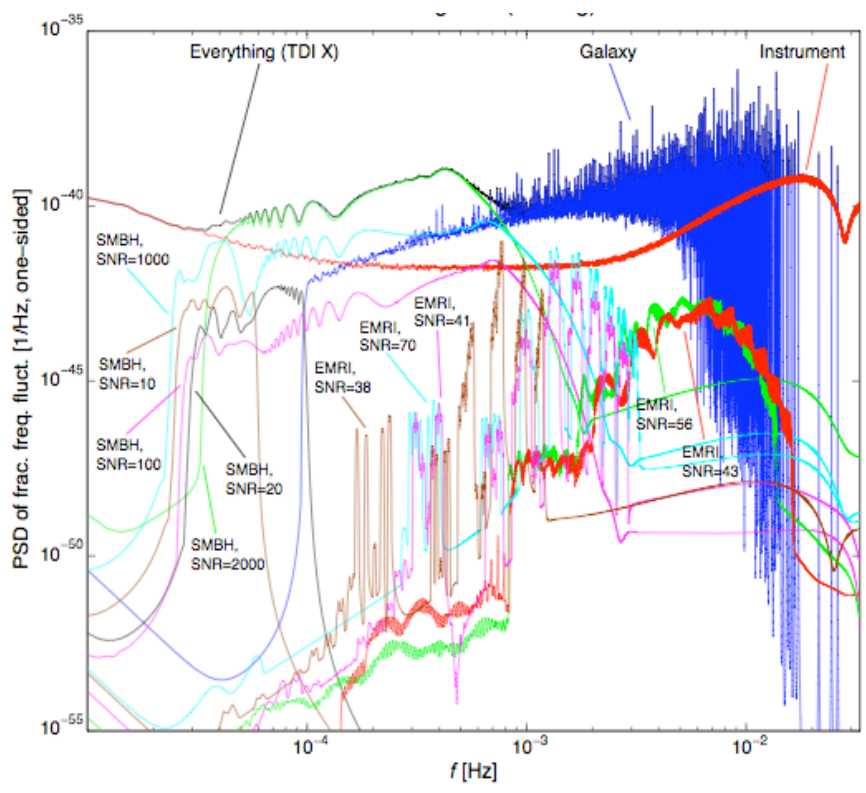
Location and character of ISCO



Periapsis and orbital-plane precession



LISA Data: An embarrassment of riches



[Arnaud et al., 2007, CQG 24 S551]

What has already been accomplished?

	MLDC 1	MLDC 2	MLCD 1B	MLDC 3
GB	<ul style="list-style-type: none"> Verification ✓ Unknown, isolated ✓ Unknown, interfering ✓ 	<ul style="list-style-type: none"> Galaxy of 3×10^6 ✓ 	<ul style="list-style-type: none"> Verification ✓ Unknown, isolated ✓ Unknown, confused ✓ 	<ul style="list-style-type: none"> Galaxy of 6×10^7 <i>chirping</i> ✓
MBH	<ul style="list-style-type: none"> Isolated ✓ 	<ul style="list-style-type: none"> 4–6x, over Galaxy and EMRIs ✓ 	<ul style="list-style-type: none"> Isolated ✓ 	<ul style="list-style-type: none"> Over Galaxy <i>spinning, precessing</i> ✓
EMRI		<ul style="list-style-type: none"> Isolated ✓ 4–6x, over Galaxy and SMBHs 	<ul style="list-style-type: none"> Isolated ✓ 	<ul style="list-style-type: none"> 5 together, weaker ✓
New				<ul style="list-style-type: none"> Cosmic string cusp <i>bursts</i> ✓ Cosmological <i>background</i> ✓

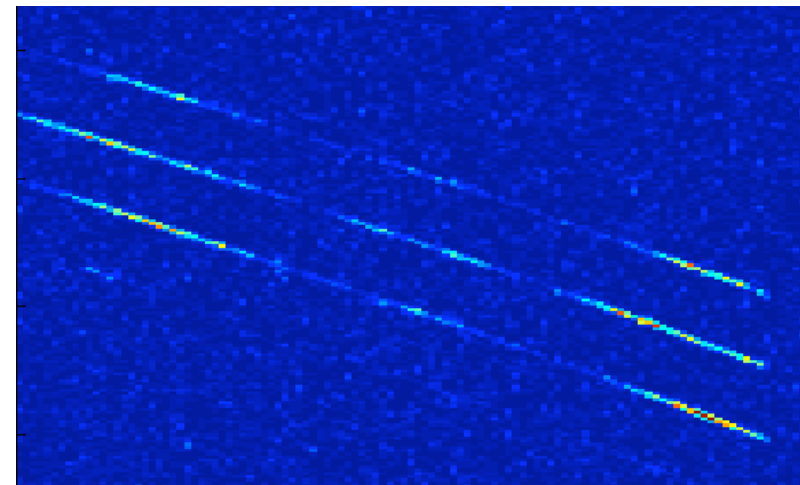
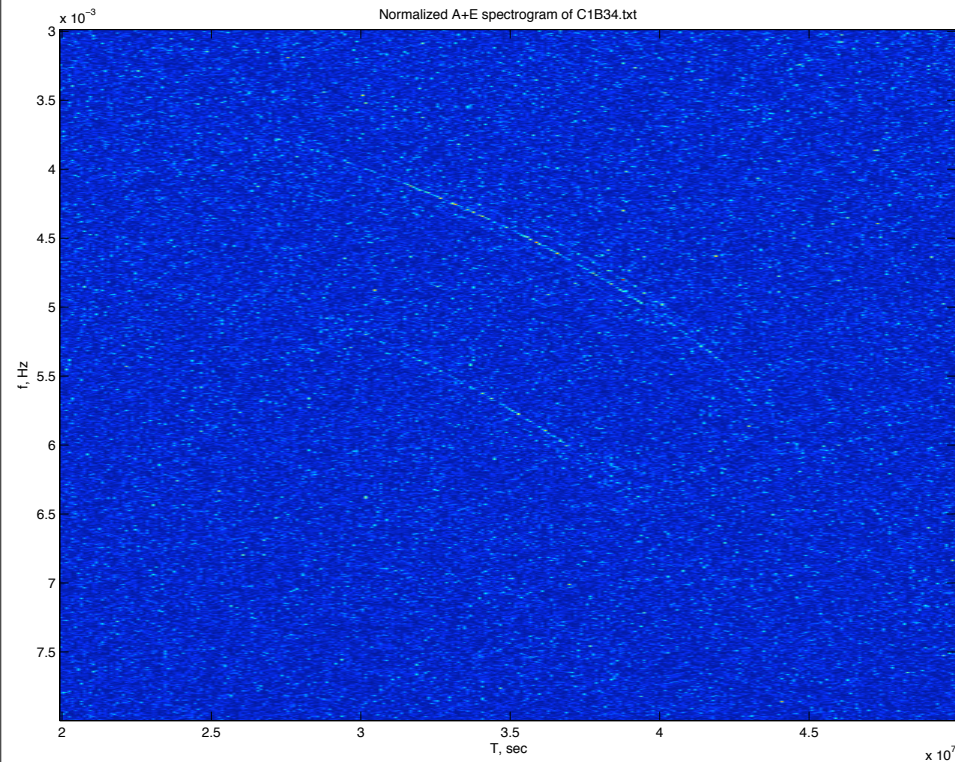
Table by M. Vallisneri

Mock LISA Data Challenges; LISA Parameter-Estimation task force

EMRI detection and analysis

$$h(t) = h(M_1, M_2, \vec{S}_1, \vec{S}_2, \theta, \phi, D_L, e, \dots; t) \quad 17 \text{ parameters}$$

Need innovative search techniques to separate many overlapping signals: Markov-Chain Monte Carlo, MultiNest, time-frequency searches



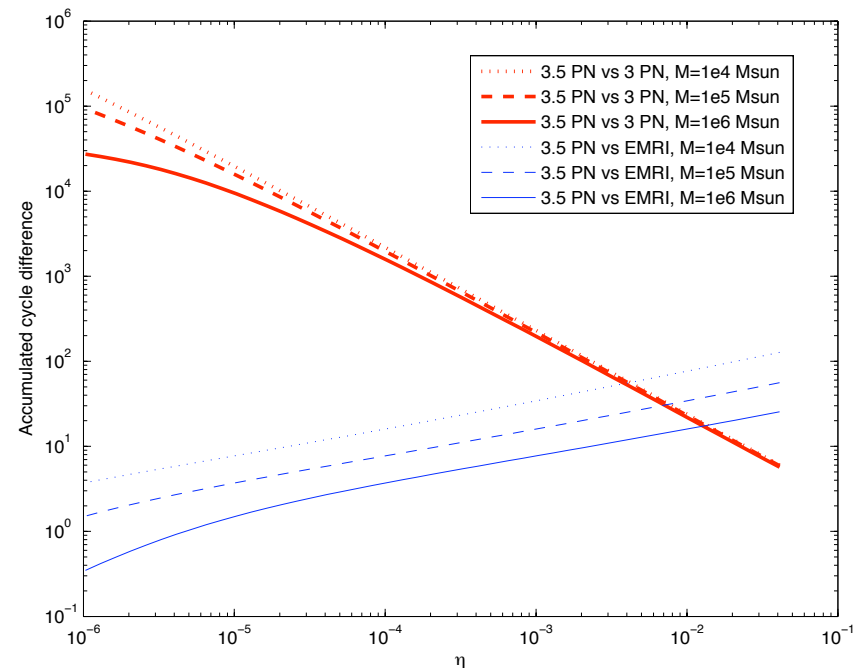
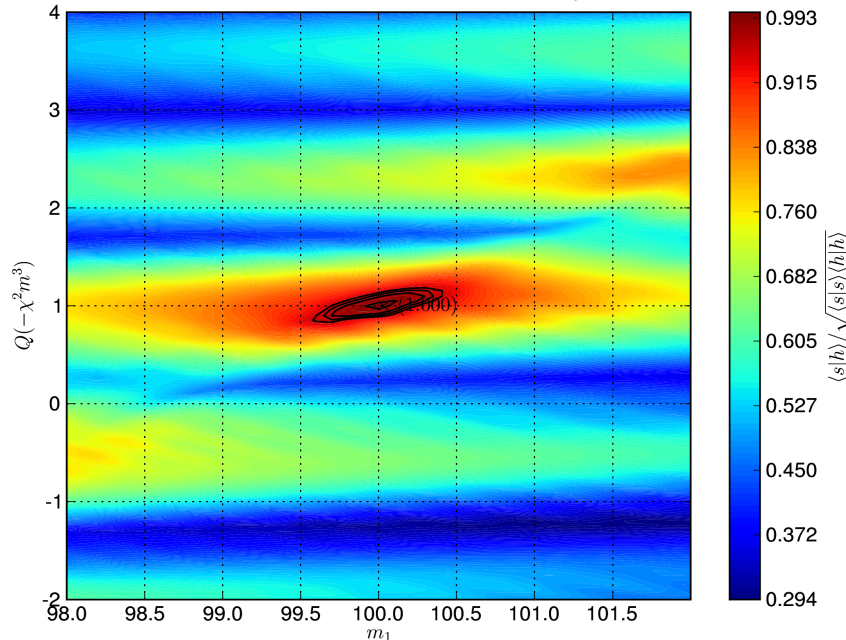
[Gair, IM, Wen, 2008, CQG 25 184031]

Intermediate-mass-ratio Inspirals

Can measure mass quadrupole moment to around 20% of Kerr value with Advanced LIGO [Brown et al., PRL 99, 201102]

Waveforms are a problem: both post-Newtonian and self-force waveforms currently fail in the intermediate regime [IM and Gair, 2005, PRD 72 084025]

$(100.0, 1.4)M_{\odot}, m_2 = 1.4, \chi = 0.90, Q = 1.00, s_x = 0.00, s_y = 0.00, s_z = 1.00$



Summary

- Extreme- or intermediate- mass-ratio inspirals are great probes of strong gravity
- They give us an opportunity to test General Relativity
- Focus on null-hypothesis tests: are the massive, compact objects consistent with being Kerr black holes?
- Several smoking guns are possible, from the emergence of chaos to changes in the frequencies and characteristics of the ISCO and precessions