Can we detect intermediate-mass-ratio inspirals?

NORTHWESTERN UNIVERSITY

Ilya Mandel (Northwestern), Jonathan R. Gair (Cambridge)

Abstract

Gravitational waves emitted during intermediate-mass-ratio inspirals (IMRIs) of intermediate-mass black holes (IMBHs) into supermassive black holes could represent a very interesting source for LISA. Similarly, IMRIs of stellar-mass compact objects into IMBHs could be detectable by Advanced LIGO. At present, however, it is not clear what waveforms could be used for IMRI detection, since the post-Newtonian approximation breaks down as an IMRI approaches the innermost stable circular orbit, and the perturbative solution is only known to the lowest order in the mass ratio. We study the expected mismatches between approximate and true waveforms, and the choice of the best available waveform as a function of the mass ratio and the total mass of the system. We also discuss the significance of the spin of the smaller body and the need for its inclusion in the waveforms. [based on arXiv:0811.0138]

Introduction

There is a growing body of evidence for the existence of intermediate-mass black holes (IMBHs, between one hundred & a few thousand solar masses). Although the evidence is still inconclusive, mergers involving IMBHs could make for very interesting sources of gravitational waves (GWs) detectable by ground-based (Advanced LIGO & Virgo) or space-based (LISA) GW detectors. Mergers involving IMBHs could lead to GW events in several ways:

1. Compact object + IMBH (LIGO IMRI event) [Mandel et al., 2008]
2. IMBH + IMBH (LISA, comparable mass) [Fregeau et al., 2006]
3. IMBH + MBH (LISA IMRI event) [Miller, 2005]

The first and third of these could be intermediate-mass-ratio inspirals (IMRIs), which fall between extreme-mass-ratio inspirals (EMRIs) and comparable-mass binaries for the purposes of data analysis. While EMRIs are well-described by perturbative waveforms that are expansions in the mass ratio $\eta = mM/(M+m)^2$, and comparable-mass inspirals are approximated by the post-Newtonian (pN) expansion in v/c, IMRIs fall somewhere in the middle between these approximations. Is either of them accurate enough for IMRI detection and parameter estimation?

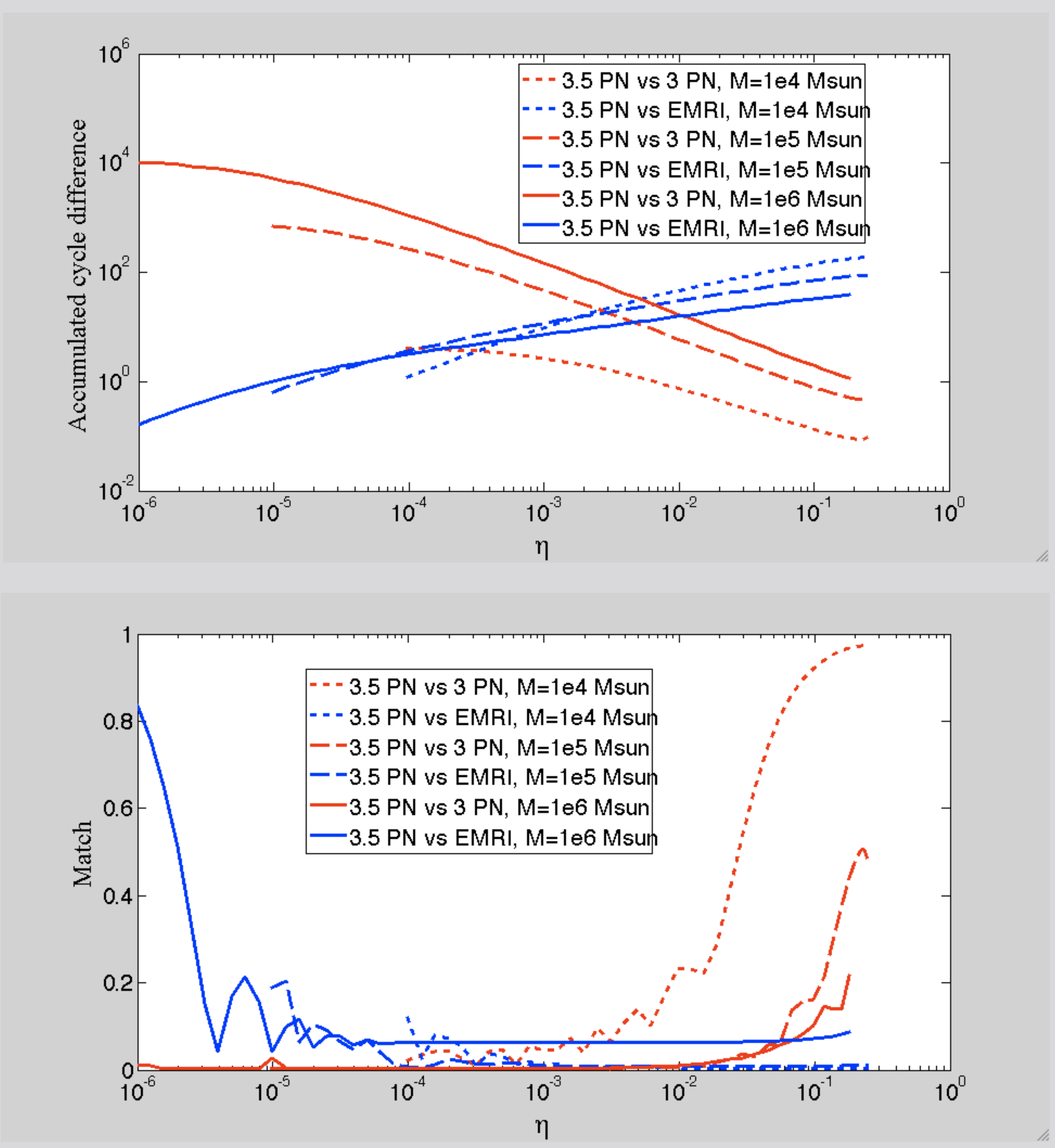

Figure 1 – The difference between approximate and "true" waveforms over the last year of inspiral before ISCO (or during the time it takes the GW frequency to increase from 0.01 mHz to the ISCO, if less), as a function of the symmetric mass ratio η. The top figure shows the difference in cycles $(\Psi_{true}-\Psi_{approx})/(2\pi)$. The bottom figure shows the match $M(h_{true}, h_{approx})$. Red curves correspond to approx=3pN; blue curves correspond to approx=EMRI. Solid, dashed, and dotted curves refer to inspirals into Schwarzschild black holes of mass 10^6, 10^5, and 10^4 solar masses, respectively.

Comparison of pN and EMRI waveform families

We don't know the "true" IMRI waveforms: numerical relativity can not yet handle such unequal mass ratios. Therefore, we attempt to gauge the accuracy of the pN and EMRI waveform families in the intermediate mass regime as follows. We treat the 3.5 pN waveform [Blanchet, 2006] as if it were the correct theoretical waveform. We then compare it with either:

a. the 3 pN waveform to estimate the errors in the pN expansion; or

b. the 3.5 pN waveform expanded to the lowest order in η ("EMRI-fied" 3.5 pN) to estimate the errors in the EMRI waveform, which is currently known only to the lowest order in η.

In the top panel of Figure 1, we show the number of excess cycles accumulated between the 3 pN (red) or EMRI-fied 3.5 pN (blue) waveforms and the standard 3.5 pN waveform during the last year of a LISA inspiral, or during the time it takes for the GW frequency to change from 0.01 mHz to the innermost stable circular orbit (ISCO), if less. In the bottom panel of Figure 1, we show the match factor, or the overlap between the normalized waveforms in LISA noise $S_n(f)$:

$$\langle a|b\rangle = 4\mathcal{R}\int_0^\infty \frac{a(f)b^*(f)}{S_n(|f|)}\mathrm{d}f$$

We also consider the effects of the small-body spin by comparing 3.5 pN waveforms with and without a (maximal) spin-spin coupling term [Poisson & Will, 1995]. We plot the match between waveforms that include and omit this coupling term in Figure 2.

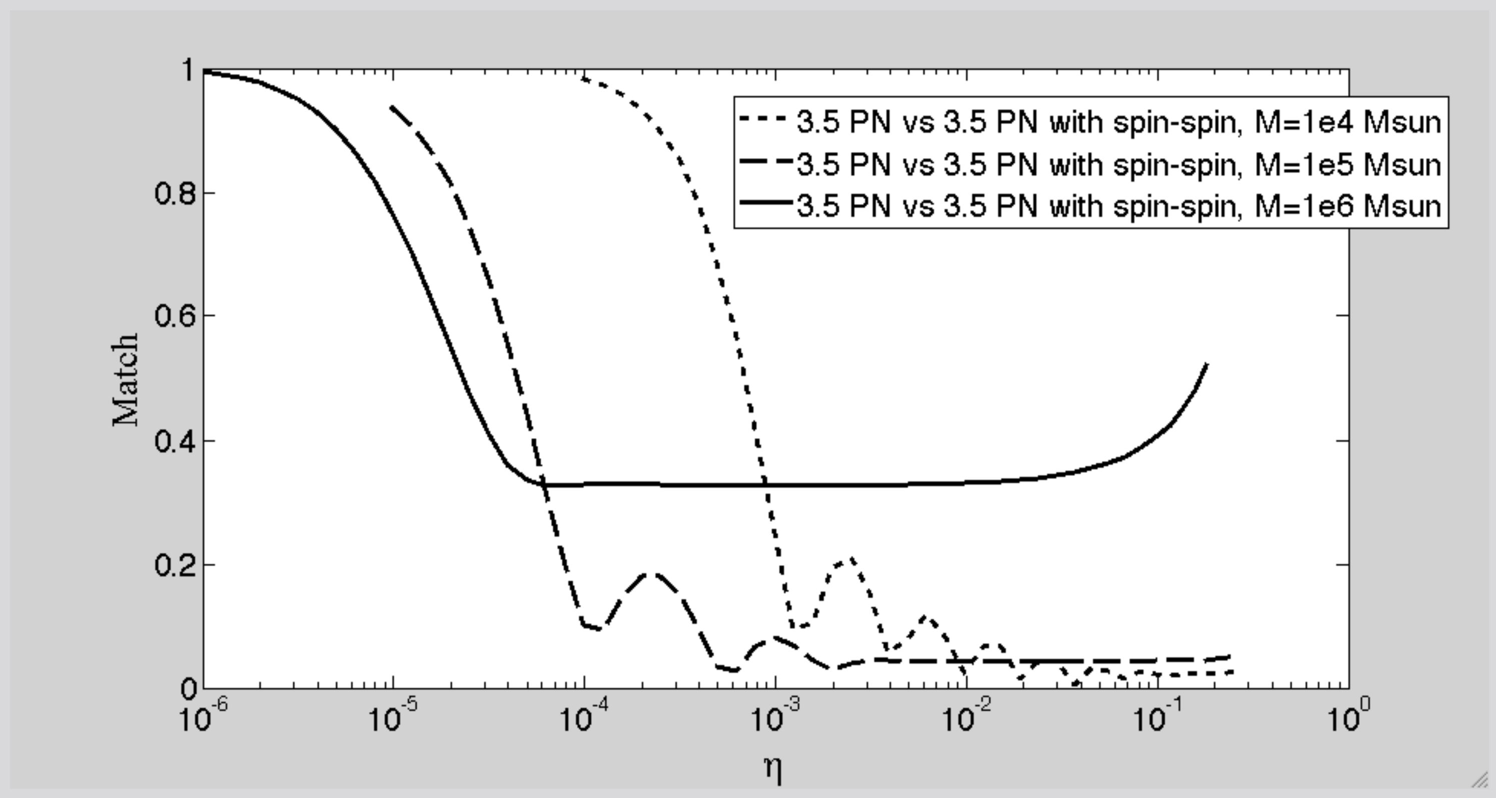

Figure 2 – The match between 3.5 pN waveforms with and without the spin-spin coupling term over the last year of inspiral before ISCO (or during the time it takes the GW frequency to increase from 0.01 mHz to the ISCO, if less), as a function of the symmetric mass ratio η: $M(h_{true}, h_{spin})$. Solid, dashed, and dotted curves refer to inspirals into black holes of mass 10^6, 10^5, and 10^4 solar masses, respectively.

Discussion and Future Prospects

We find, as expected, that the pN approximation performs well at high η, when the binary spends only a few cycles near ISCO, so those last cycles when v/c is high and the pN approximation fails do not contribute significantly. Meanwhile, the EMRI waveforms are more faithful at low η: the EMRI approximation error in $\dot{f}$ scales as $O(\eta^2 T^2)$, where T is the signal duration, so as long as T is limited by the LISA observation time, the EMRI waveform performs well. However, for higher values of η, the signal duration is bandwidth-limited and T scales as η^{-1}, so the EMRI waveform fails. Thus, our most significant result is that *for a wide range of intermediate mass ratios, neither the post-Newtonian nor the EMRI approximation is likely to be faithful.* We also note that spin-spin coupling is not as irrelevant as generally supposed even for significantly unequal mass ratios.

This is merely a first step in the study of IMRI waveforms, and a number of issues remain to be explored; these include the following questions:

- Do either pN or EMRI waveforms constitute an "effective" template family in the sense of a dense coverage of the space of true IMRI waveforms?
- Can we build faithful hybrid waveforms by combining pN and EMRI expansions?
- What are the systematic vs statistical errors of parameter estimation with approximate waveform families in the IMRI regime?