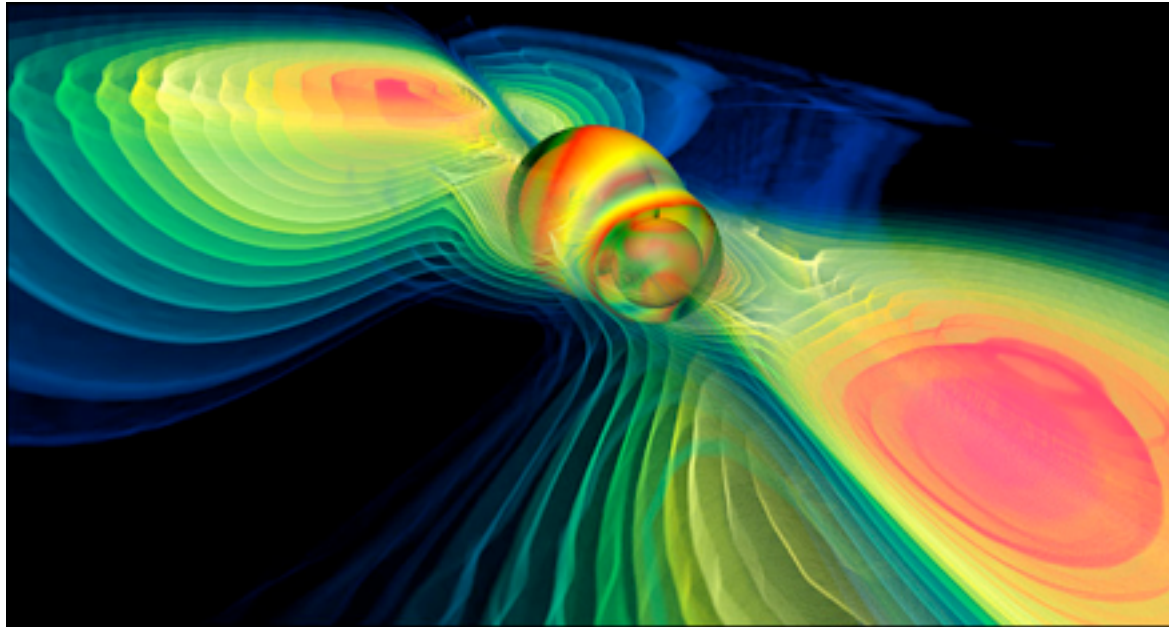


# GWastrophysics



(Image: MPI for Gravitational Physics / W.Benger-ZIB)

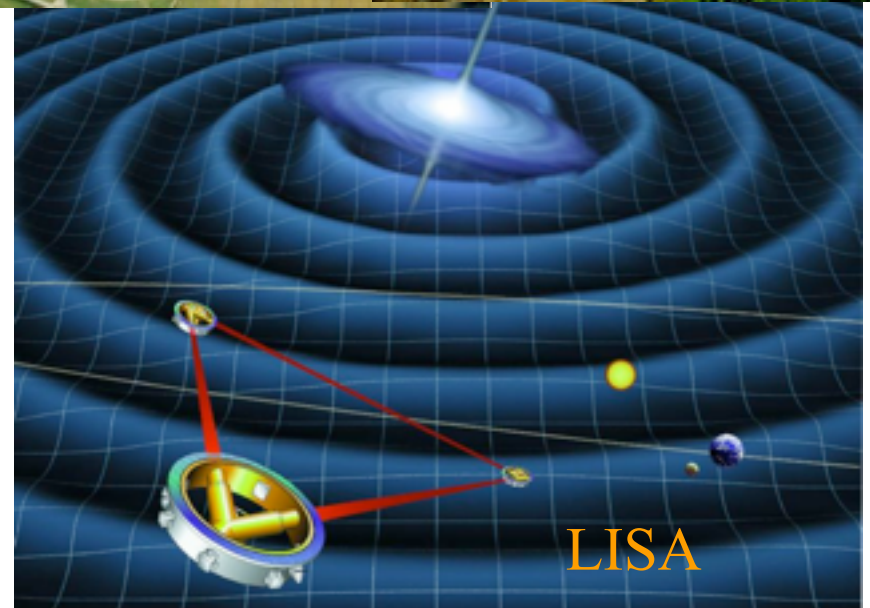
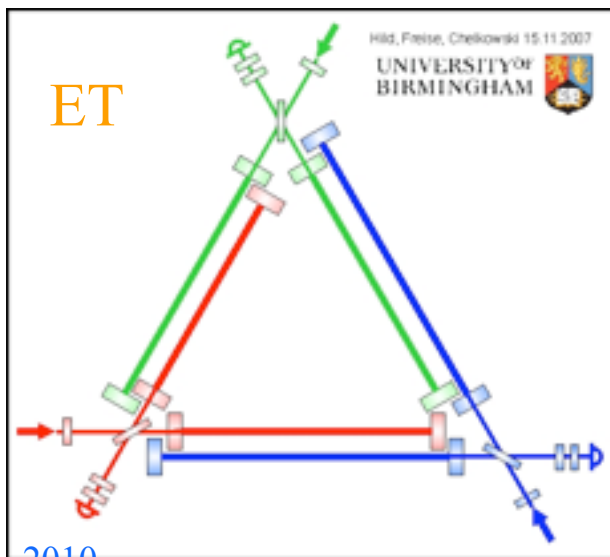
Ilya Mandel

(NSF Astronomy and Astrophysics Postdoc Fellow,  
Northwestern University / MIT)

July 14, 2010

University of Birmingham

# Gravitational-wave observatories



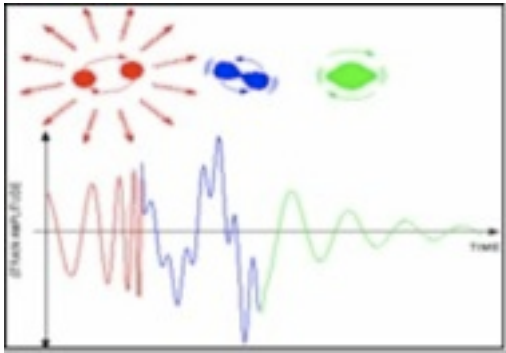
Birmingham: July 14, 2010

Tuesday, November 9, 2010

# Types of GW sources



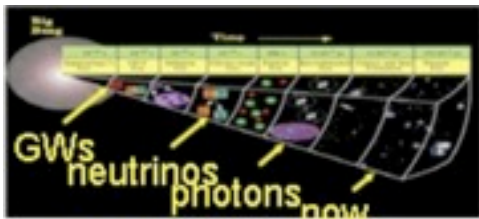
- Continuous sources [sources with a slowly evolving frequency]: e.g., non-axisymmetric neutron stars, slowly evolving binaries



- Coalescence sources [known waveforms, matched filtering]: compact object binaries



- Burst events [unmodeled waveforms]: e.g., asymmetric SN collapse, cosmic string cusps



- Stochastic GW background [early universe]

- ??? [expect the unexpected]

# Outline

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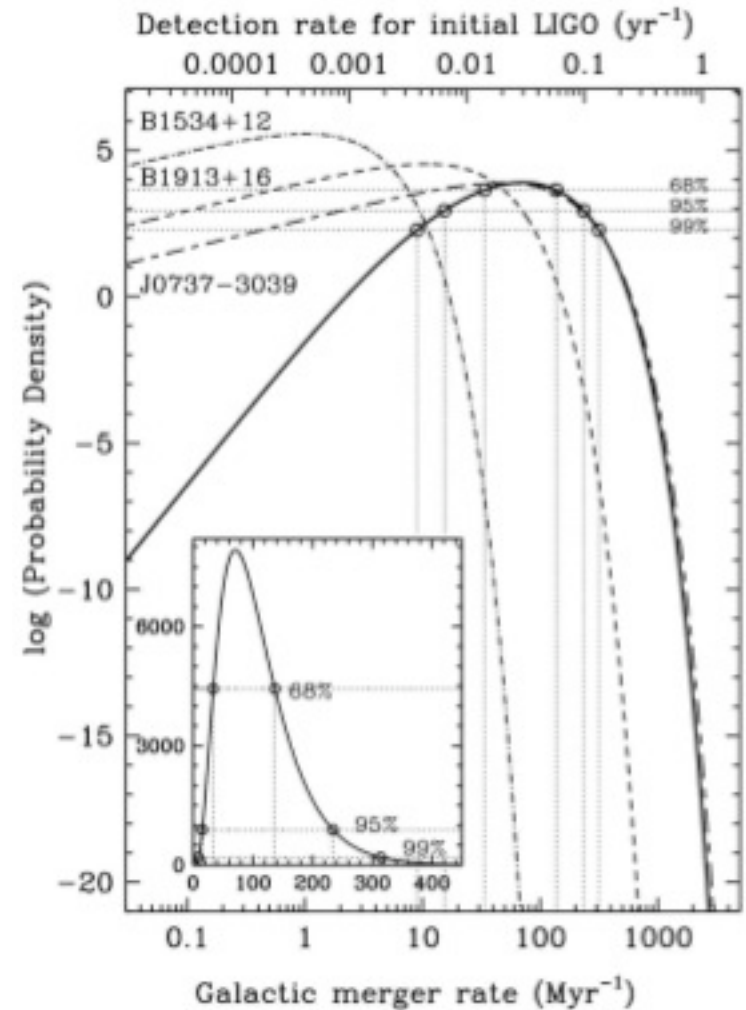
1. Compact-binary-coalescence rate predictions for ground-based observatories
2. Constraining astrophysics with observed event rates, upper limits, and population parameter distributions
3. Testing general relativity with intermediate- and extreme-mass-ratio inspirals

# Coalescence rate predictions

- Based on [IM and O'Shaughnessy, 2010; Abadie et al., 2010]
- Ground-based interferometric detectors (LIGO, Virgo, GEO 600, AIGO, LCGT) are sensitive @ tens/hundreds Hz: ideal for detecting NS-NS, NS-BH, BH-BH binaries
- Coalescence rate predictions from:
  - » extrapolation from observed binary pulsars
  - » simulations of isolated binary evolution
  - » dynamical-formation models
  - » intermediate-mass-black holes ?
- These estimates are still significantly uncertain at present
- “There are known knowns. There are things we know that we know. There are **known unknowns**. That is to say, there are things that we now know we don't know. But there are also unknown unknowns. There are things we do not know we don't know.” [Donald Rumsfeld]

# Extrapolation from BNS observations

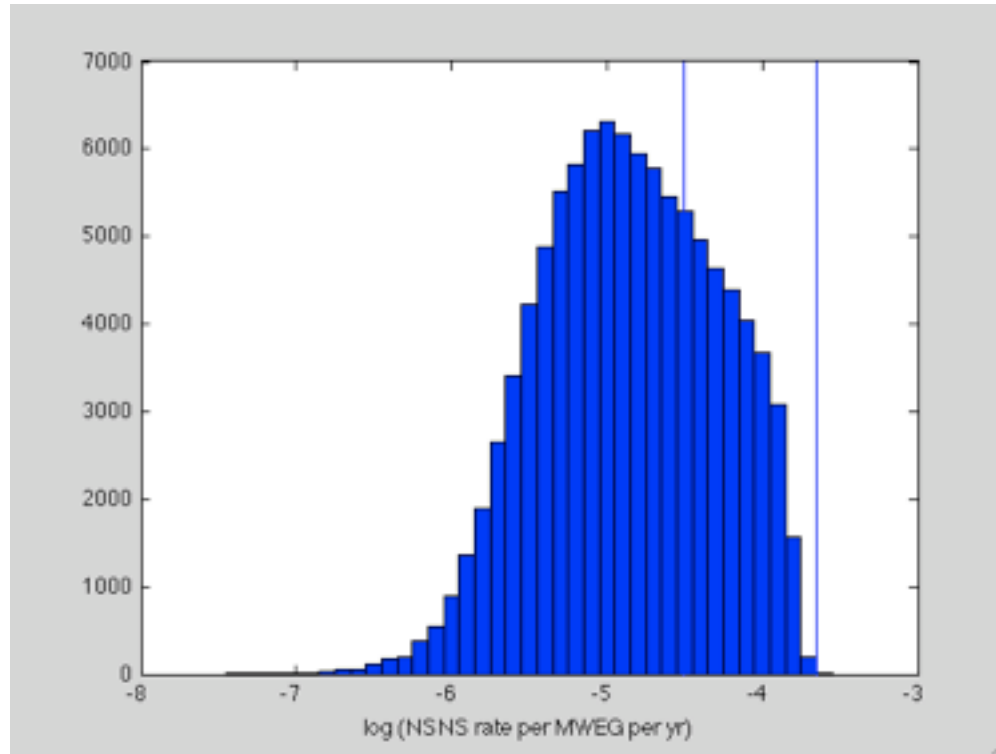
- Best NS-NS merger-rate estimates come from observed Galactic binary pulsars
- Small-number statistics ( $\sim 10$  total,  $\sim 5$  merging in 15 Gyr)
- Selection effects (pulsar luminosity distribution)
- [Kim et al., 2003 ApJ 584 985, 2006 astro-ph/0608280; Kalogera et al., 2004, ApJ 601 L179]



# Population synthesis models

- No observed NS-BH or BH-BH binaries
- Predictions based on population-synthesis models for isolated binary evolution with StarTrack [Belczynski et al., 2005, astro-ph/0511811] or similar codes
- Thirty poorly constrained parameters
- [O'Shaughnessy et al., 2005 ApJ 633 1076, 2008 ApJ 672 479] vary seven most important parameters:
  1. power-law index in binary mass ratio
  - 2, 3, 4. supernovae kicks described by two independent Maxwellians and their relative contribution
  5. strength of massive stellar wind
  6. common-envelope efficiency
  7. fractional mass retention during nonconservative mass transfer

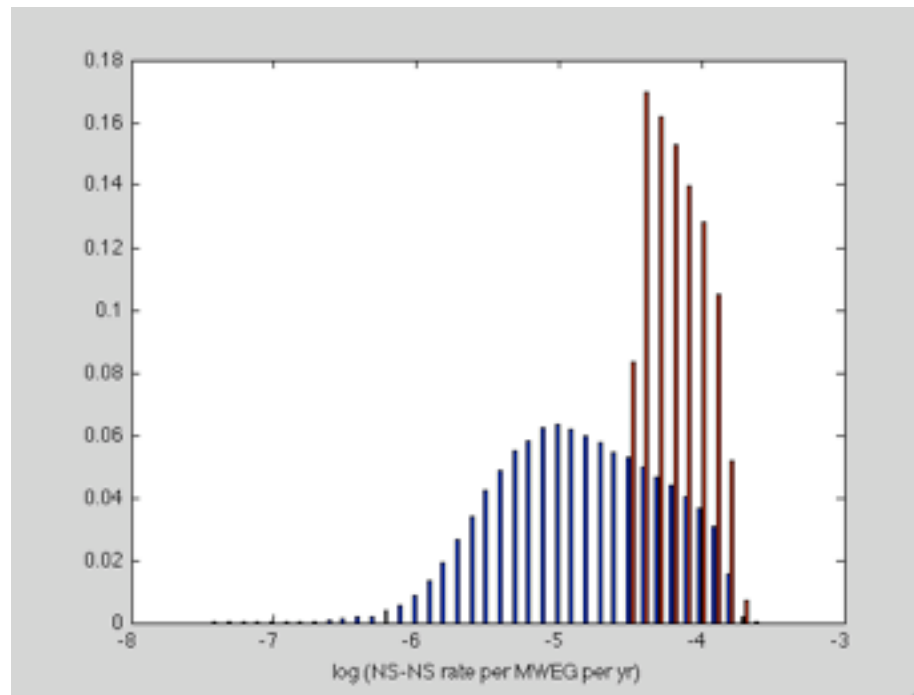
# Constraining models



- Add constraints from observations; binary pulsars: NS-NS, NS-WD, supernovae, etc.
- Average over models that satisfy constraints

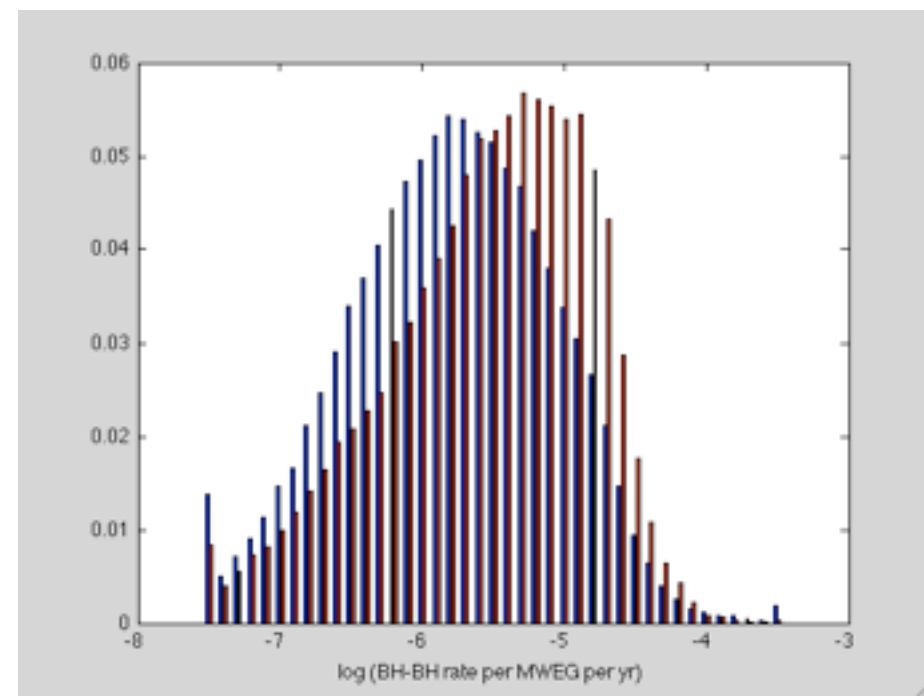
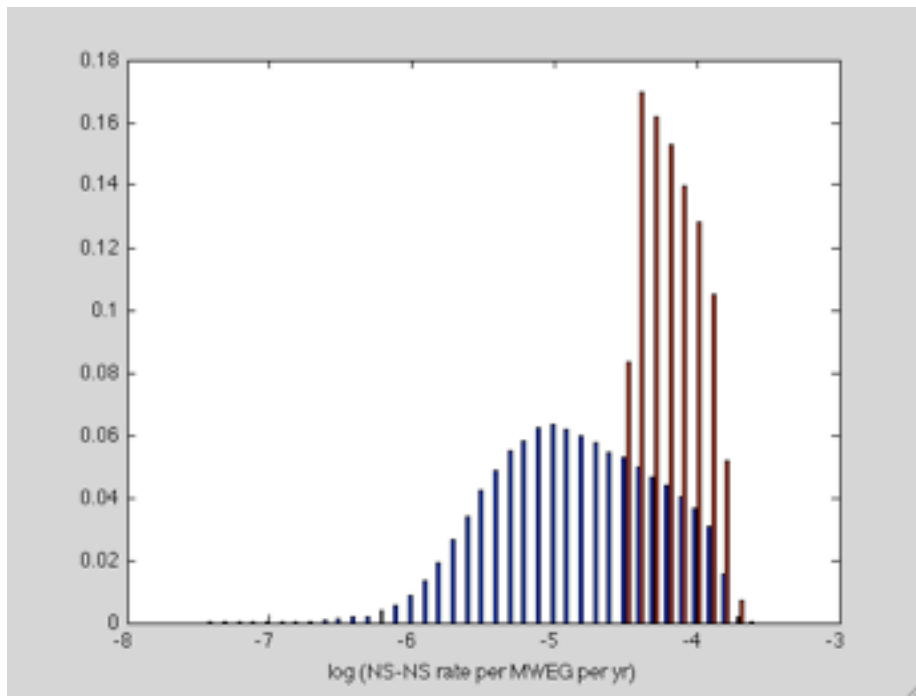


# Effect of adding constraints, 1



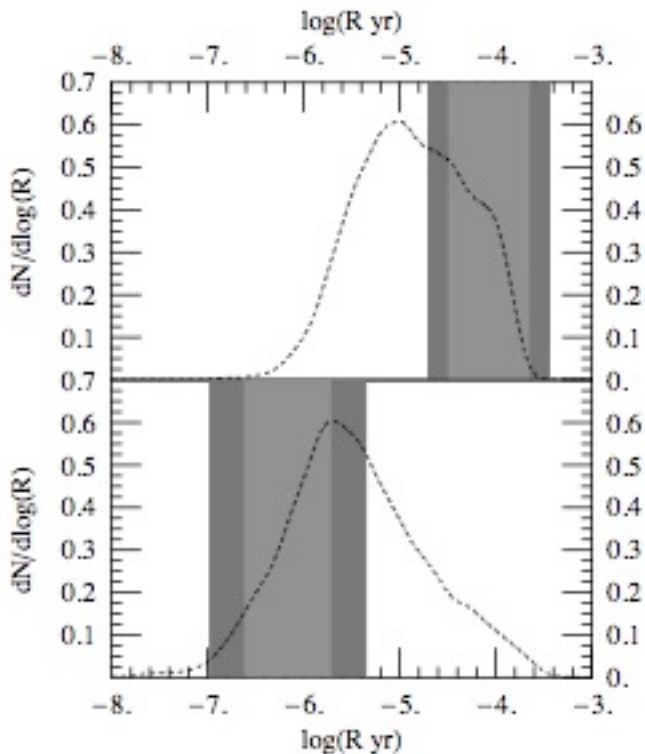
Single constraint satisfaction - no accounting for sampling uncertainties or model fitting errors

# Effect of adding constraints, 1

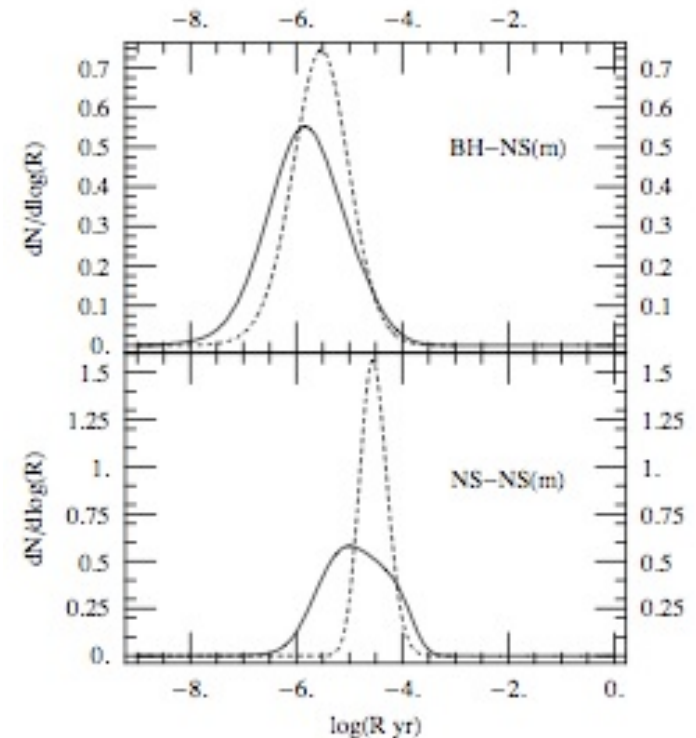


Single constraint satisfaction - no accounting for sampling uncertainties or model fitting errors

# Effect of adding constraints, 2



Constraints from  
observed binary pulsars



BH-NS and NS-NS  
rate/MWEG predictions

[plots from O'Shaughnessy et al., 2008, ApJ 672 479]

# LIGO sensitivity

[plot from Kopparapu et al., 2008 ApJ 675 1459]

$\dot{N} = R \times N_G$   
 (merger rate) =  
 (merger rate per  $L_{10}$ ) \*  
 ( $N_G$  in  $L_{10}$ 's)

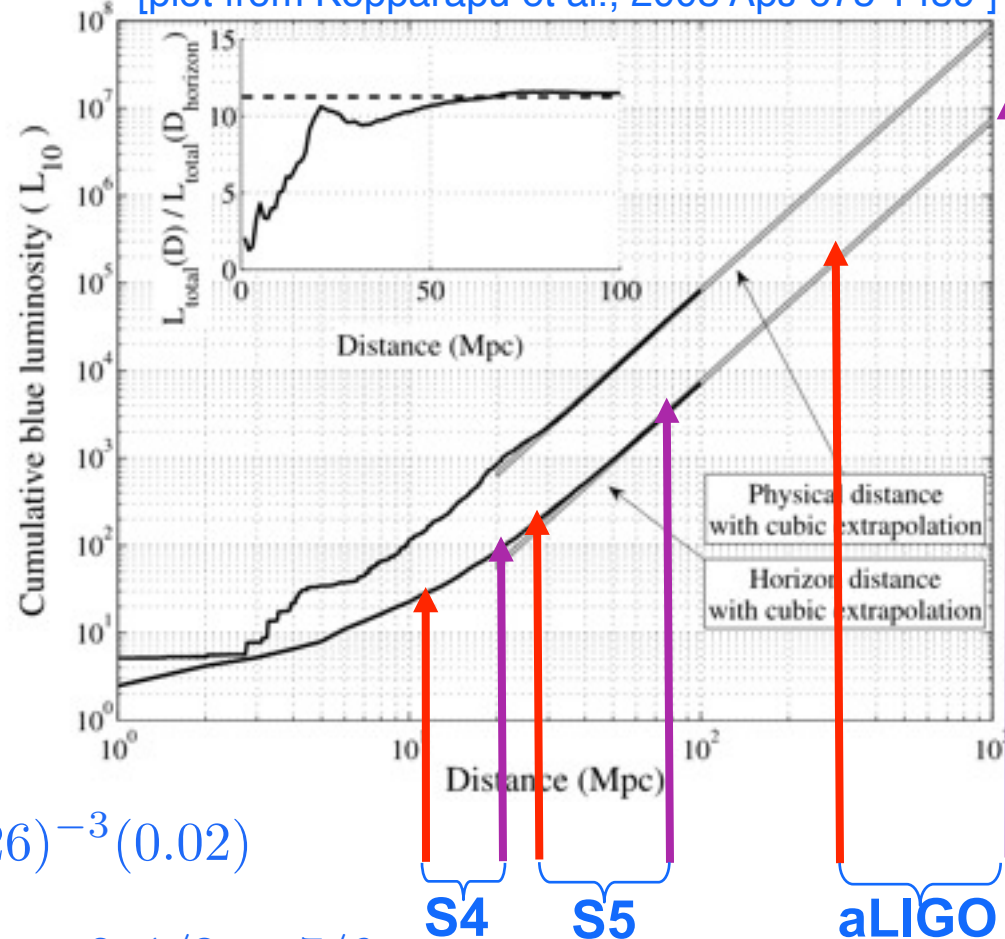
$$\rho \equiv \sqrt{4 \int_0^{f_{\text{ISCO}}} \frac{|\tilde{h}(f)|^2}{S_n(f)} df}$$

$$\rho(D_{\text{horizon}}) \equiv 8$$

1/2.26 -- sky and orientation averaging; 0.02  $L_{10}$  per  $\text{Mpc}^3$

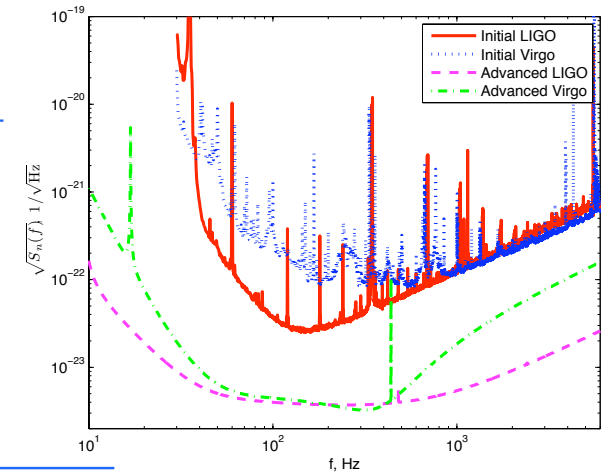
$$N_G(L_{10}) = \frac{4}{3} \pi \left( \frac{D_{\text{horizon}}}{\text{Mpc}} \right)^3 (2.26)^{-3} (0.02)$$

$$|\tilde{h}(f)| = 2/D * (5\mu/96)^{1/2} (M/\pi^2)^{1/3} f^{-7/6}$$



# Merger and Detection Rates

Source	$R_{\text{low}}$	$R_{\text{re}}$	$R_{\text{high}}$
NS-NS ( $\text{MWEG}^{-1} \text{ Myr}^{-1}$ )	1	100	1000
NS-BH ( $\text{MWEG}^{-1} \text{ Myr}^{-1}$ )	0.05	3	100
BH-BH ( $\text{MWEG}^{-1} \text{ Myr}^{-1}$ )	0.01	0.4	30



IFO	Source	$\dot{N}_{\text{low}}$ $\text{yr}^{-1}$	$\dot{N}_{\text{re}}$ $\text{yr}^{-1}$	$\dot{N}_{\text{high}}$ $\text{yr}^{-1}$
Initial	NS-NS	$2 \times 10^{-4}$	0.02	0.2
	NS-BH	$7 \times 10^{-5}$	0.004	0.1
	BH-BH	$2 \times 10^{-4}$	0.007	0.5
Advanced	NS-NS	0.4	40	400
	NS-BH	0.2	10	300
	BH-BH	0.4	20	1000

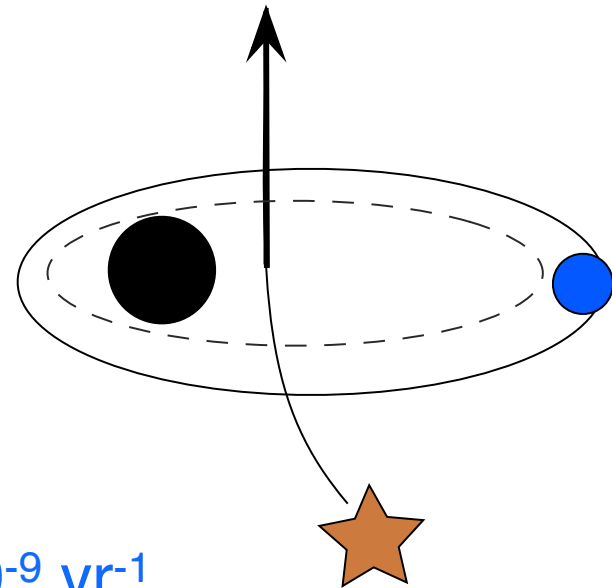
[IM & O'Shaughnessy, 2010, CQG 27 114007; Abadie et al., 2010, arXiv:1003.2480]

# Dynamical Formation

- BH-BH mergers in dense black-hole subclusters of globular clusters
  - » [O'Leary, O'Shaughnessy, Rasio, 2007 PRD 76 061504]
  - » Predicted rates  $10^{-4}$  to 1 per Mpc<sup>3</sup> per Myr
  - » Plausible optimistic values could yield 0.5 events/year for Initial LIGO
- BH-BH scattering in galactic nuclei with a density cusp caused by a massive black hole (MBH)
  - » [O'Leary, Kocsis, Loeb, 2009 arXiv:0807.2638]
  - » Based on a number of optimistic assumptions
  - » Predicted detection rates of 1 to 1000 per year for Advanced LIGO
- BH-BH mergers in nuclei of small galaxies without an MBH
  - » [Miller and Lauburg, 2009 ApJ 692 917]
  - » Predicted rates of a few X 0.1 per Myr per galaxy
  - » Tens of detections per year with Advanced LIGO

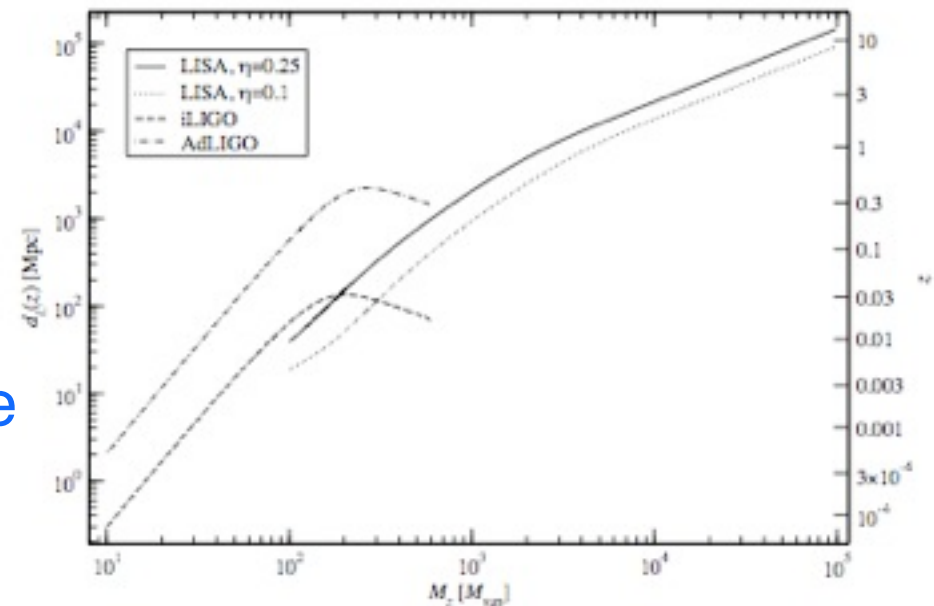
# Inspirals into IMBHs

- Intermediate-mass-ratio inspirals of compact objects (1.4 solar-mass NSs or 10 solar-mass BHs) into intermediate-mass black holes in globular clusters
- Dominant mechanism: IMBH swaps into binaries, 3-body interactions tighten IMBH-CO binary, merger via GW radiation reaction [IM et al., 2008 ApJ 681 1431]
- Rate per globular cluster: few  $\times 10^{-9}$  yr $^{-1}$
- Predicted Advanced LIGO event rates between 1/few years and  $\sim 30$ /year



# Inspirals of two IMBHs

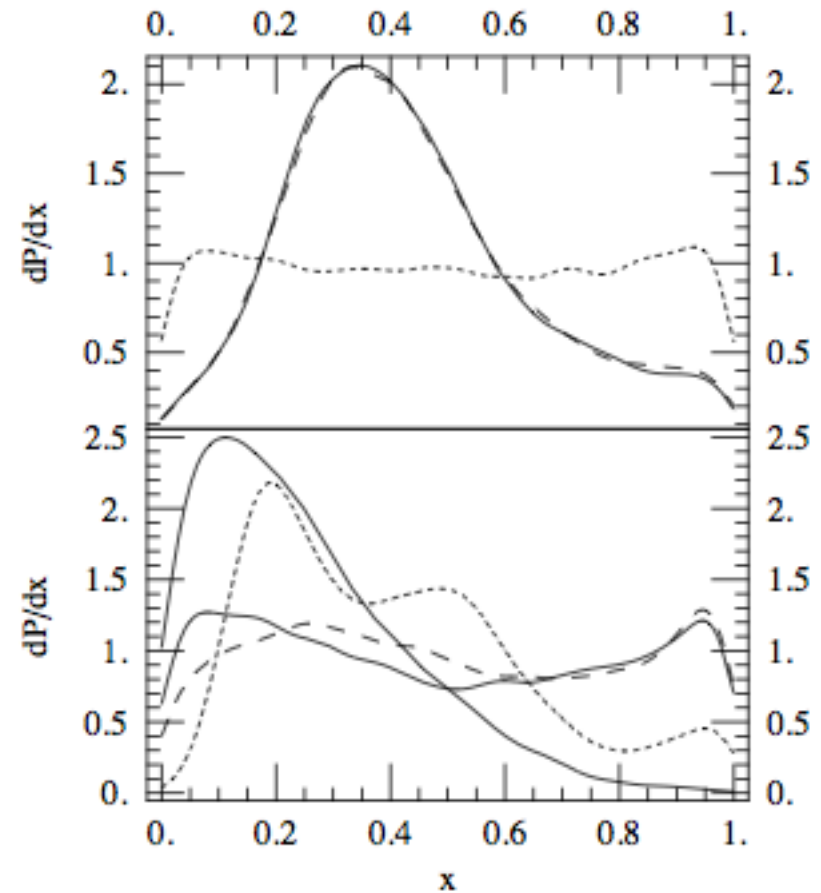
- Two very massive stars could form in globular clusters with sufficient binary fraction, then grow through runaway collision to form two IMBHs in same GC
- Rates of order 1/year are possible for Advanced LIGO [Fregeau et al., 2006 ApJ 646 L135]
- IMBH binaries could also form when two GCs merge [Amaro-Seoane and Freitag, 2006, ApJ 653 L53]





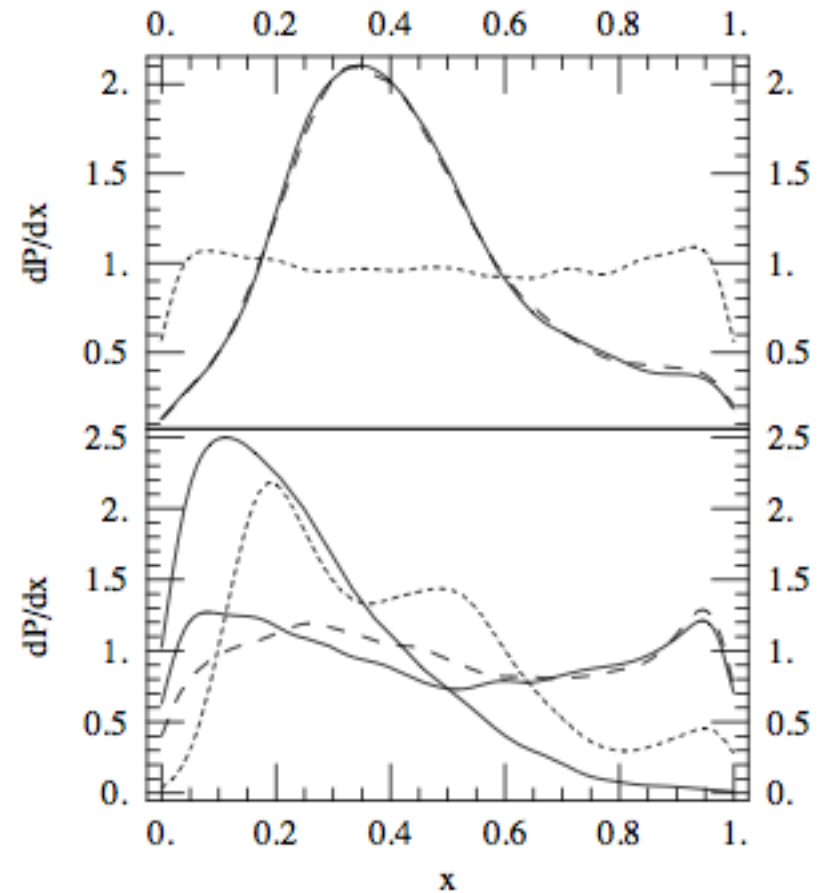
# Astrophysics with GW searches

- Constraints on astrophysical parameters from existing electromagnetic observations [O'Shaughnessy et al., 2008 ApJ 672 479]:



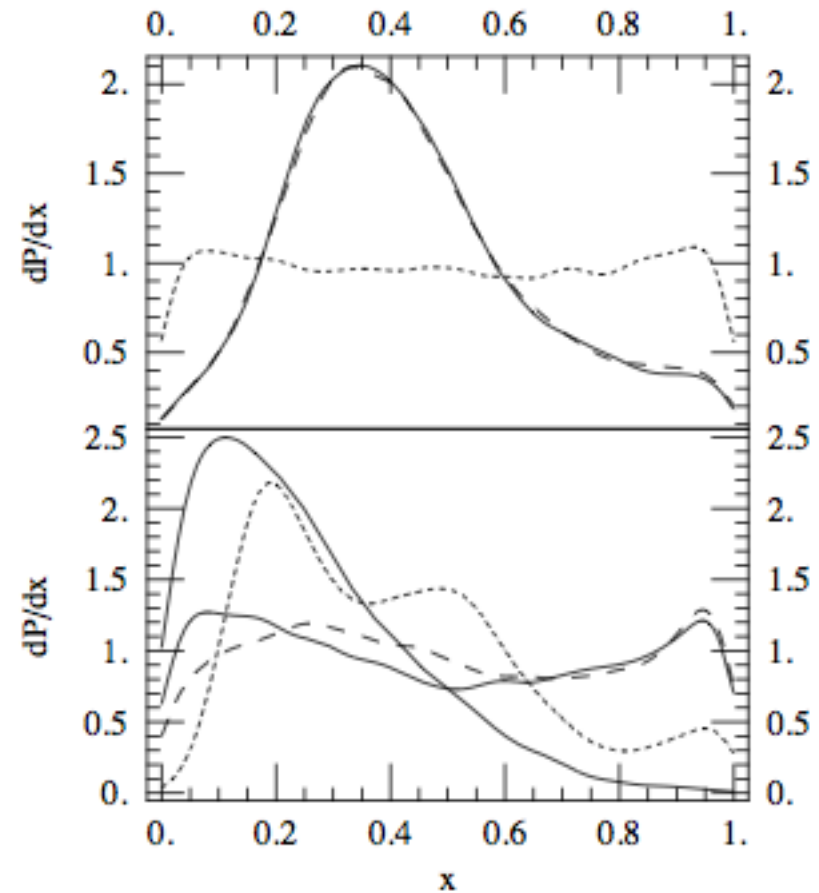
# Astrophysics with GW searches

- Constraints on astrophysical parameters from existing electromagnetic observations [O'Shaughnessy et al., 2008 ApJ 672 479]:
- Observed GW event rates can be compared with models to determine important astrophysical parameters;

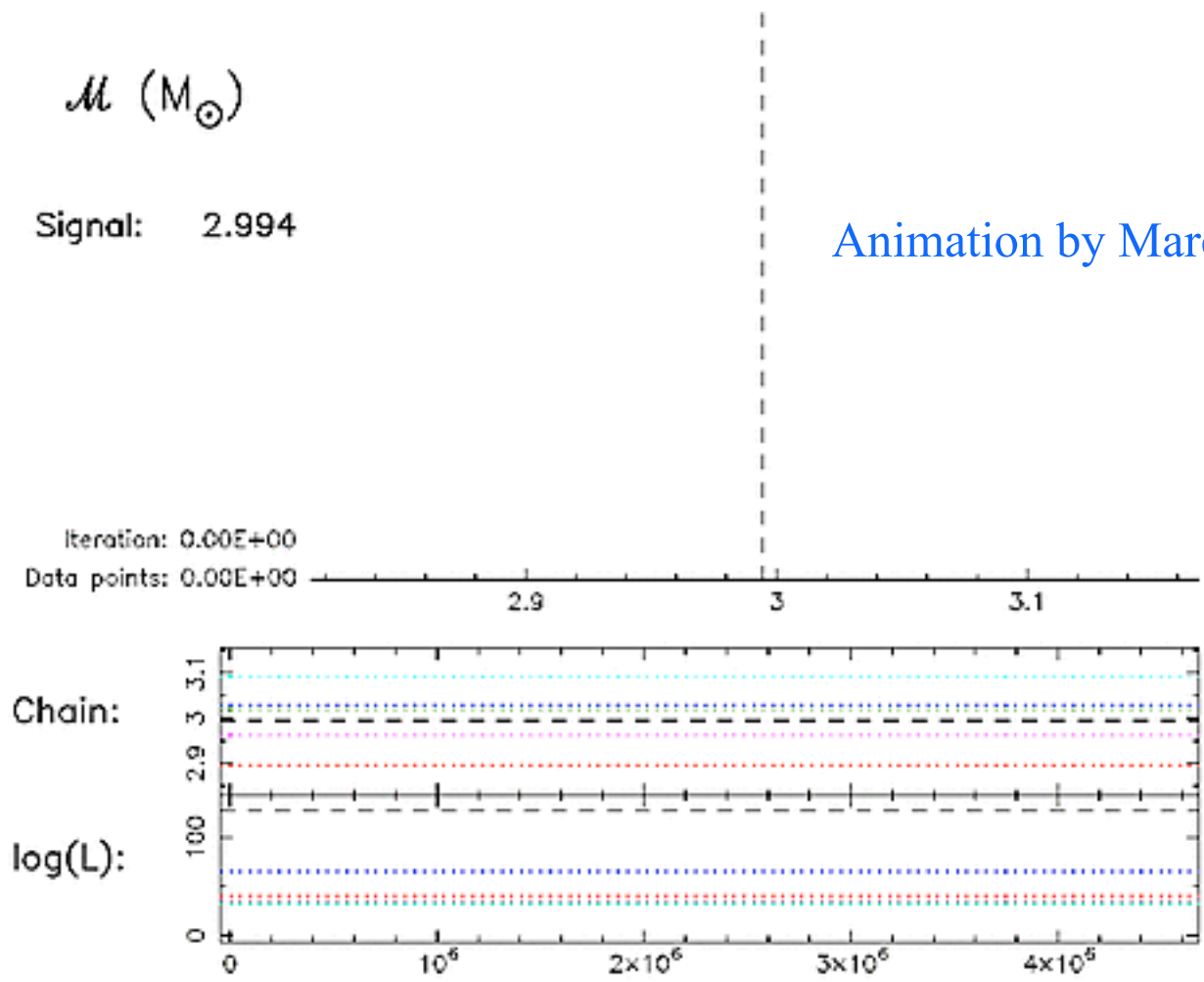


# Astrophysics with GW searches

- Constraints on astrophysical parameters from existing electromagnetic observations [O'Shaughnessy et al., 2008 ApJ 672 479]:
- Observed GW event rates can be compared with models to determine important astrophysical parameters;
- Could match measured mass distributions, etc. to models (requires accurate parameter determination)



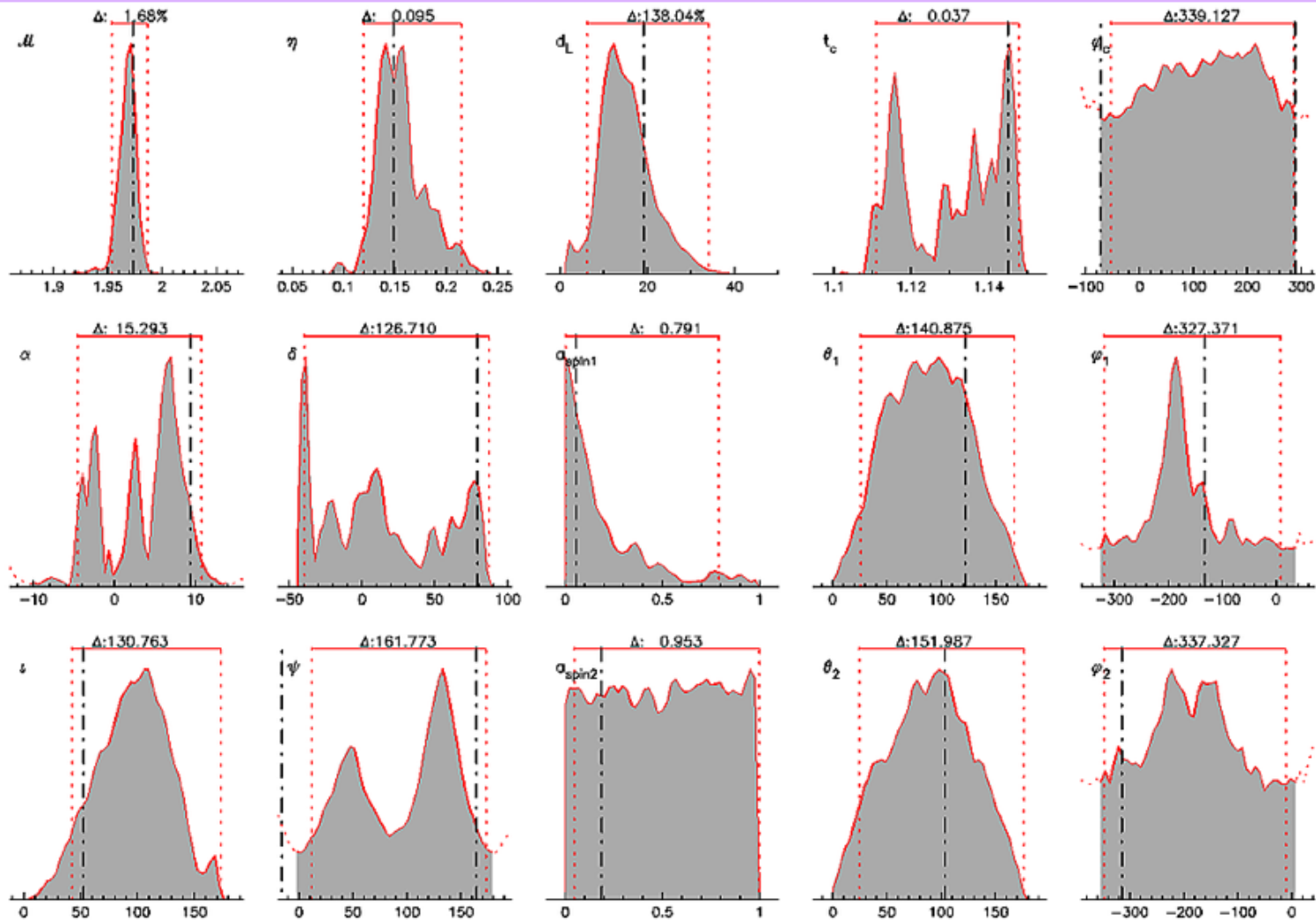
# Markov Chain Monte Carlo



Animation by Marc van der Sluys

van der Sluys, IM, Raymond, et al., 0905.1323

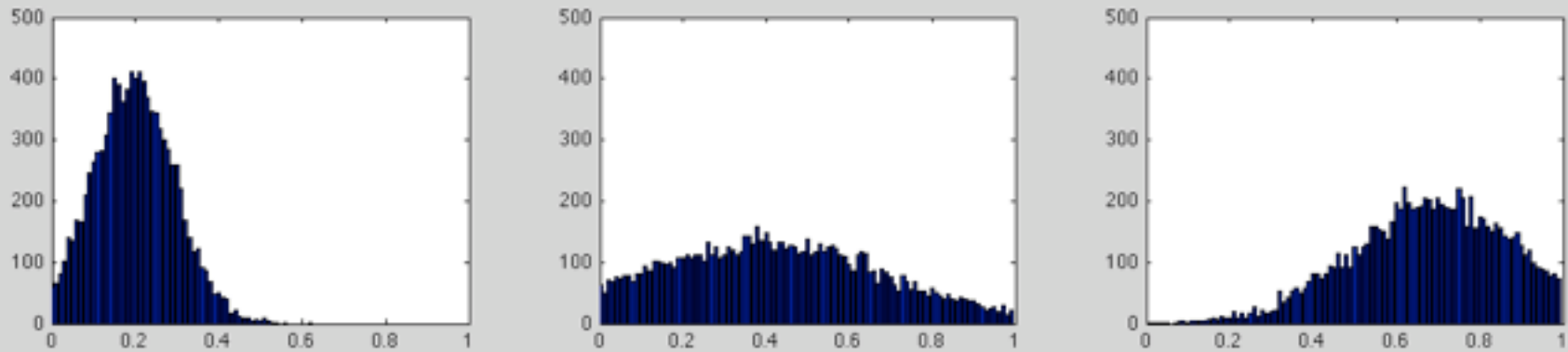
# Accurate Parameter Estimation



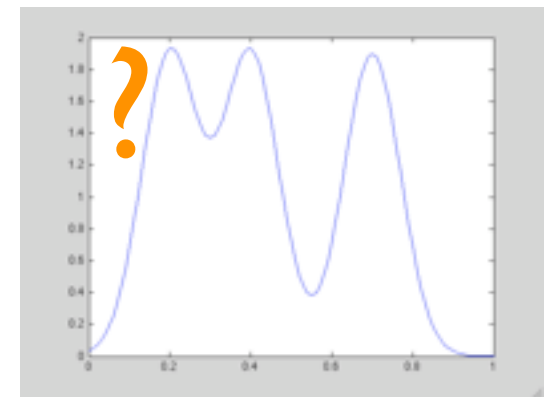
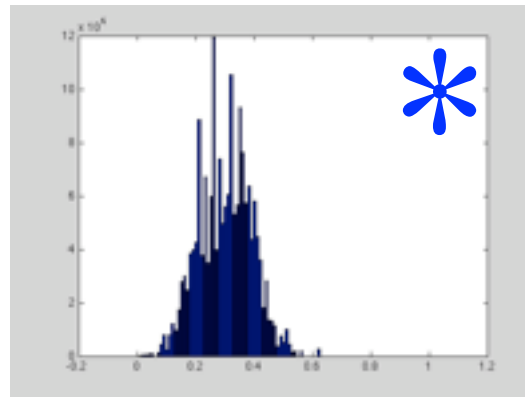
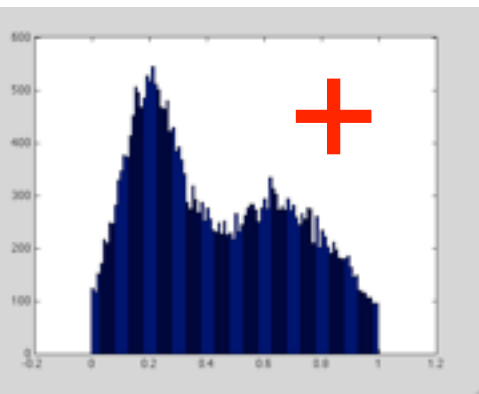
van der Sluys, IM, Raymond, et al., 0905.1323

# Combining events into populations

Parameter estimation on multiple GW detections:  
yield a set of individual marginalized posterior probability density functions

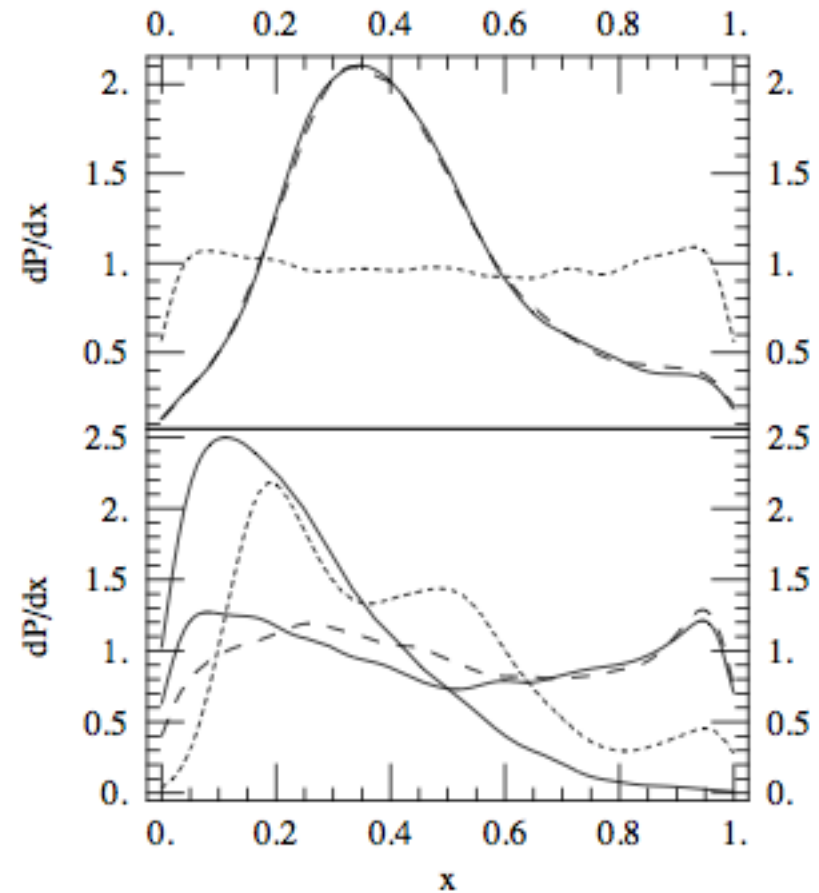


How do we combine these to make a statement about parameter distribution of the population being sampled?



# Astrophysics with GW searches

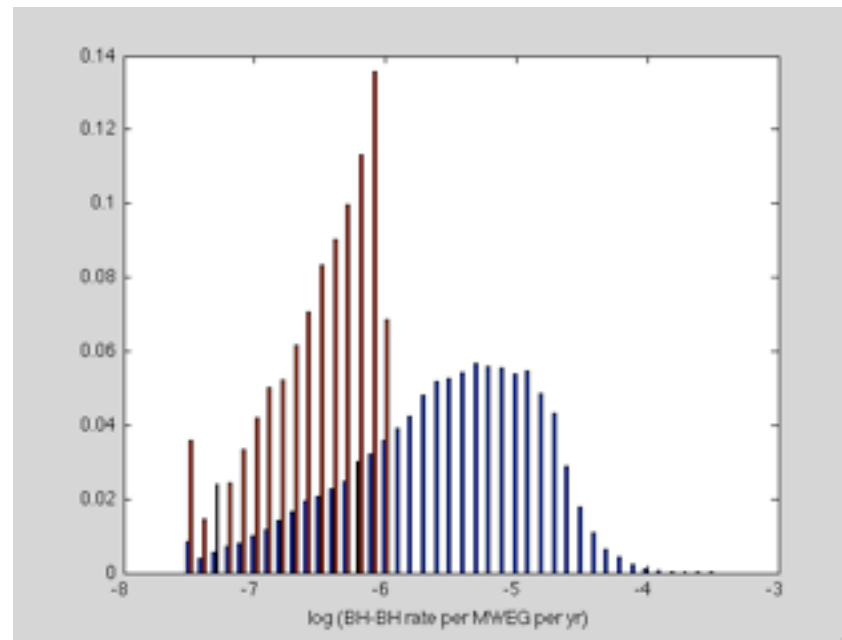
- Constraints on astrophysical parameters from existing electromagnetic observations [O'Shaughnessy et al., 2008 ApJ 672 479]:
- Observed GW event rates can be compared with models to determine important astrophysical parameters;
- Could match measured mass distributions, etc. to models (requires accurate parameter determination)
- As detector sensitivity improves, even upper limits can be useful in constraining parameter space for birth kicks, common-envelope efficiency, winds, etc.



# Astrophysics with GW searches

- Observed GW event rates can be compared with models to determine important astrophysical parameters;
- Could match measured mass distributions, etc. to models (requires accurate parameter determination)

## Constraints from upper limits - example



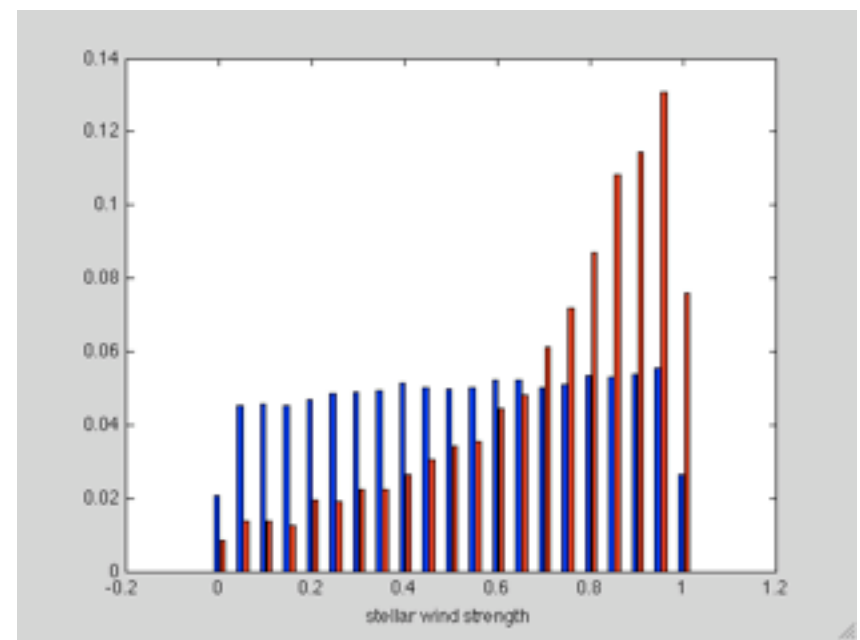
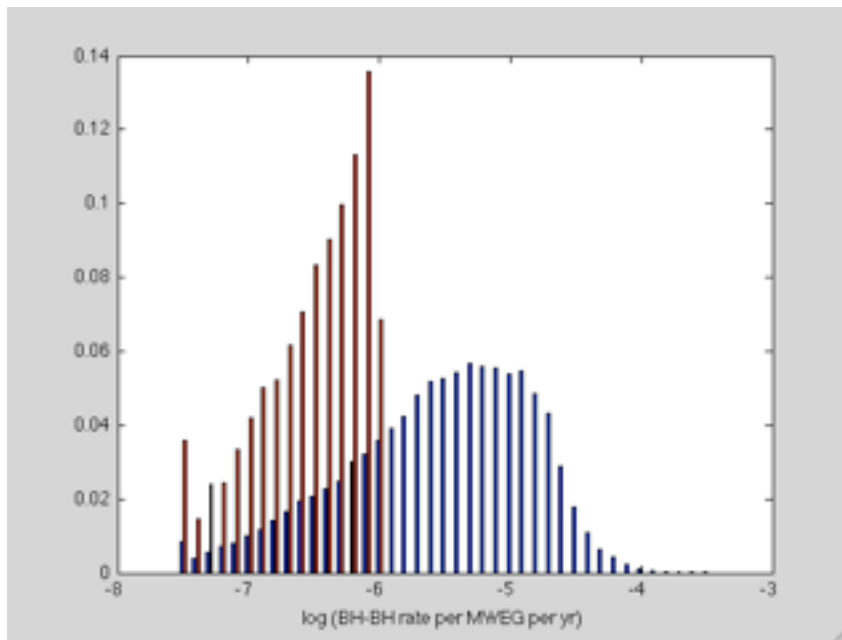
[IM & O'Shaughnessy, 2010, CQG 27 114007]



# Astrophysics with GW searches

- Observed GW event rates can be compared with models to determine important astrophysical parameters;
- Could match measured mass distributions, etc. to models (requires accurate parameter determination)

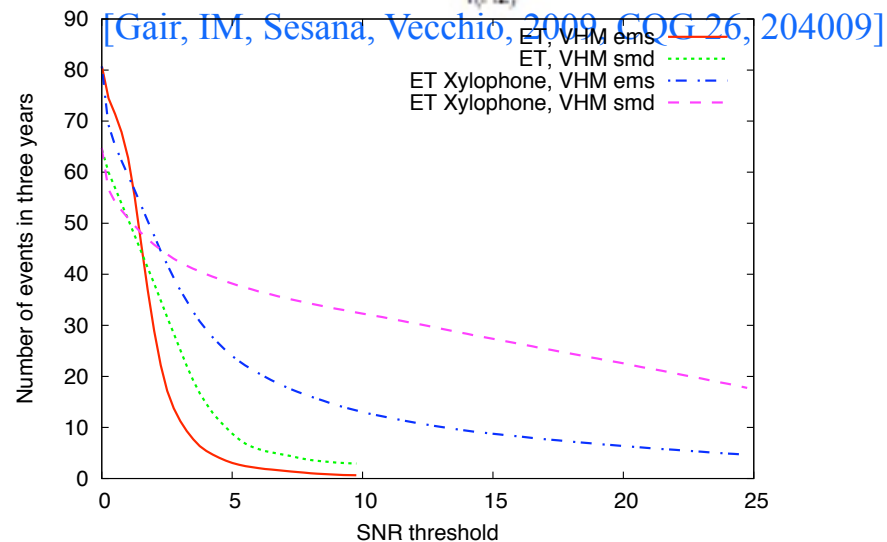
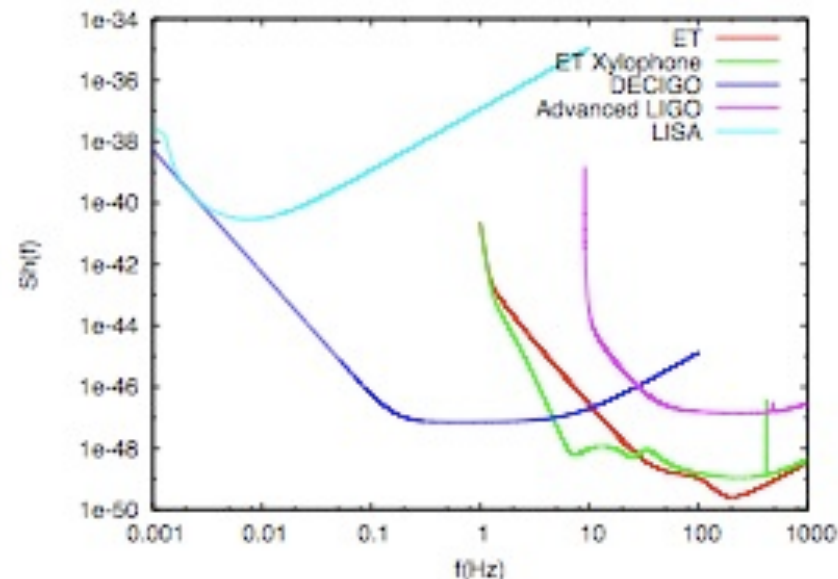
## Constraints from upper limits - example



[IM & O'Shaughnessy, 2010, CQG 27 114007]

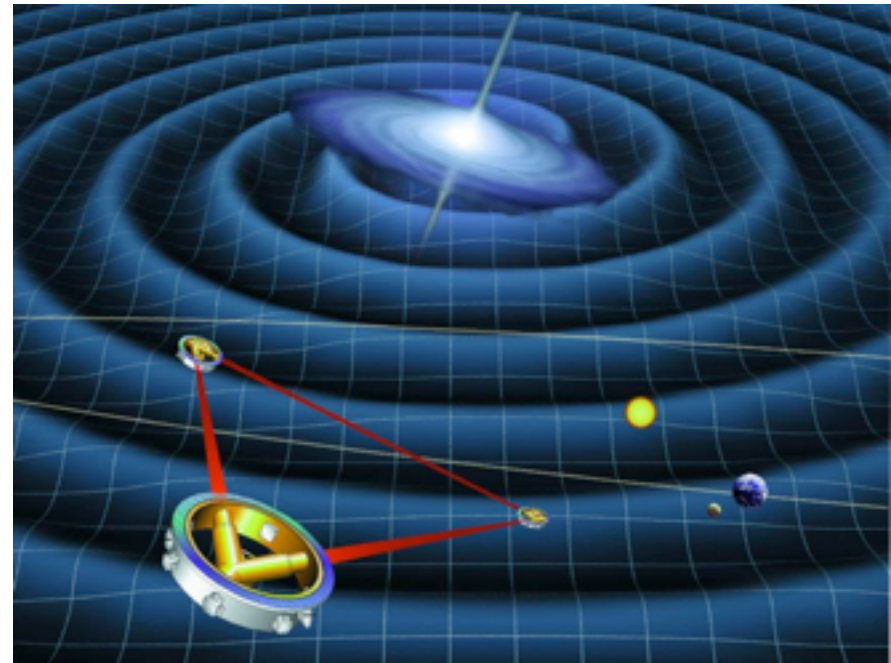
# The future: 3rd-generation detectors

- The Einstein Telescope:
  - » Underground, sensitive to 1 Hz
  - » Exciting science example: mergers of light seeds of massive black holes at high redshifts [Sesana, Gair, IM, Vecchio, 2009, ApJL 698 121]
  - » May detect thousands of IMBH-IMBH mergers and hundreds of IMRIs into IMBHs in globular clusters (see review [Gair, IM, Miller, Volonteri, 2010, arXiv:0907.5450])



# LISA Binary Sources: EMRIs

- LIGO sensitive @ a few hundred Hz
  - » NS-NS, NS-BH, BH-BH binaries
- LISA sensitive @ a few mHz
  - » massive black-hole binaries
  - » galactic white dwarf (and compact object) binaries
  - » extreme-mass-ratio inspirals of WDs/NSs/BHs into SMBHs
    - could see tens to hundreds to  $z \sim 1$  [e.g., Amaro-Seoane et al., 2007, CQG 24 R113]

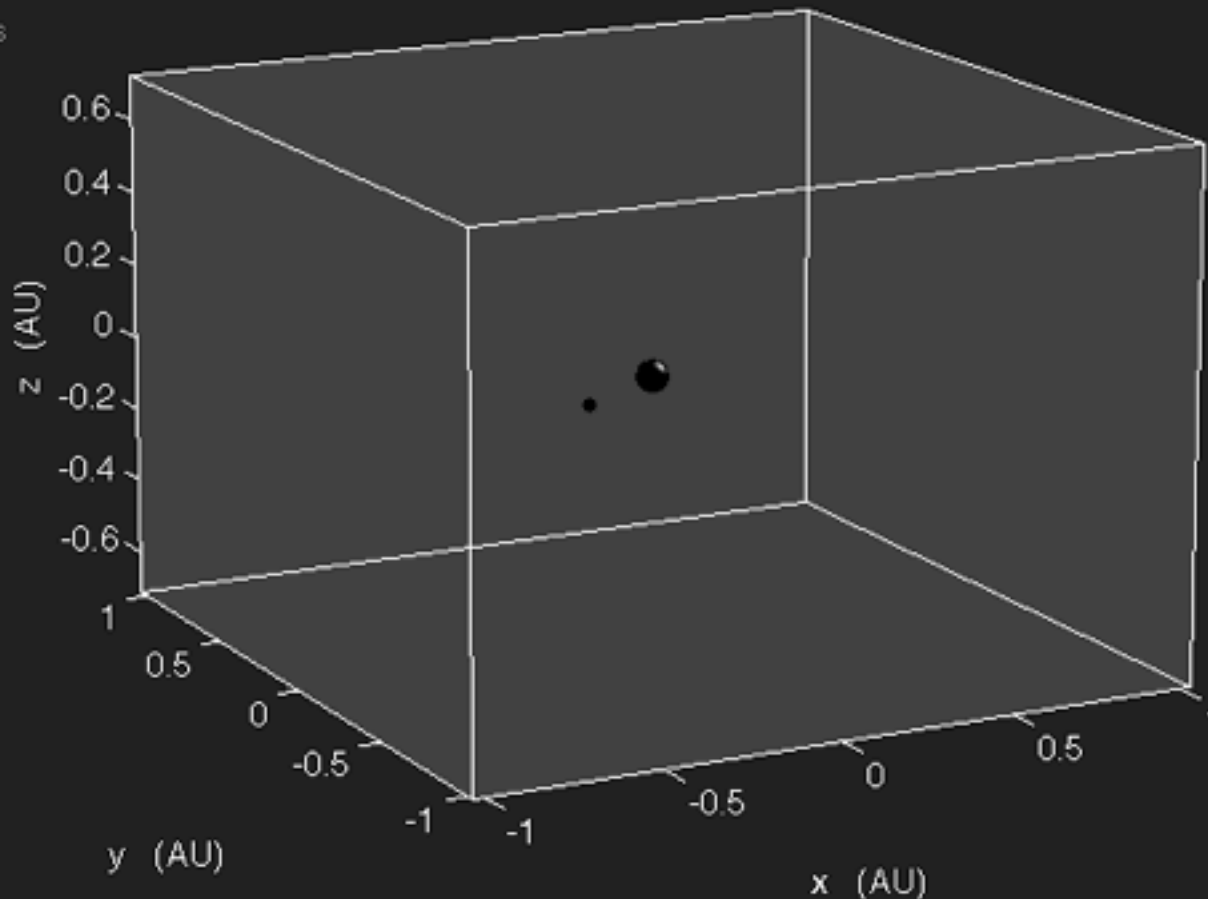


# Extreme Mass Ratio Inspirals

Large black hole:  
shown to scale  
3,000,000 solar masses  
90% maximal spin

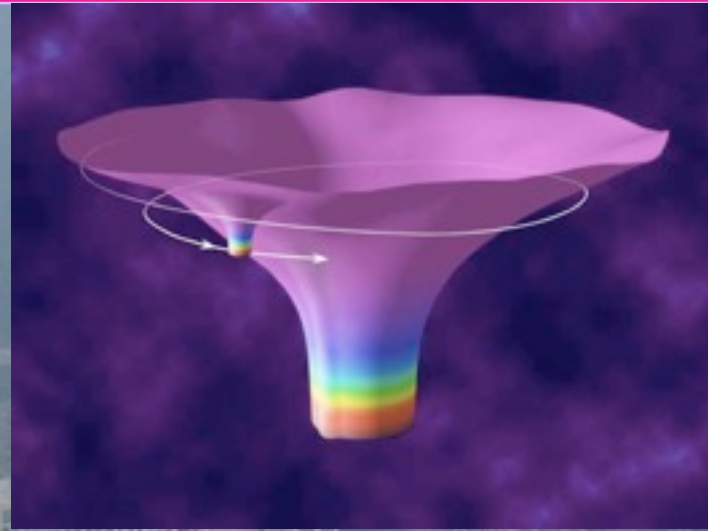
Small black hole:  
shown enlarged  
540 solar masses  
negligible spin

Trace duration:  
1 day



Steve Drasco  
Max Planck Institute  
for Gravitational Physics  
(Albert Einstein Institute)  
sdrasco@aei.mpg.de

# Exploring the spacetime...



# ... taking lots of pictures



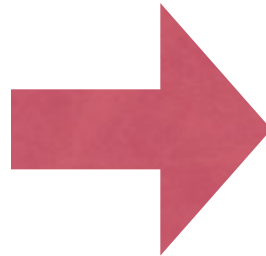
Birmingham: July 14, 2010

Tuesday, November 9, 2010

# Testing the “no-hair” theorem

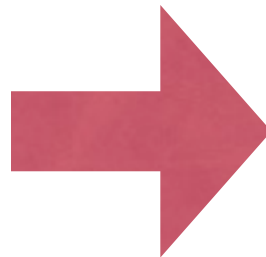
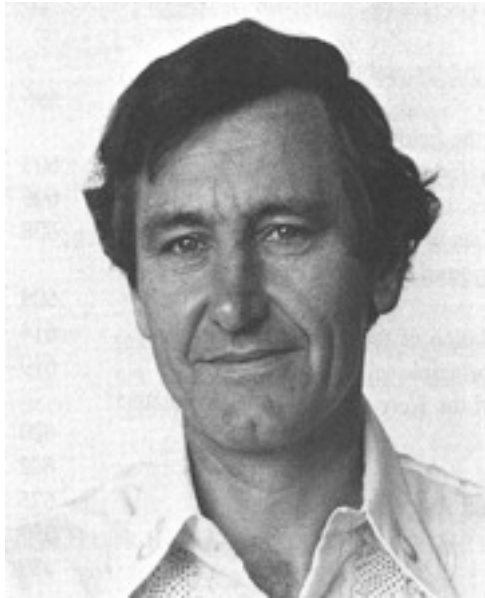
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# Testing the no-hair theorem



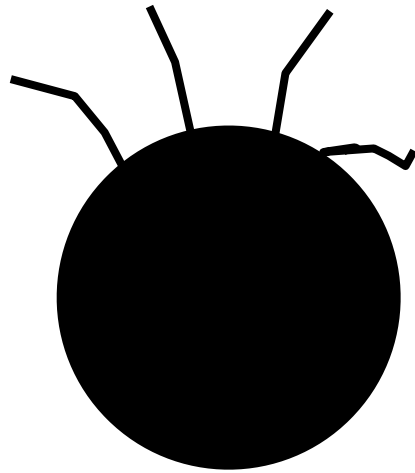


# Testing the no-hair theorem?



Stationary, vacuum, asymptotically flat spacetimes in which the singularity is fully enclosed by a horizon with no closed timelike curves outside the horizon are described by the Kerr metric

# Do black holes have hair?



$$M_n + iS_n \neq M(ia)^n$$

**Ryan's theorem** [1995]: GWs from nearly circular, nearly equatorial orbits in stationary, axisymmetric spacetimes encode all of the spacetime multipole moments... *in principle*

Manko-Novikov spacetime, an exact solution of Einstein's equations:

$$ds^2 = -f(\rho, z) (dt - \omega(\rho, z) d\phi)^2 + \frac{1}{f(\rho, z)} \left[ e^{2\gamma(\rho, z)} (d\rho^2 + dz^2) + \rho^2 d\phi^2 \right]$$

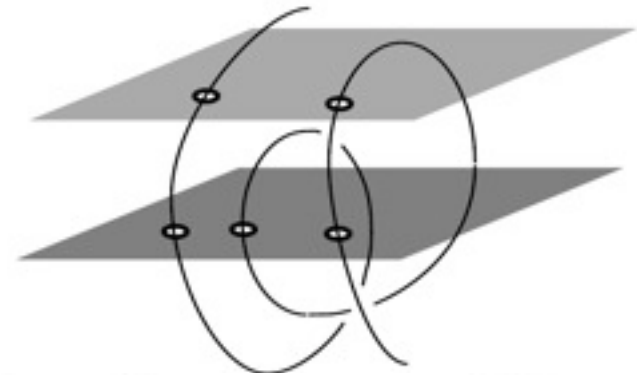
Search for observable imprints of a “bumpy” spacetime, such as deviations from the full set of isolating integrals (energy, angular momentum, Carter constant) in Kerr [Gair, Li, IM, 2009, PRD 77:024035]

# The emergence of chaos

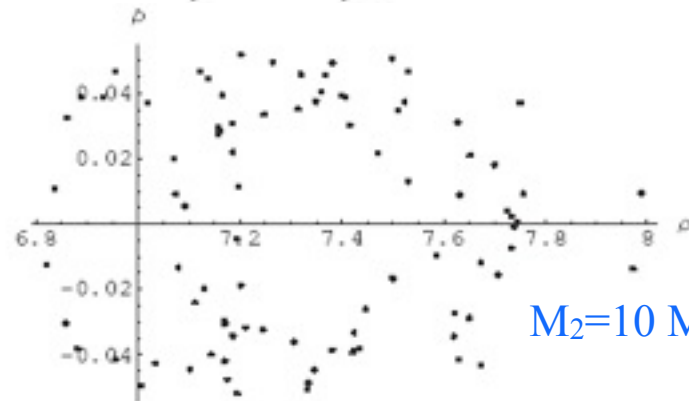
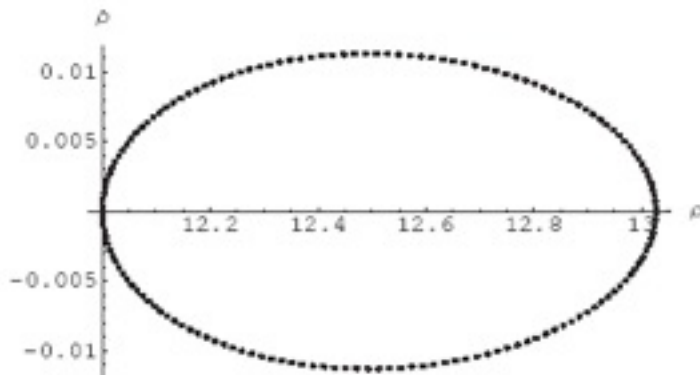
Solve the geodesic equation  
and study Poincare maps:

- Plot  $dp/dt$  vs.  $\rho$  for  $z=z_0$  crossings
- Phase space plots should be closed curves for all  $z_0$  iff there is a third isolating integral [Carter constant]

$$\frac{\partial^2 x^\alpha}{\partial \tau^2} = -\Gamma_{\beta\gamma}^\alpha \frac{\partial x^\beta}{\partial \tau} \frac{\partial x^\gamma}{\partial \tau}$$

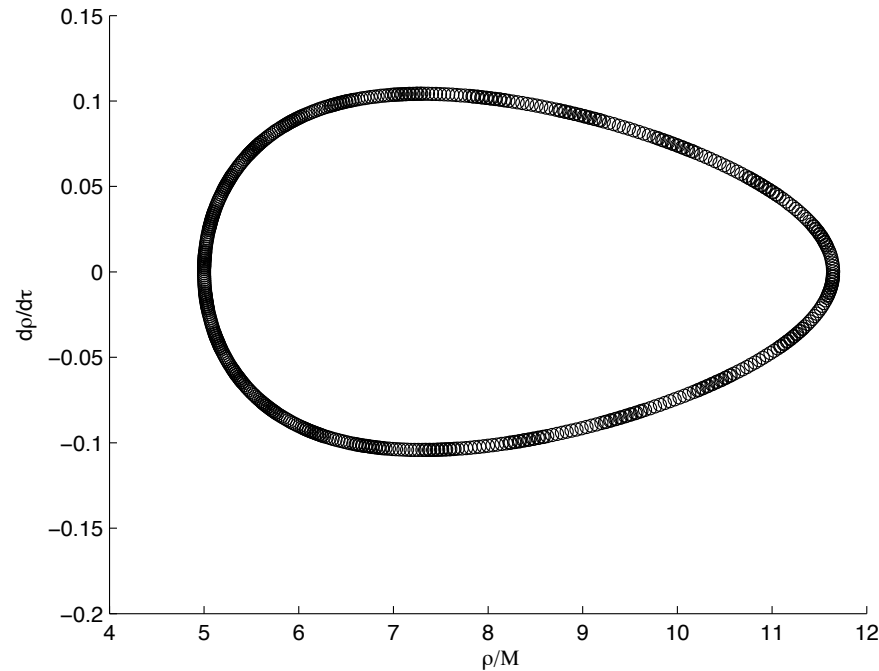


Newtonian+hexadecapole:  $V(r, \theta) = -\frac{M_0}{r} + \frac{M_2}{r^3} P_2(\cos \theta) + \frac{M_4}{r^5} P_4(\cos \theta)$



$M_2=10 M_0$ ;  $M_4=400 M_0$

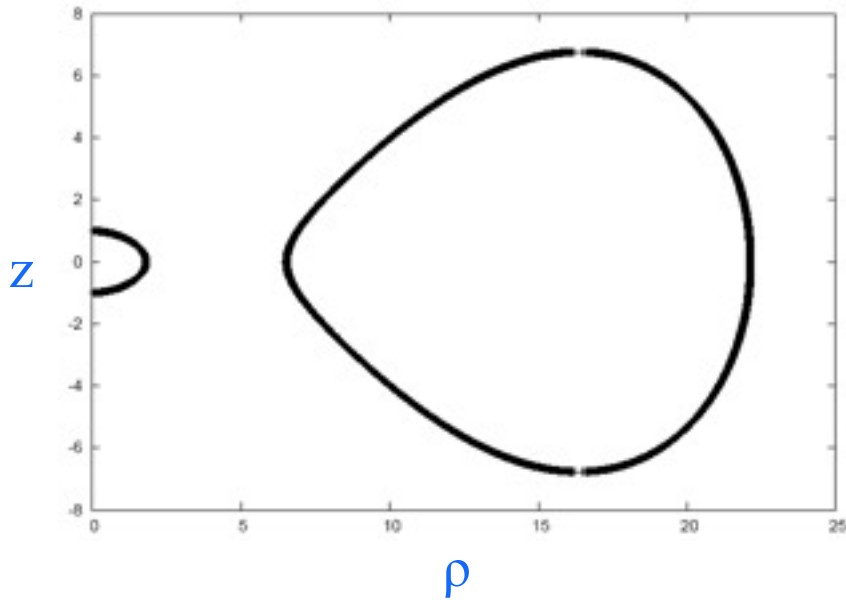
# All is regular in “bumpy” spacetimes



Poincare map for  $E=0.95$ ,  $L_z=-3$ ,  $a/M=0.9$ ,  $q=0.95$

# Or is it?...

Effective potential  $(\dot{\rho}^2 + \dot{z}^2) = V(E, L_z, \rho, z)$  defines allowed bound orbits



$E=0.95, L_z=-3, a/M=0.9, q=0$

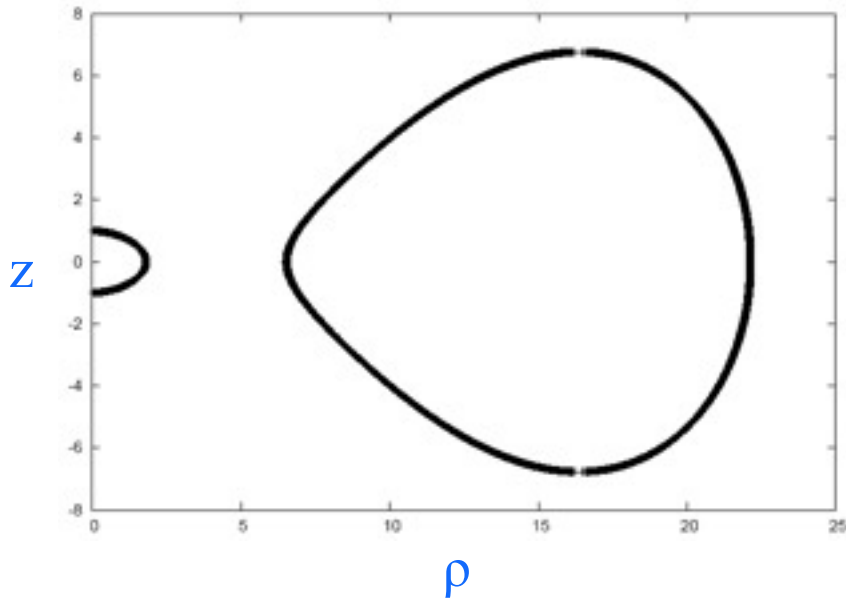
$z$

$\rho$

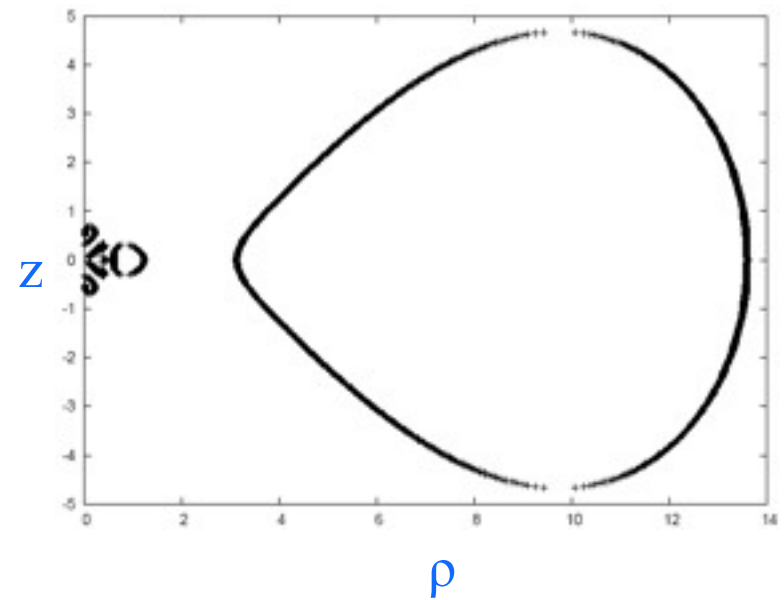
$E=0.95, L_z=-3, a/M=0.9, q=0.95$

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Effective potential  $(\dot{\rho}^2 + \dot{z}^2) = V(E, L_z, \rho, z)$  defines allowed bound orbits

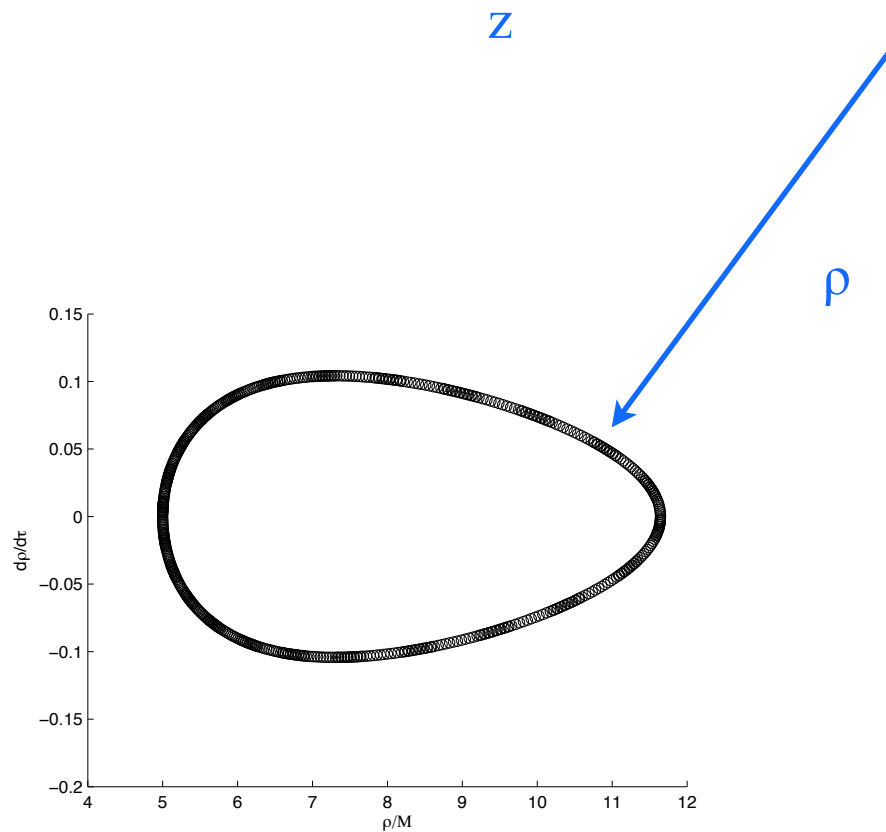


$E=0.95, L_z=-3, a/M=0.9, q=0$



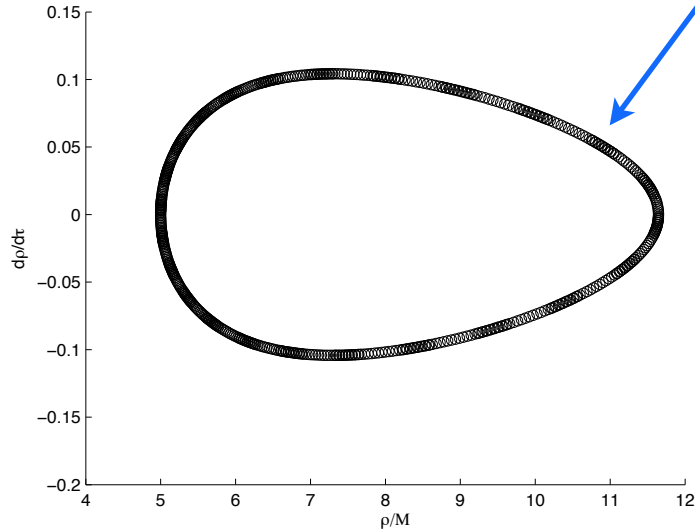
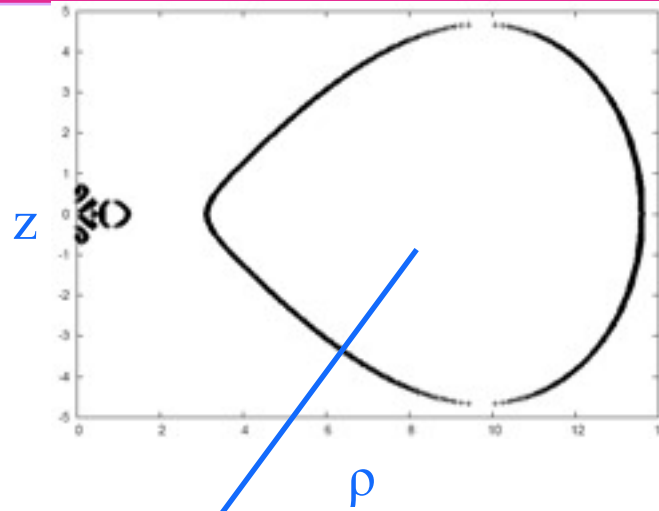
$E=0.95, L_z=-3, a/M=0.9, q=0.95$

# It's a mad, mad, mad, mad geodesic



$$E=0.95, L_z=-3, a/M=0.9, q=0.95$$

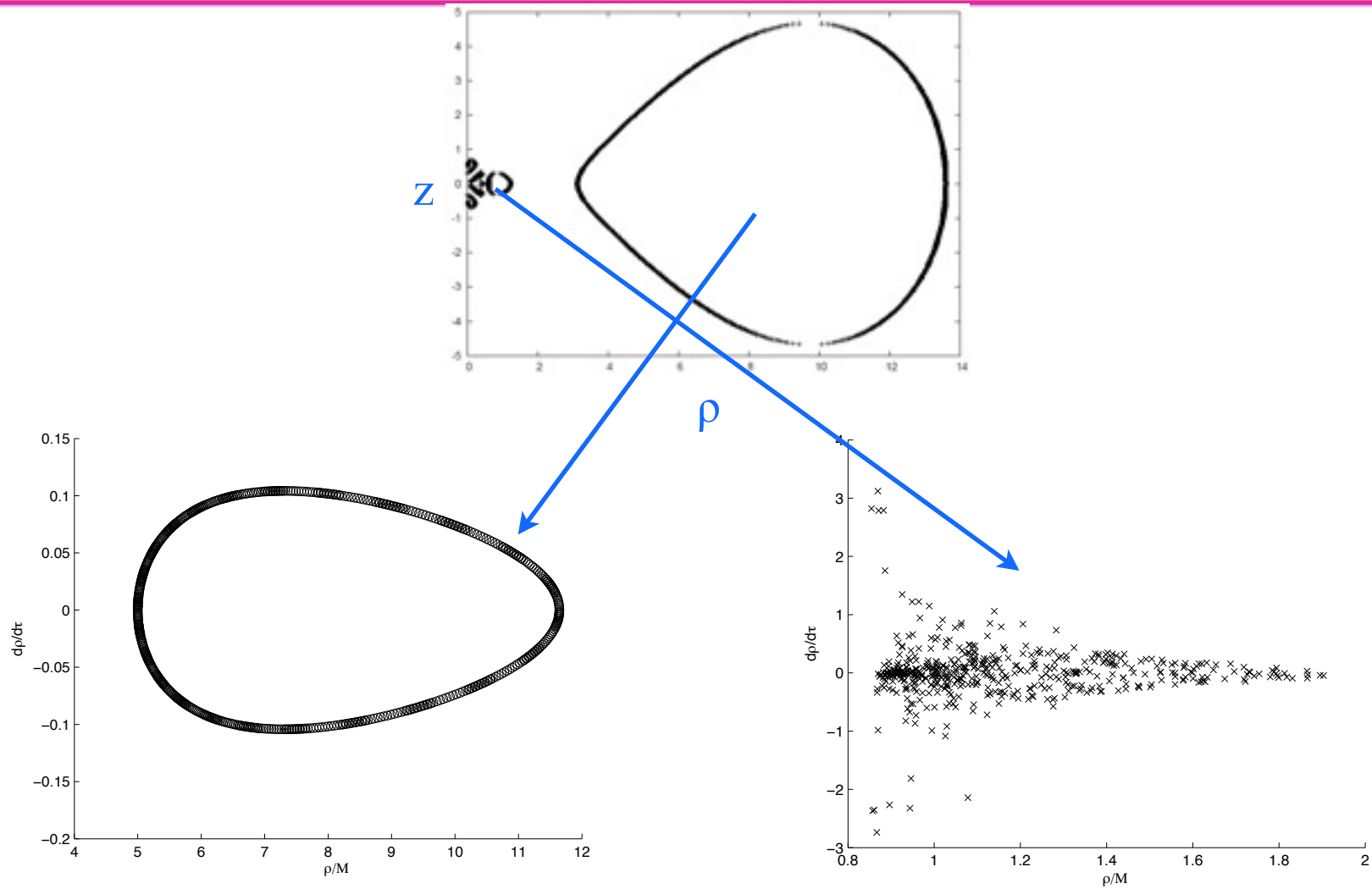
# It's a mad, mad, mad, mad geodesic



$$E=0.95, L_z=-3, a/M=0.9, q=0.95$$

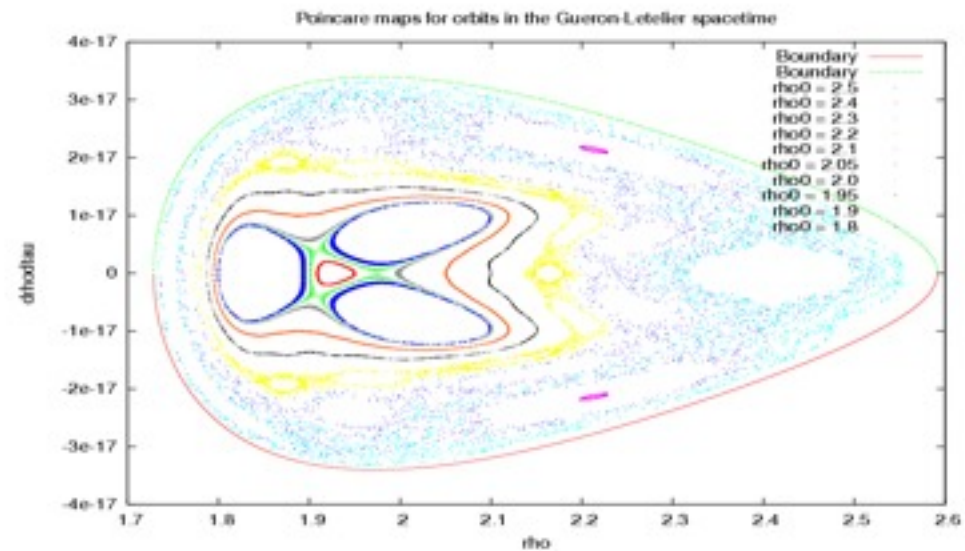
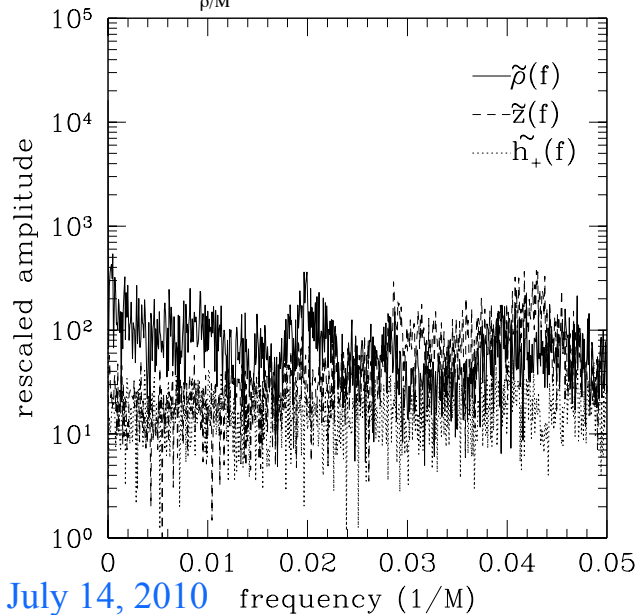
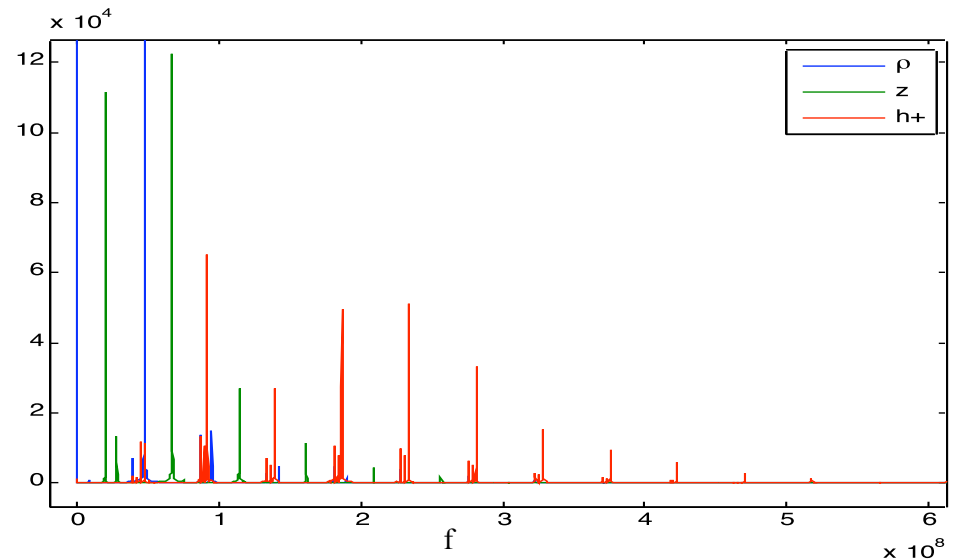
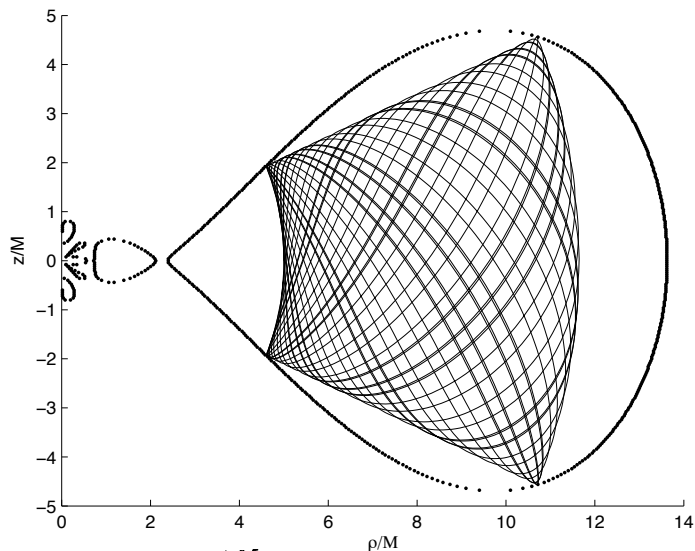


# It's a mad, mad, mad, mad geodesic



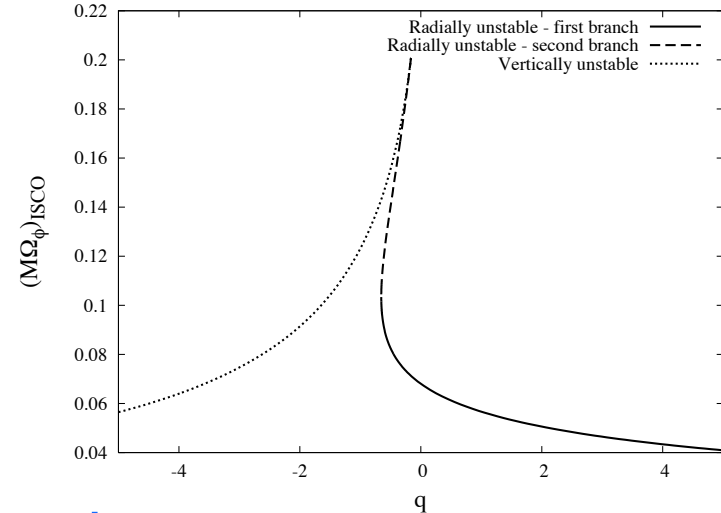
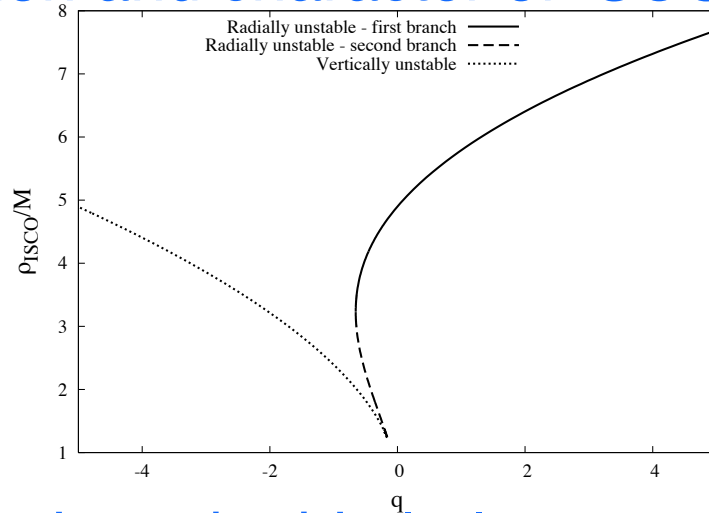
$$E=0.95, L_z=-3, a/M=0.9, q=0.95$$

# Order and Chaos, side by side

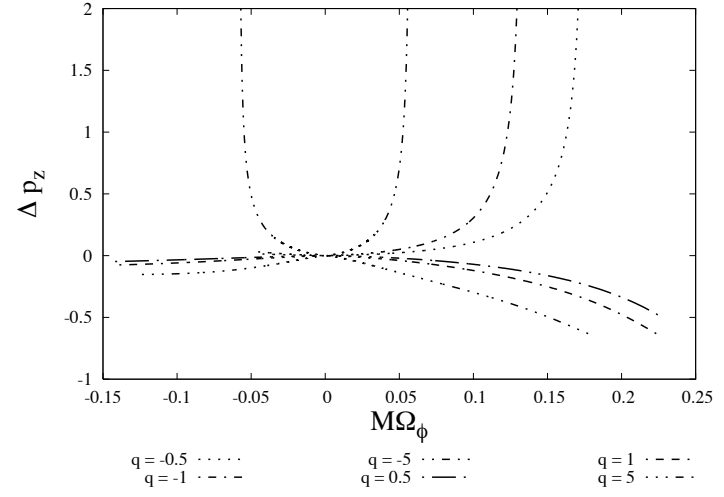
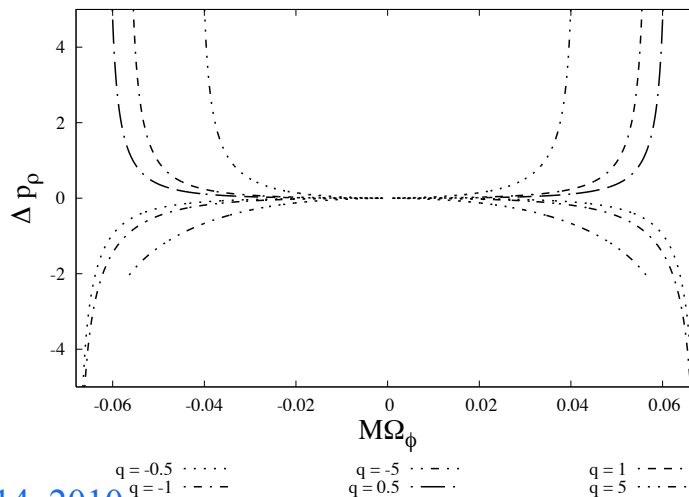


# Other signs of non-Kerr spacetimes

## Location and character of ISCO



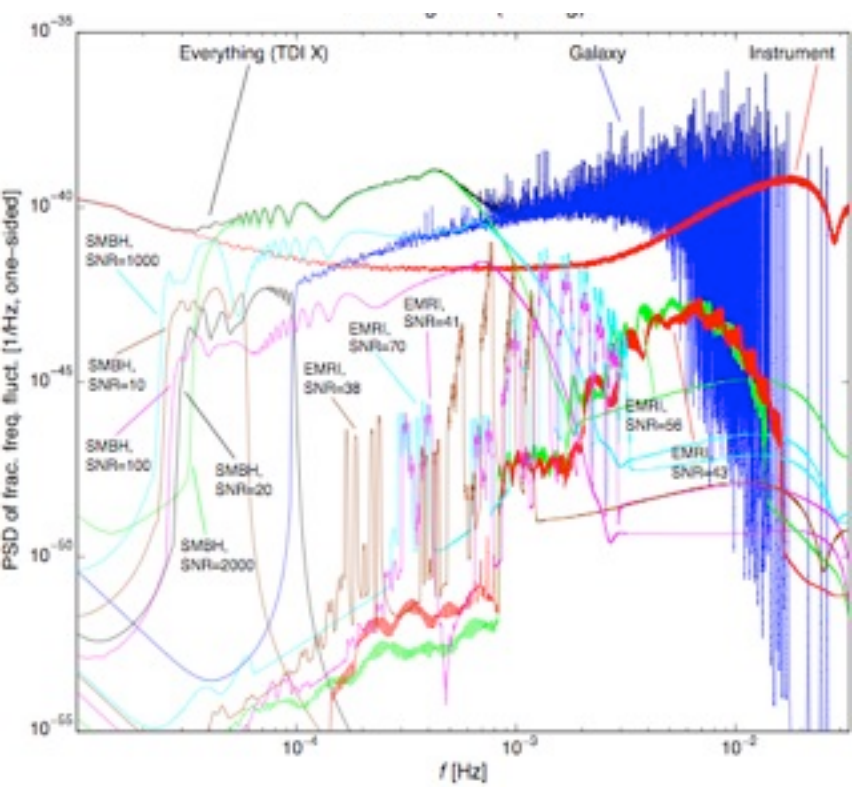
## Periapsis and orbital-plane precession



$q = -0.5$  .....       $q = -5$  .....       $q = 1$  .....  
 $q = -1$  .....       $q = 0.5$  .....       $q = 5$  .....

$q = -0.5$  .....       $q = -5$  .....       $q = 1$  .....  
 $q = -1$  .....       $q = 0.5$  .....       $q = 5$  .....

# LISA Data: An embarrassment of riches



[Arnaud et al., 2007, CQG 24 S551]

## What has already been accomplished?

	MLDC 1	MLDC 2	MLCD 1B	MLDC 3
GB	<ul style="list-style-type: none"> <li>Verification ✓</li> <li>Unknown, isolated ✓</li> <li>Unknown, interfering ✓</li> </ul>	<ul style="list-style-type: none"> <li>Galaxy of <math>3 \times 10^6</math> ✓</li> </ul>	<ul style="list-style-type: none"> <li>Verification ✓</li> <li>Unknown, isolated ✓</li> <li>Unknown, confused ✓</li> </ul>	<ul style="list-style-type: none"> <li>Galaxy of <math>6 \times 10^7</math> chirping ✓</li> </ul>
MBH	<ul style="list-style-type: none"> <li>Isolated ✓</li> </ul>	<ul style="list-style-type: none"> <li>4–6x, over Galaxy and EMRIs ✓</li> </ul>	<ul style="list-style-type: none"> <li>Isolated ✓</li> </ul>	<ul style="list-style-type: none"> <li>Over Galaxy spinning, precessing ✓</li> </ul>
EMRI		<ul style="list-style-type: none"> <li>Isolated ✓</li> <li>4–6x, over Galaxy and SMBHs</li> </ul>	<ul style="list-style-type: none"> <li>Isolated ✓</li> </ul>	<ul style="list-style-type: none"> <li>5 together, weaker ✓</li> </ul>
New				<ul style="list-style-type: none"> <li>Cosmic string cusp bursts ✓</li> <li>Cosmological background ✓</li> </ul>

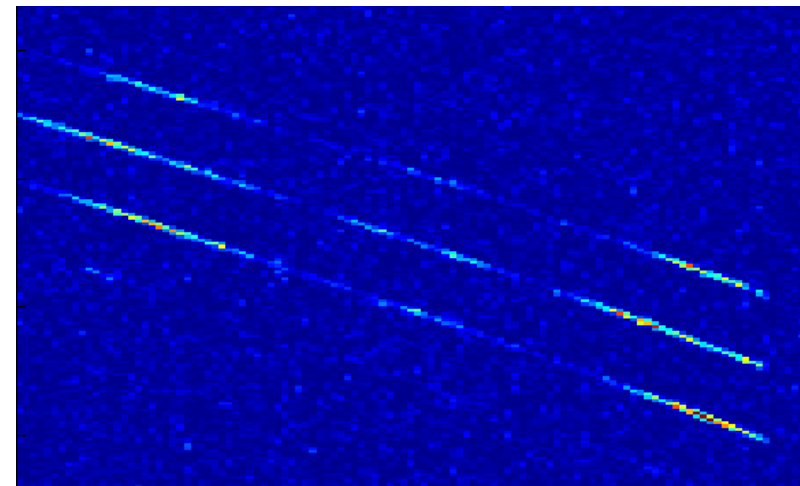
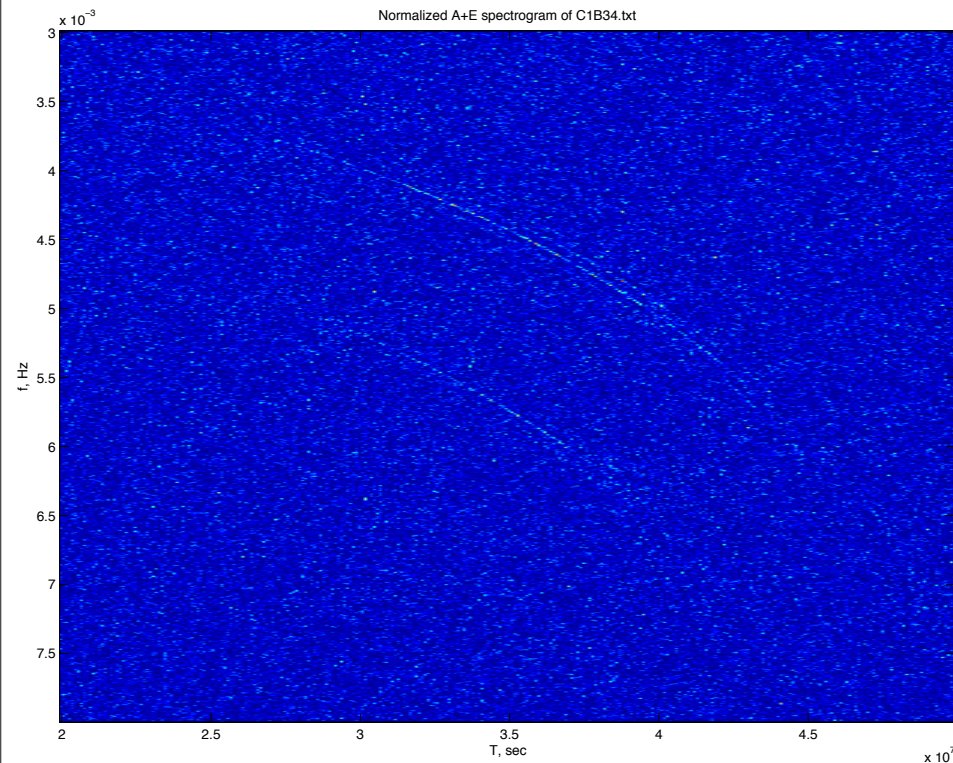
Table by M. Vallisneri

## Mock LISA Data Challenges; LISA Parameter-Estimation task force

# EMRI detection and analysis

$$h(t) = h(M_1, M_2, \vec{S}_1, \vec{S}_2, \theta, \phi, D_L, e, \dots; t) \quad 17 \text{ parameters}$$

Need innovative search techniques to separate many overlapping signals: Markov-Chain Monte Carlo, MultiNest, time-frequency searches

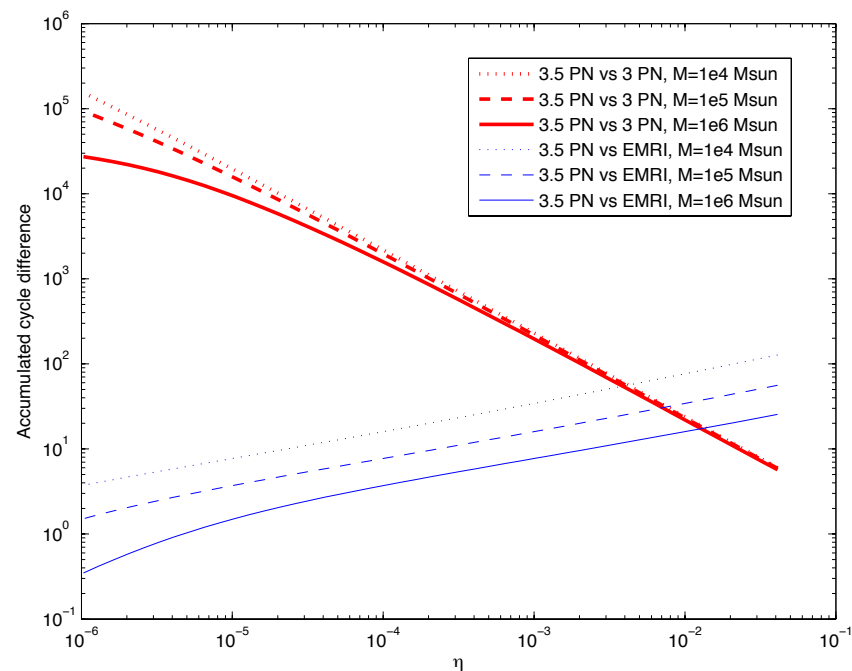
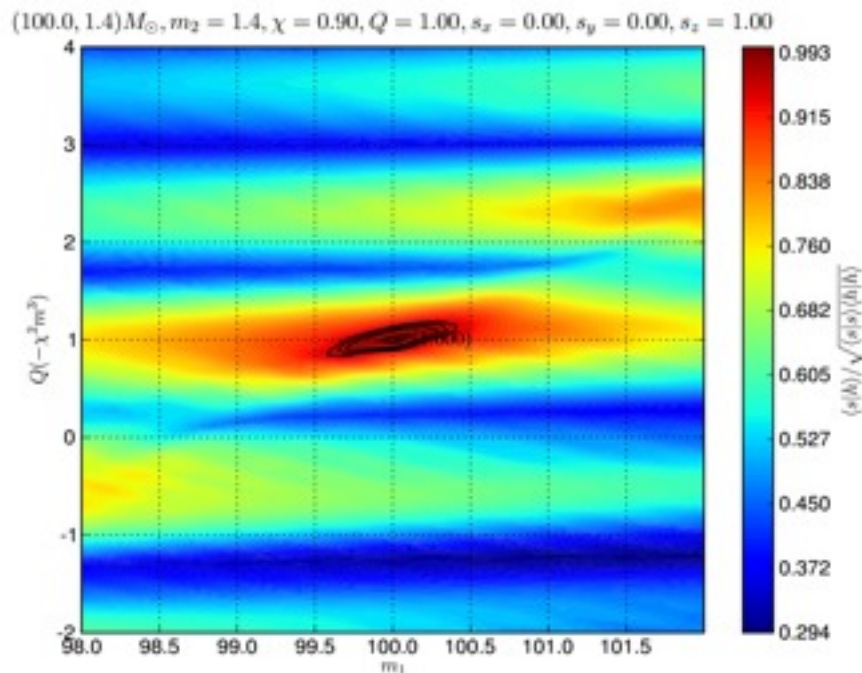


[Gair, IM, Wen, 2008, CQG 25 184031]

# Intermediate-mass-ratio Inspirals

Can measure mass quadrupole moment to around 20% of Kerr value with Advanced LIGO [Brown et al., PRL 99, 201102]

Waveforms are a problem: both post-Newtonian and self-force waveforms currently fail in the intermediate regime [IM and Gair, 2005, PRD 72 084025]



# Summary

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- Advanced LIGO is likely to see NS-NS, NS-BH, BH-BH coalescences; tens or more coalescences may be seen according to some models, including dynamical formation
- GW detections and upper limits for compact-object coalescences will allow us to constrain astrophysical parameters through comparisons with model predictions
- Extreme- or intermediate- mass-ratio inspirals can serve as precise tests of General Relativity