Prospects in Gravitational-Wave Astronomy

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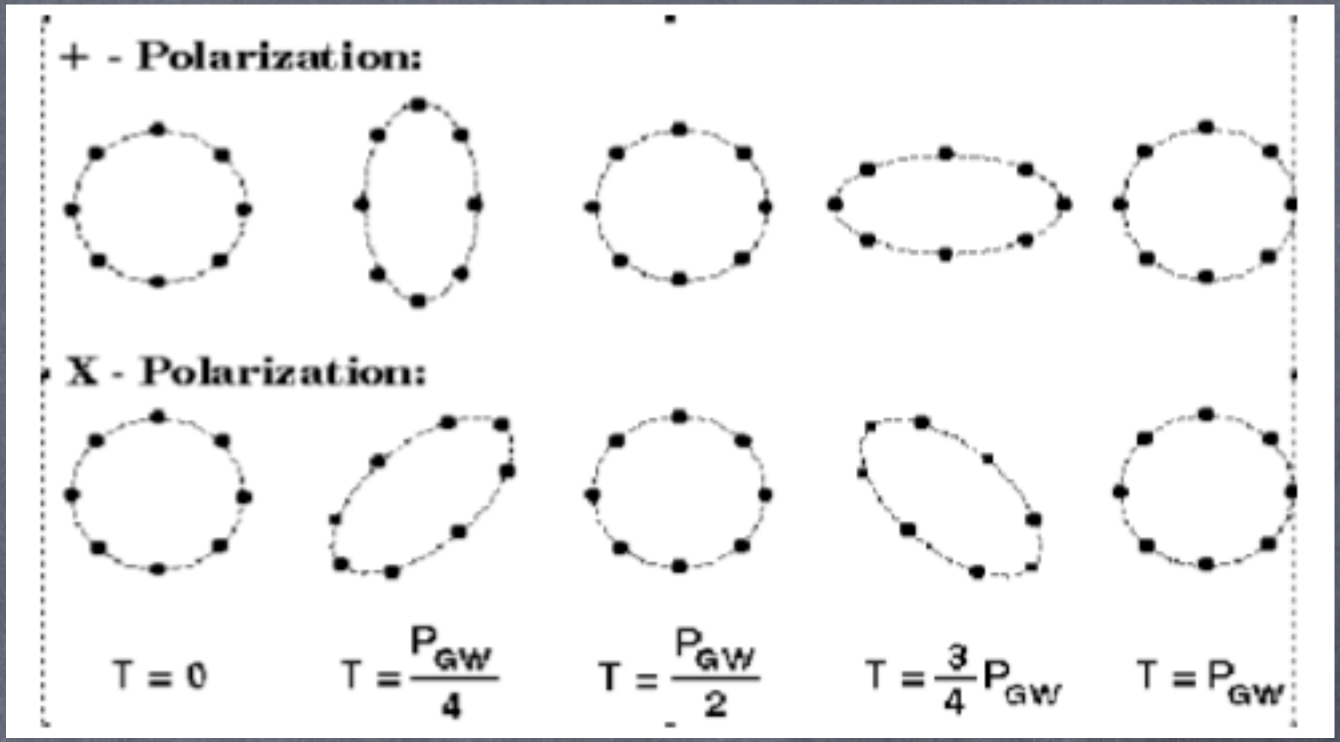
Gravitational Waves

- Ripples in spacetime:

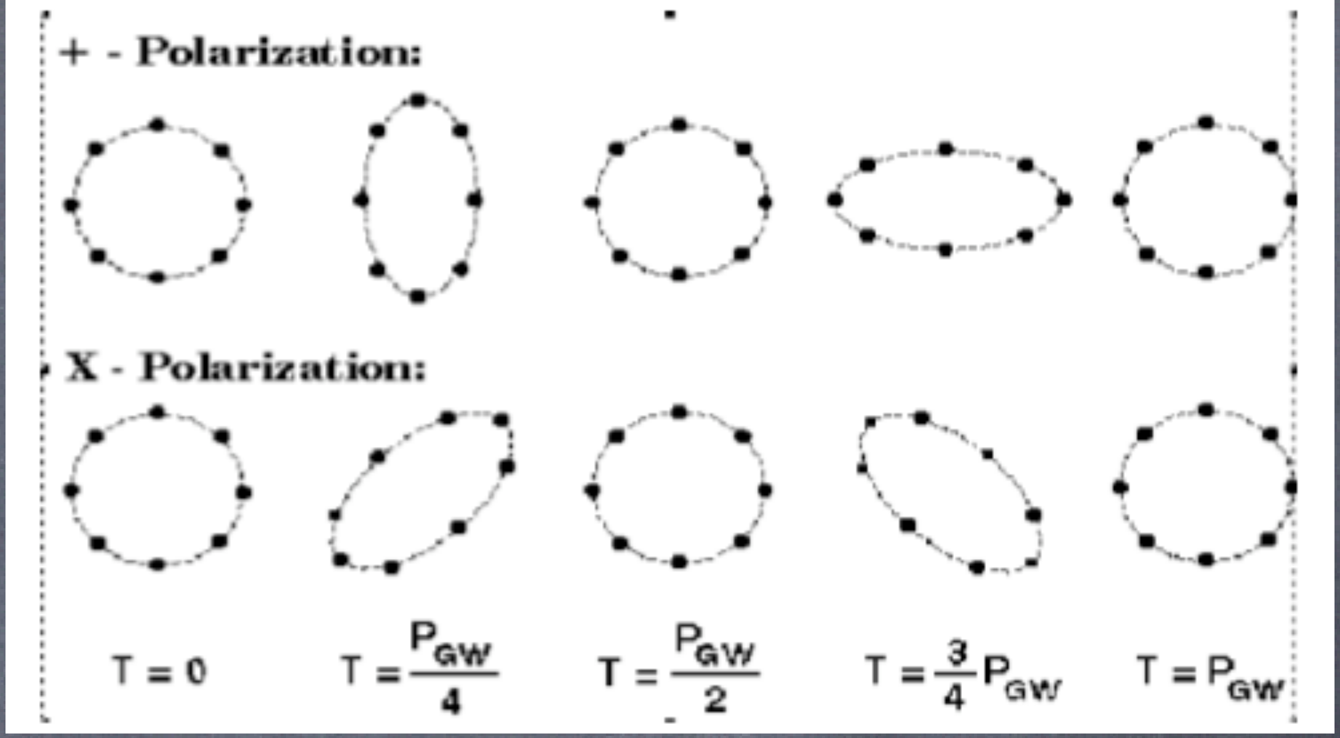


- Caused by time-varying mass quadrupole moment
- Indirectly detected by Hulse & Taylor [binary pulsar]
- Huge amounts of energy released: 5% of mass-energy of a supermassive black hole binary is more than the electromagnetic radiation emitted from an entire galaxy over the age of the universe!

Gravitational Waves

- Ripples in spacetime:

  Inspiral sound borrowed from Scott Hughes



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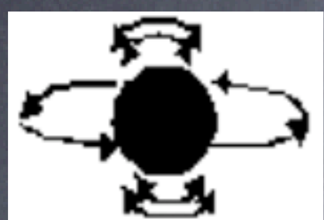
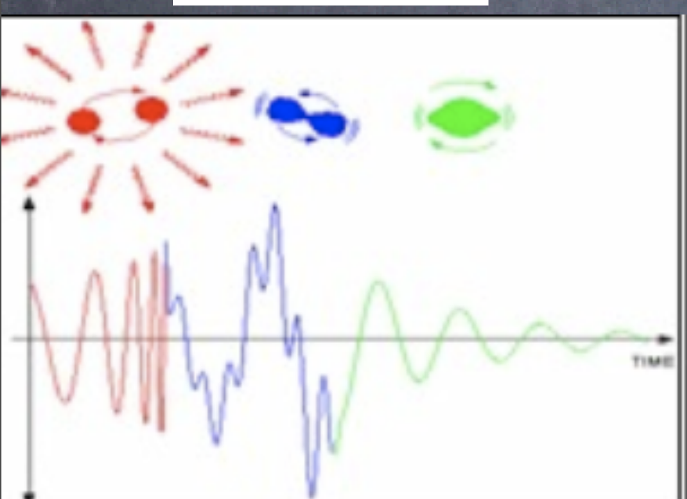
- Huge amounts of energy released: 5% of mass-energy of a supermassive black hole binary is more than the electromagnetic radiation emitted from an entire galaxy over the age of the universe!

Types of GW sources





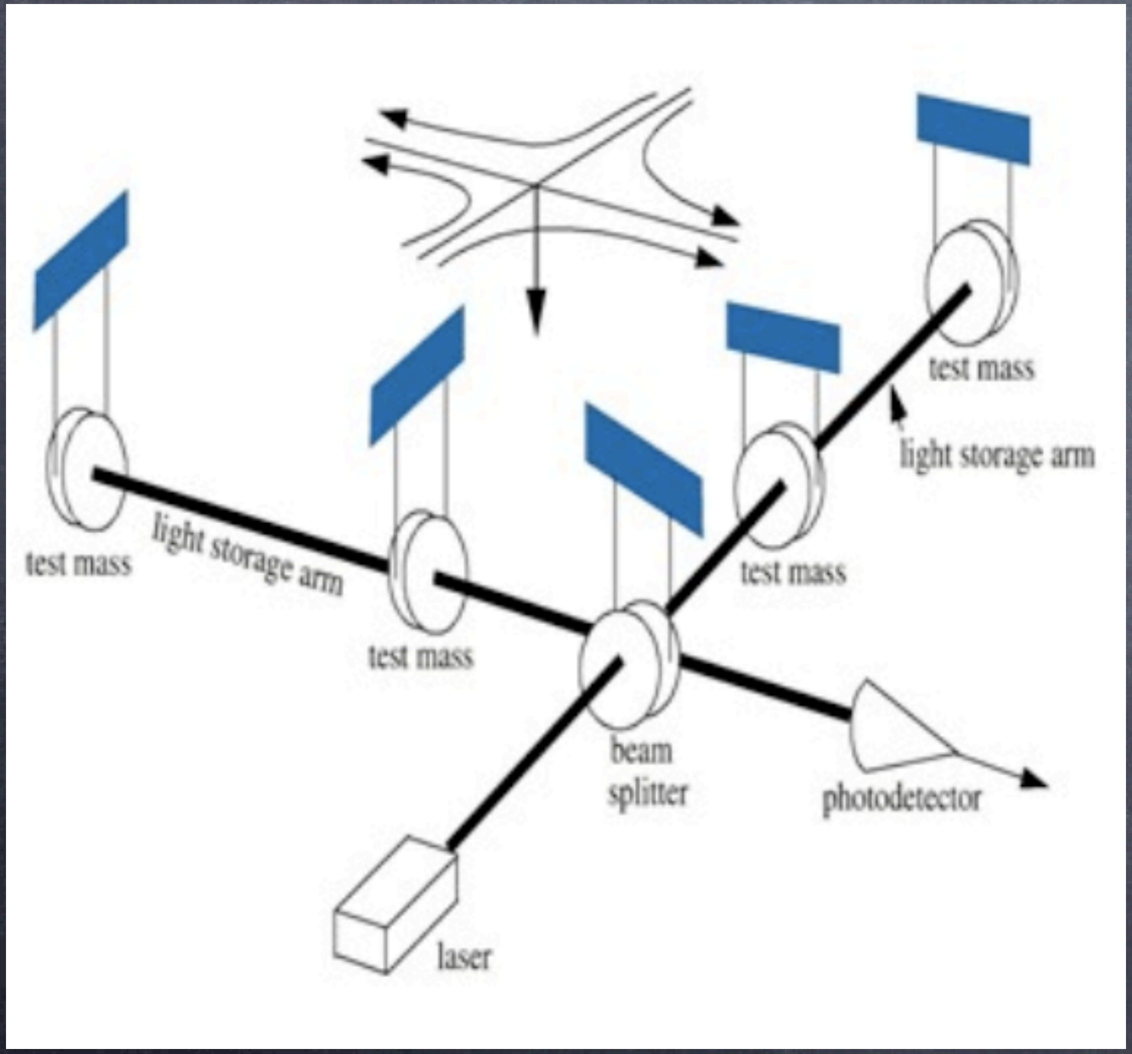
- Continuous sources [sources with a slowly evolving frequency]: e.g., non-axisymmetric neutron stars, slowly evolving binaries
- Coalescence sources: compact object binaries
- Burst events [unmodeled waveforms]: e.g., asymmetric SN collapse, cosmic string cusps
- Stochastic GW background [early universe]
- ??? [expect the unexpected]

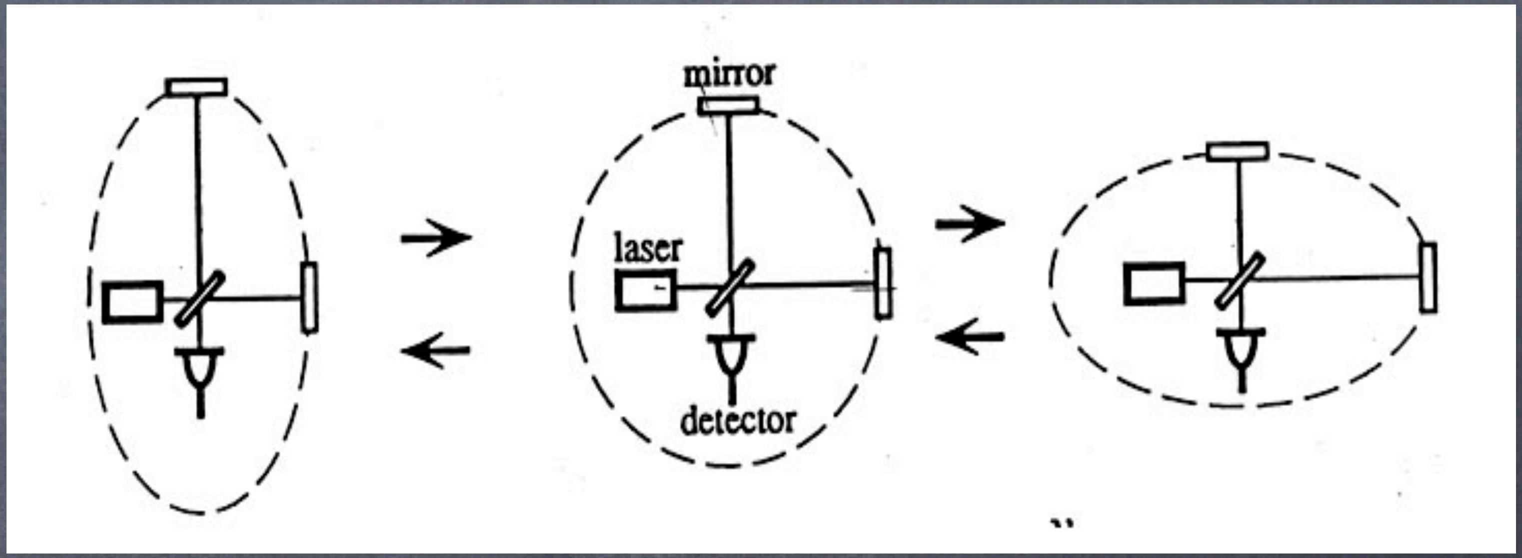
Why do we want to see GWs?

- Probing stellar dynamics and evolution via stellar-mass compact-object binary measurements (NS-NS, NS-BH, BH-BH)
- Studying galactic structure formation by measuring mass and spin distributions of massive black holes (MBHs); measuring high-redshift mergers of MBH progenitors; understanding galactic mergers (e.g., kicks)
- Direct probes of early-universe cosmology by measuring GWs emitted soon after the Big Bang
- Mapping cosmology with GW events as standard candles (especially with electromagnetic counterparts to binary mergers)
- Studying structure of neutron stars and white dwarfs
- Studying compact objects falling into massive black holes in galactic nuclei

Opportunity and Challenge

GWs carry a lot of energy, but interact weakly: can pass through everything, including detectors!



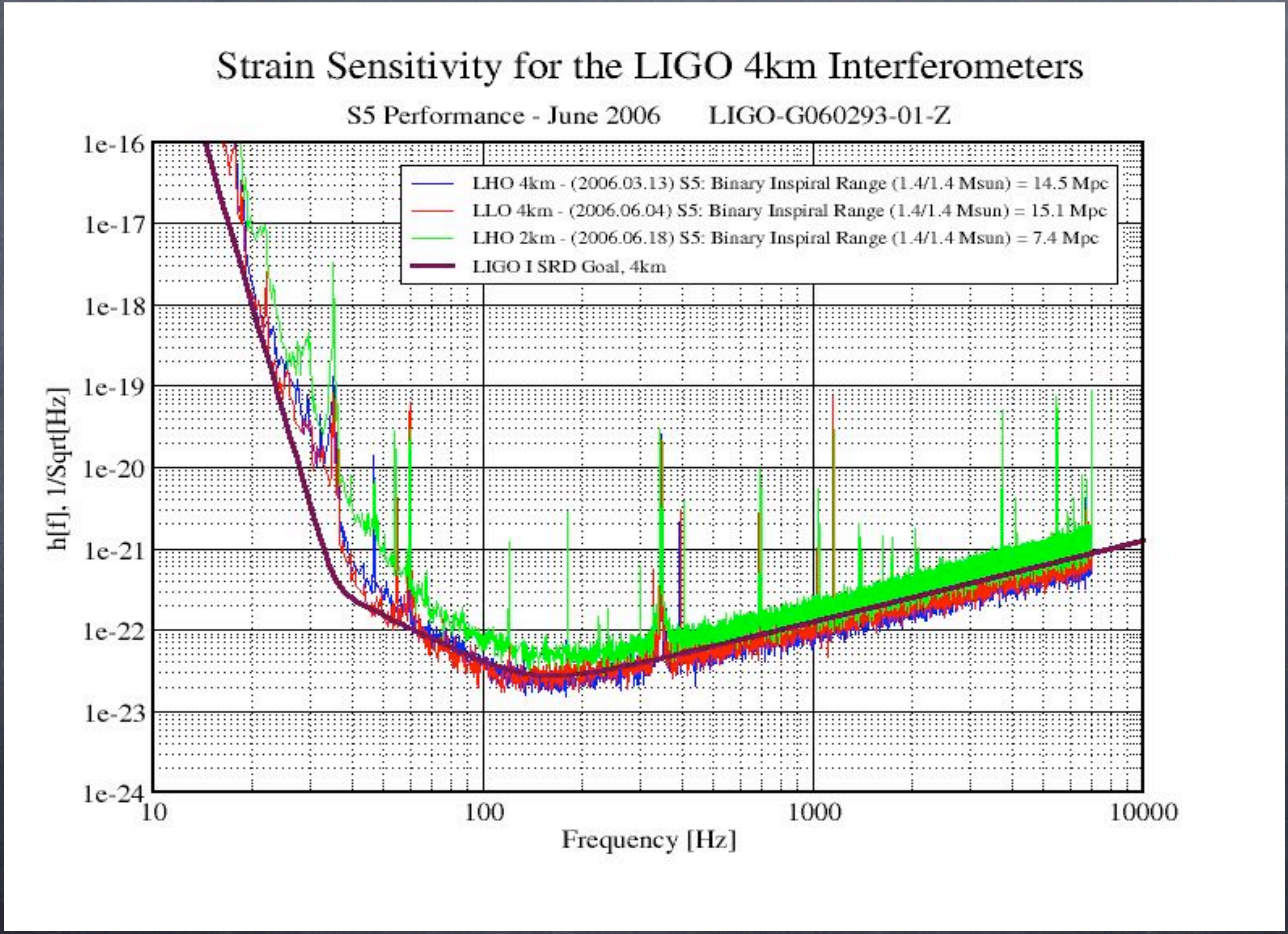


Michelson-type interferometers

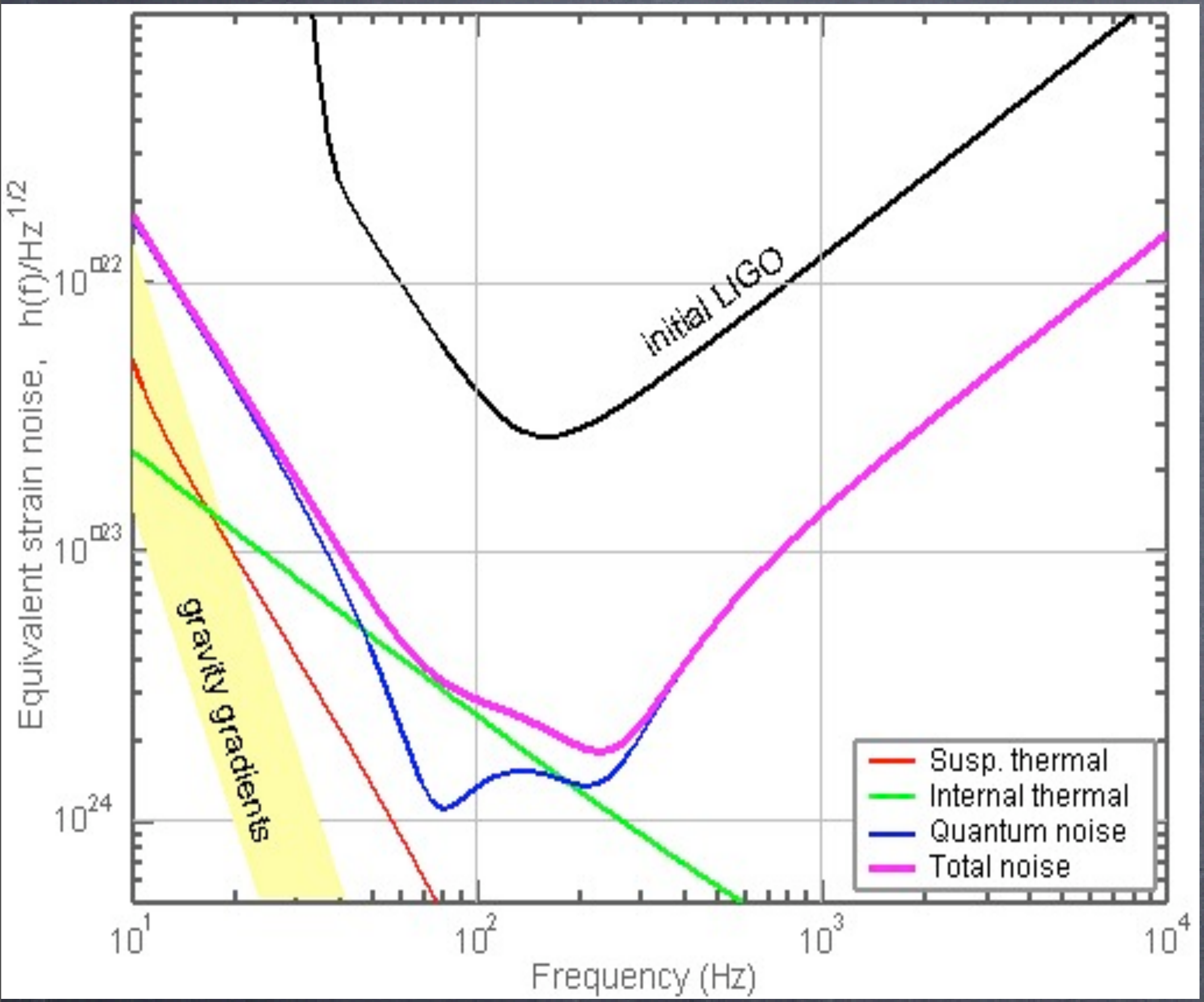
LIGO (Laser Interferometer GW Observatory

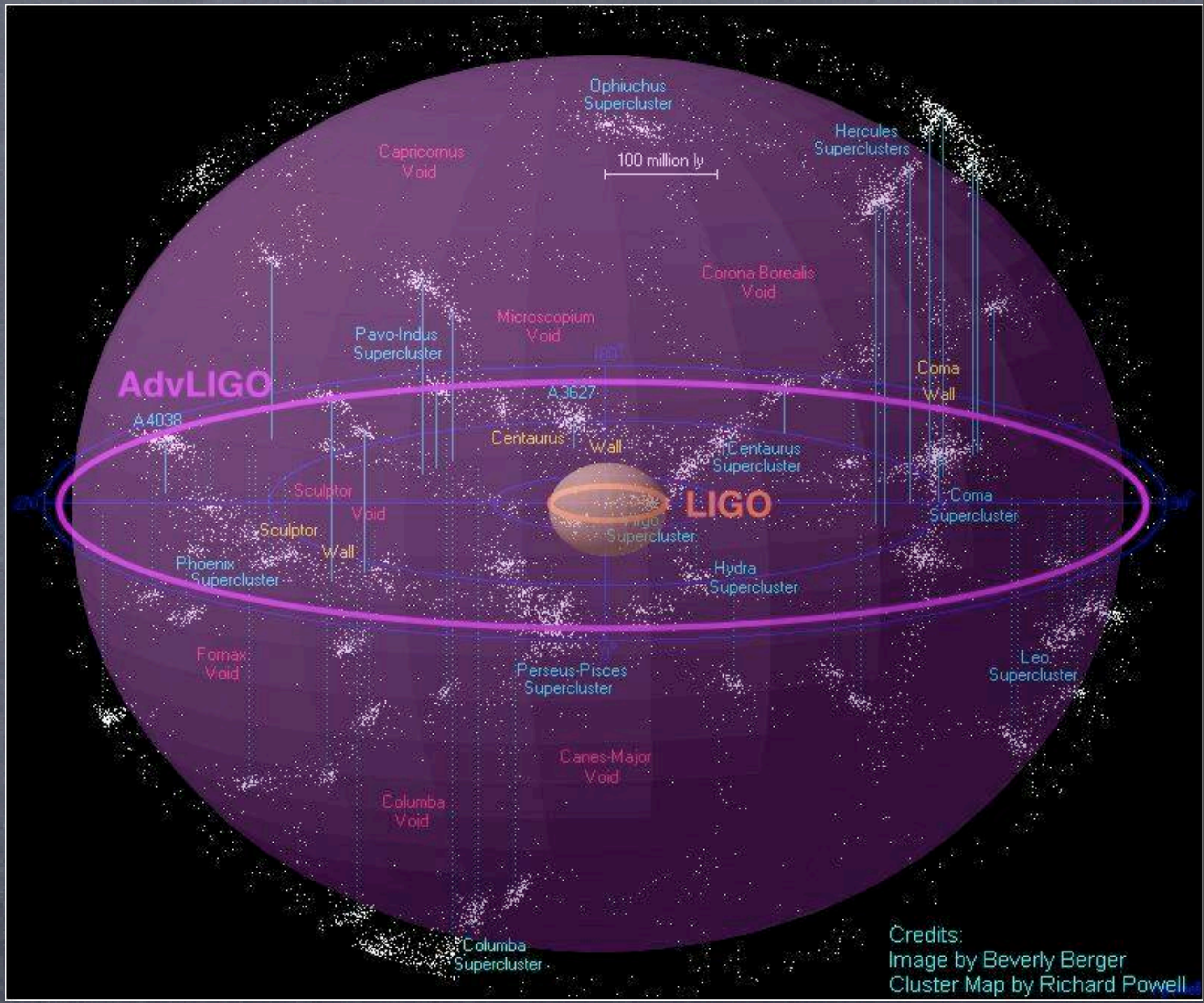
- 4 km long arms
- Typical strains $h = \Delta L / L \sim 10^{-22}$ (NS-NS in Virgo)
- Needs to measure $\Delta L = hL \sim 10^{-17}$ m
- 2 LIGO detectors in US + Virgo, GEO

LIGO Noise Curve



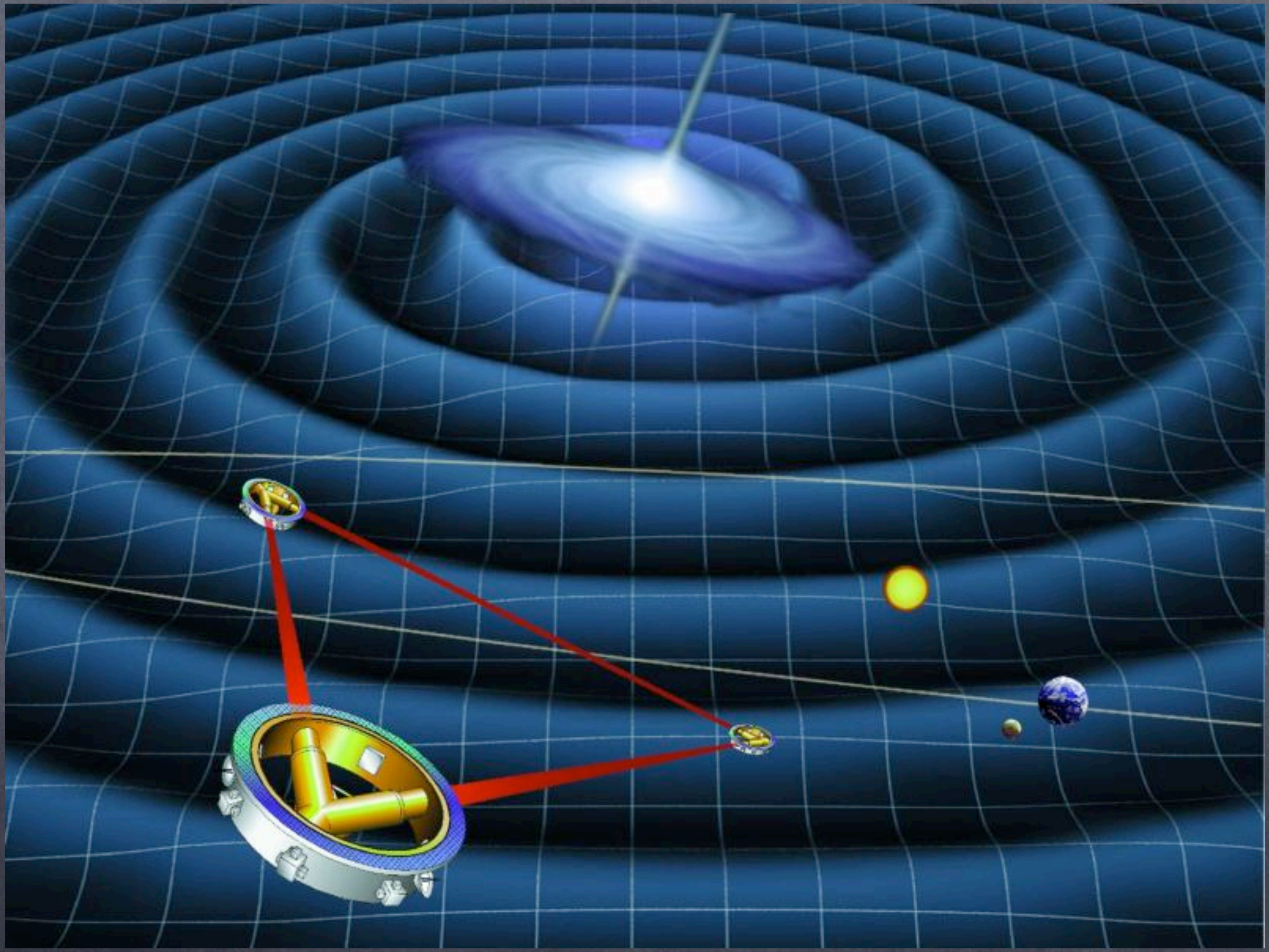
Advanced LIGO





- x10 in range -> x1000 in event rate
- 10 Hz low frequency cutoff

LISA (Laser Interferometer Space Antenna)



- 3 spacecraft following Earth around sun, 5 million km apart

LIGO and LISA Binary Sources

- LIGO sensitive @ a few hundred Hz

  NS-NS, NS-BH, BH-BH binaries

- LISA sensitive @ a few mHz

  supermassive black-hole binaries ($10^6\,\mathrm{M}_\odot$)
  galactic white dwarf binaries
  extreme-mass-ratio inspirals
  of WDs/NSs/BHs into SMBHs

Intermediate-mass-ratio inspirals (IMRIs)

- IMRIs have mass ratios between 10 and 10^4
- LIGO IMRIs: Inspirals of compact objects (1.4 solar-mass Neutrons Stars to 10 solar-mass Black Holes) into intermediate mass black holes (IMBHs, 50-350 solar masses)
- Indirect evidence for IMBH existence in globular clusters (50 – 10^4 solar masses)
  - Observational evidence (e.g. Macarone et al.)
  - Simulations (e.g. McMillan et al., O'Leary et al.)
  - Simulations vs. Observations (e.g. Trenti)
- IMRIs could be the first proof of IMBH existence!

Event Rates: Mechanisms

- Three-body interactions: IMBH swaps into binaries, forms CO-IMBH binaries which are tightened via three-body interactions with other stars, then merge via GW radiation reaction
- Direct capture via energy loss to GWs
- Kozai resonances in hierarchical triple systems: inner binary eccentricity is driven up by outer companion
- Tidal capture of MS star that evolves into CO while in orbit
- Tidal interactions (orbital-vibrational coupling) for NS inspirals

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Event rates per G.C.

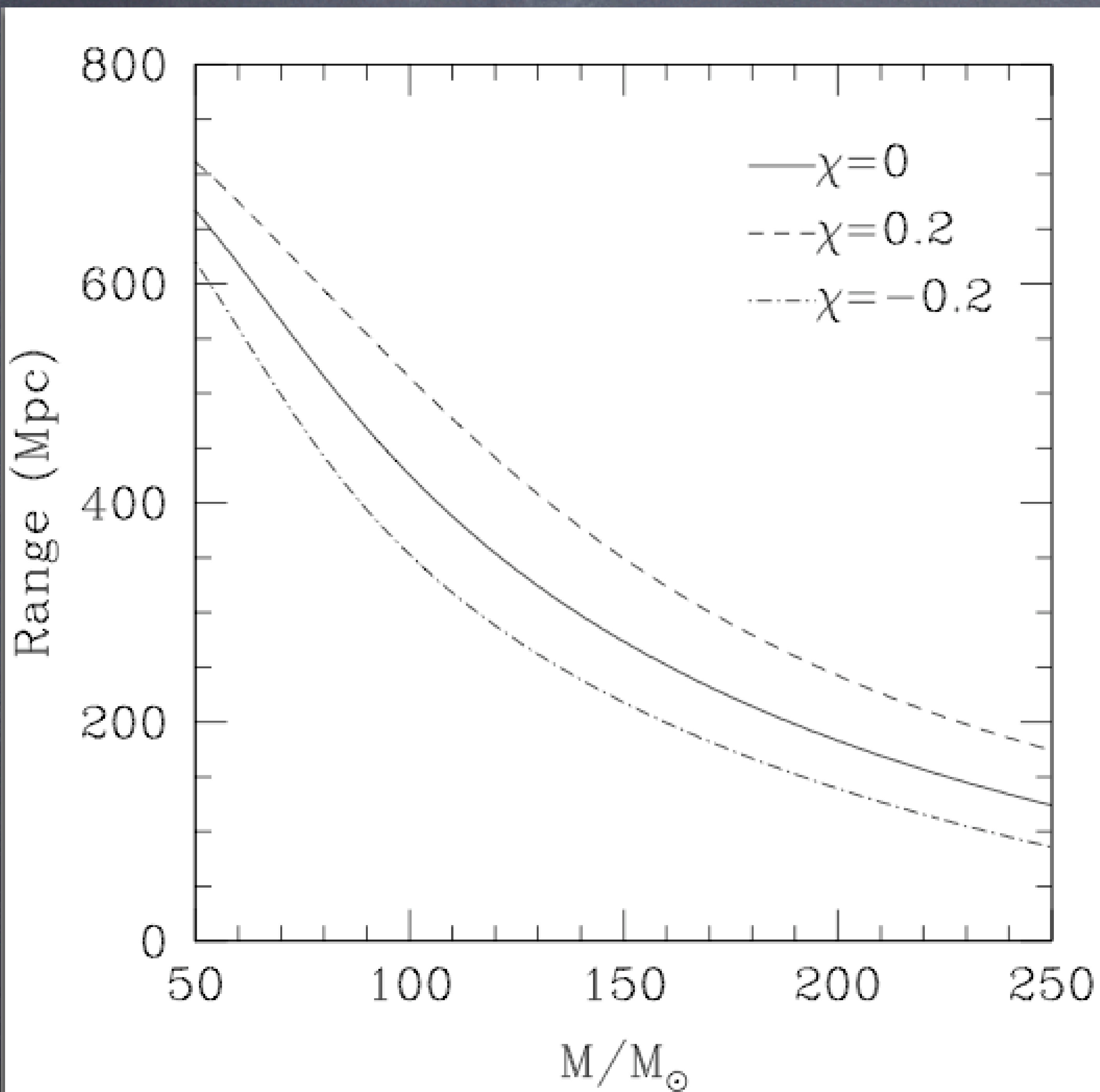
- Binary tightening via 3-body interaction
- 3-body interaction rate is $dN/dt=n\sigma v$;

  $n \sim 10^{5.5}\ pc^{-3}$; $v \sim 10$ km/s; $\sigma \sim \pi a(2GM/v^2)$
- $T_{harden} \sim O(M/m)\ (dN/dt)^{-1} \sim 1.5*10^8\ (AU/a)$ yr [Quinlan]
- $T_{merge} \sim 5*10^{17}\ M_\bullet^3/(M^2 m)\ (a/AU)^4\ (1-e^2)^{7/2}$ yr

  $\sim 5*10^8\ (M_\bullet/m)\ (100M_\bullet/M)^2\ (a/AU)^4$ yr [Peters & Mathews]
- To maximize rate, minimize $T=T_{harden}+T_{merge}$
- Rate per globular is $\sim 3*10^{-9}\ yr^{-1}$ for NS,

  $5*10^{-9}\ yr^{-1}$ for BH

Advanced LIGO IMRI sensitivity



- Use EMRI-like waveforms, including non-quadrupolar harmonics, to determine range
- Range is spin-dependent

$$R \approx \left[1+(\chi^2/2)\left(\frac{M}{100\,M_\odot}\right)^{1.5}\right]\sqrt{\frac{m}{M_\odot}}\left[800-540\left(\frac{M}{100\,M_\odot}\right)+107\left(\frac{M}{100\,M_\odot}\right)^2\right]$$

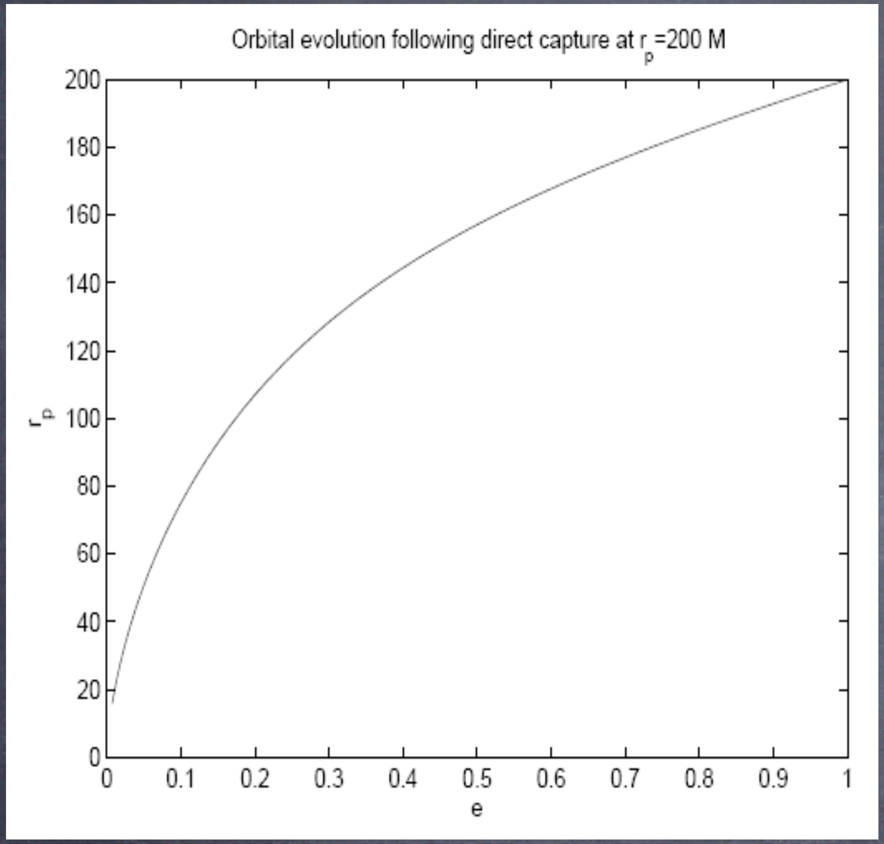
IM, arXiv:0707.0711

- Range could be increased by x1.5 by tuning Advanced LIGO

Advanced LIGO IMRI rates

- Assume 10% of all globular clusters hold suitable IMBH (typical mass 100 Msun, spin=0.2)
- If inspiraling object is 1.4 Msun NS, Advanced LIGO could detect one IMRI per 3 years
- If inspiraling object is 10 Msun BH, Advanced LIGO could detect 10 IMRIs per year
- If Advanced LIGO is IMRI-optimized, rates could go up to 1/year and 30/year

Eccentricities in AdvLIGO band



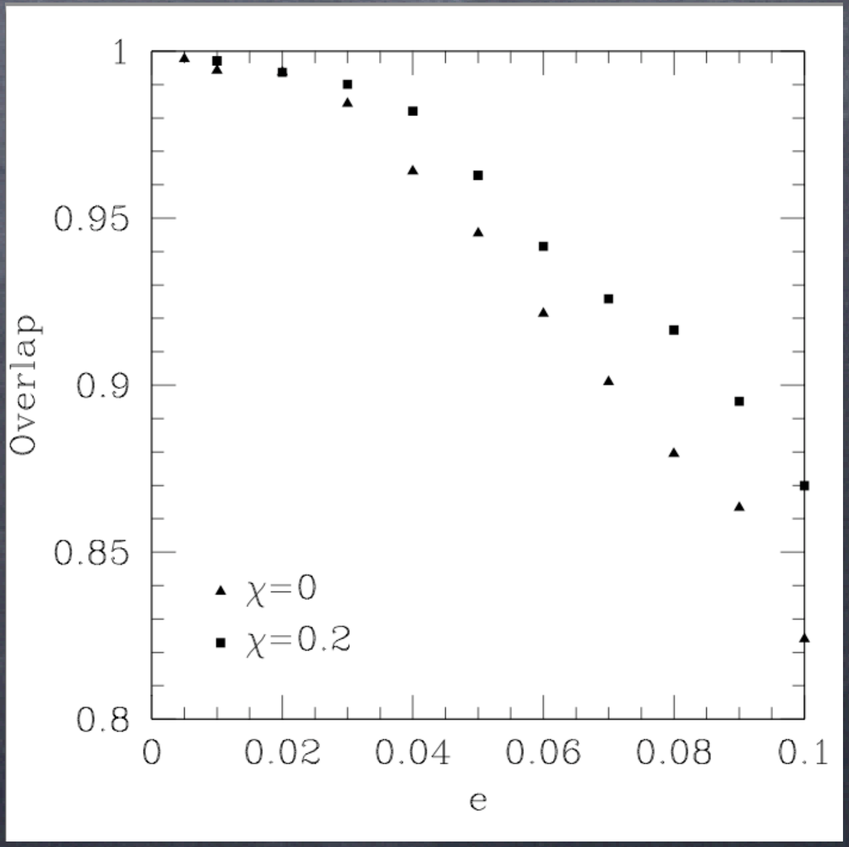
- Hardening via 3-body interactions

  Eccentricity ~ few*10^{-5} when f_{GW}=10 Hz

- Direct capture

  90% of IMRIs circularize to e<0.1 by 10 Hz, 67% circularize to e<0.01 by f_{GW}=10 Hz

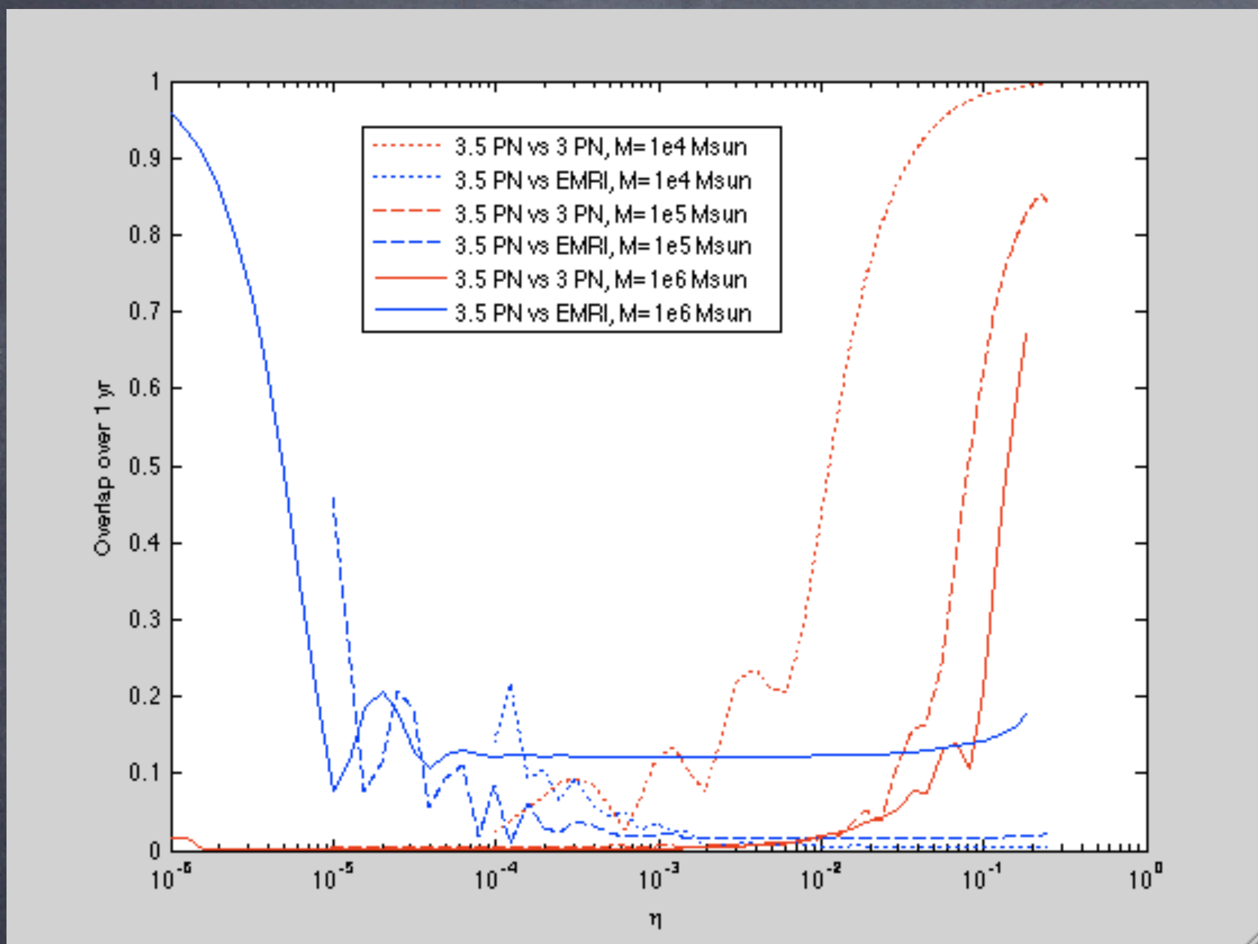
- At e=0.01, overlap between eccentric and circular templates is >0.99, so circular templates can be used for detection



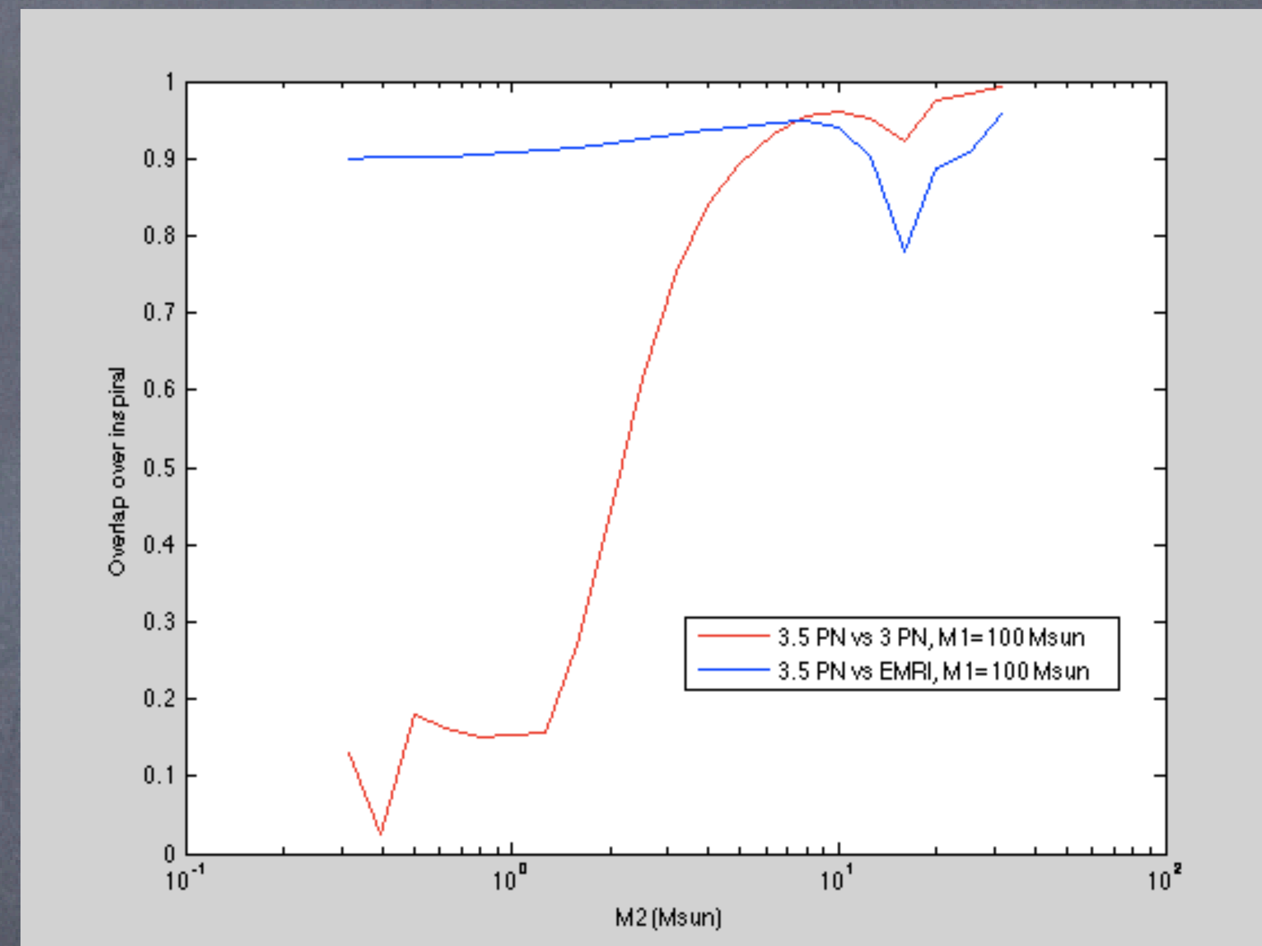
Intermediate-Mass-Ratio Inspiral Waveforms

- EMRI waveforms: expansions in $\eta = mM/(M+m)^2 \sim m/M$
- Post-Newtonian waveforms: expansions in v/c
- IMRIs fall in the middle... which one is closer? which one is easier to "patch up" to create an IMRI template?
- pN errors are concentrated near ISCO, where v/c is highest IMRIs fall between EMRIs and comparable-mass binaries for waveform generation & data analysis
- EMRI errors are spread throughout waveform:
  - $df/dt \propto m/M$; $\Delta(df/dt) \propto (m/M)^2$
  - Accumulated phase error is $\Delta\varphi \propto \Delta(df/dt)\, T^2$
  - if source is bandwidth-limited, $T \propto (M/m)$, so $\Delta\varphi = O(1)$
  - if source is limited by observation time, T is fixed (e.g., 3 yrs for LISA), and $\Delta\varphi \propto (m/M)^2$
- Threshold η depends on ISCO frequency relative to detector noise, set by M

Overlaps: $\langle a|b\rangle = 4\mathcal{R}\int_0^\infty \frac{a(f)b^*(f)}{S_n(|f|)}\mathrm{d}f$



LISA

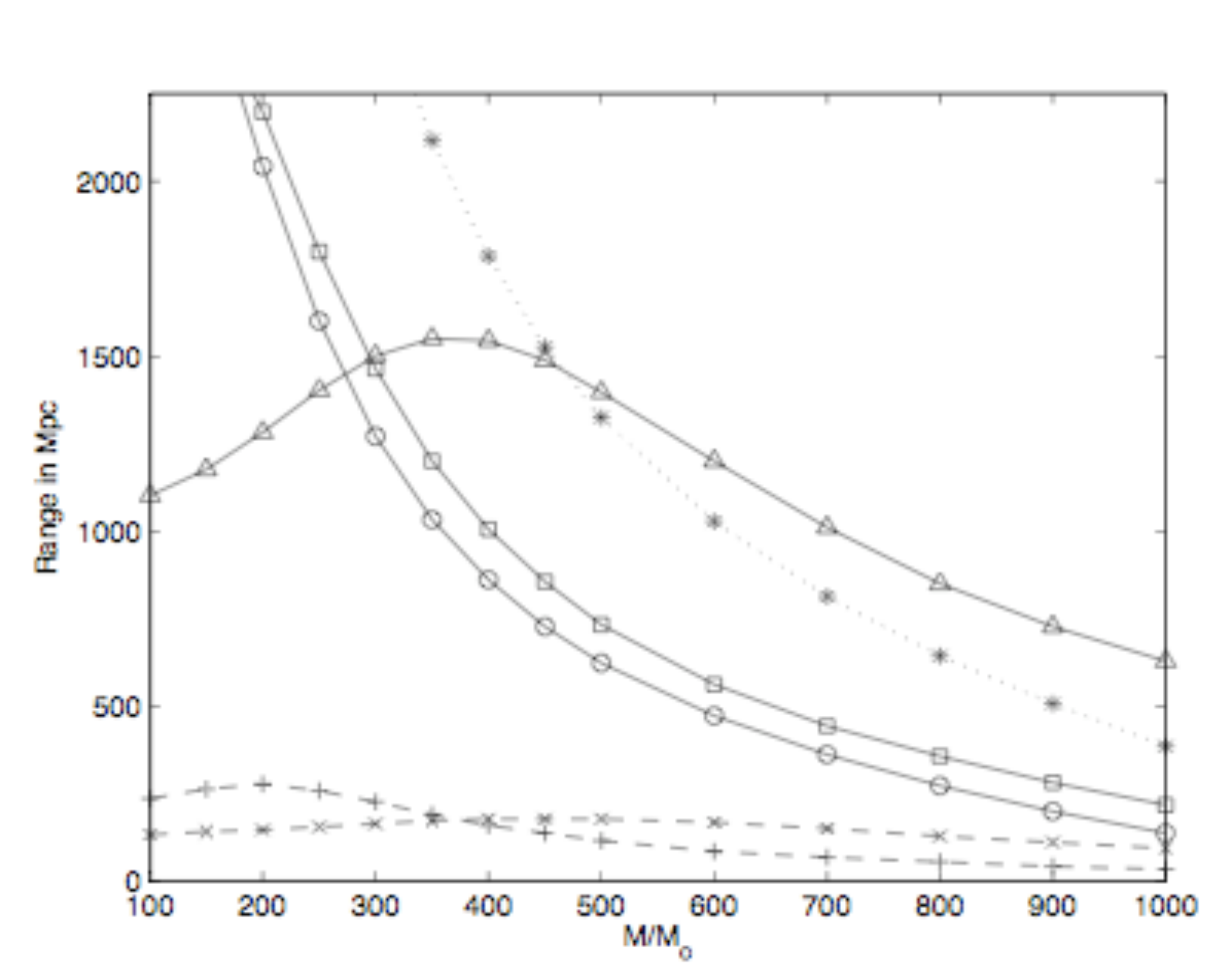


LIGO

- At low η, EMRIs are better; at high η, PN
- Neither waveform is good at intermediate η!
- Can we get better hybrid waveforms by combining EMRI and pN?

IM & Gair,arXiv:0811.0138

Ringdowns



Could complement IMRIs if higher CO and IMBH masses are prevalent

What is the “no-hair theorem”?

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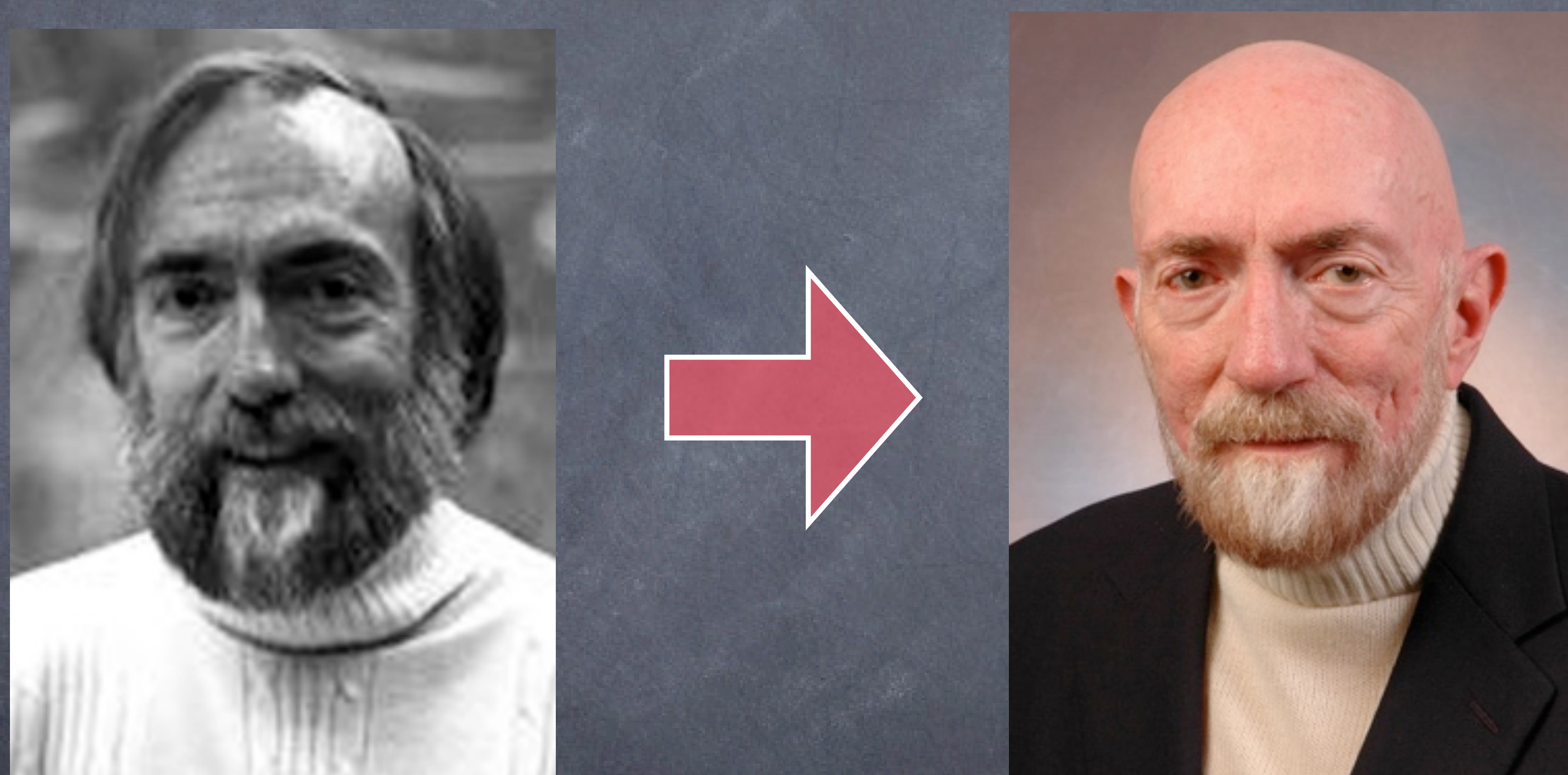
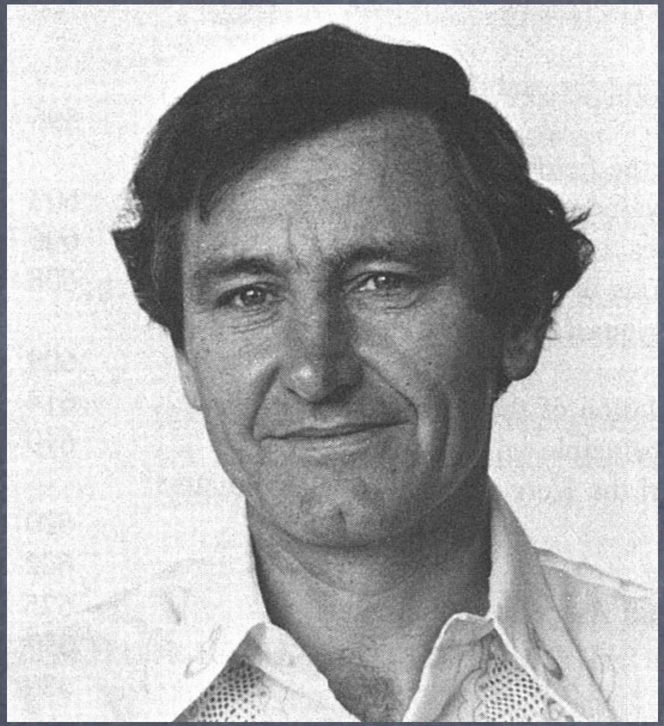


idea taken from Daniel Shaddock

What is the “no-hair theorem”?

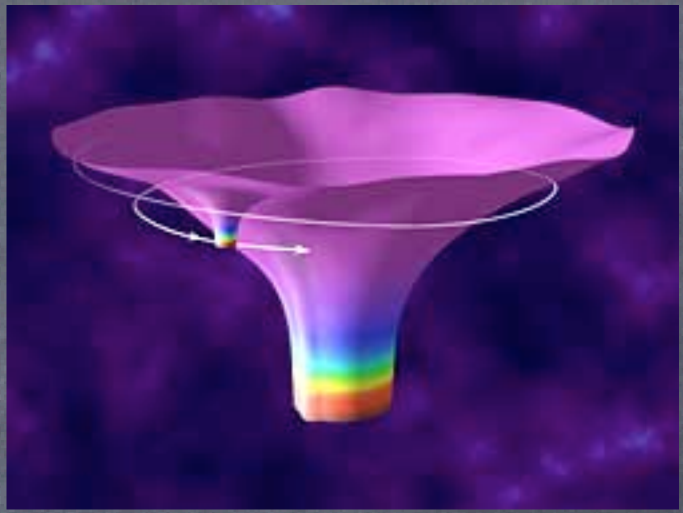


Stationary, vacuum, asymptotically flat spacetimes in which the singularity is fully enclosed by a horizon with no closed timelike curves outside the horizon are described by the Kerr metric

The no-hair theorem in English

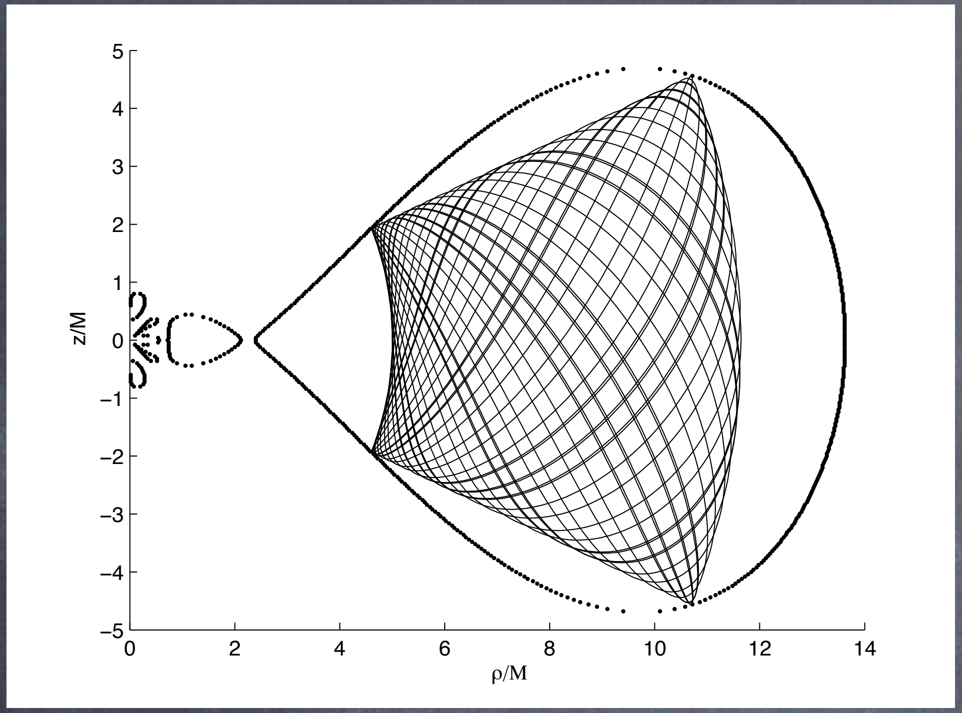
- "Black holes have no hair" means that all higher-order mass and current multipole moments are uniquely determined by the black hole mass and spin
- Conversely, an object with hair is one for which $$M_n + iS_n \neq M(ia)^n$$
- The "no-hair theorem" is a mathematical statement, so the title is a bit of a misnomer...

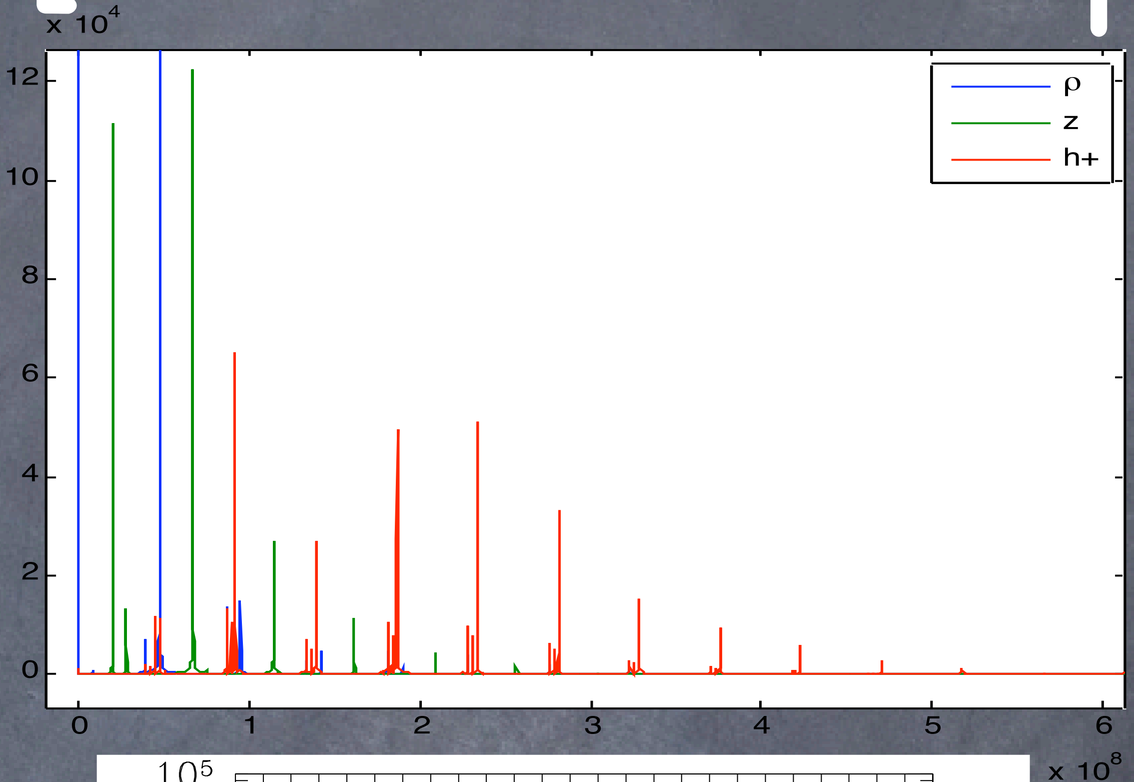
Do Black Holes Have Hair? Probing spacetime with E/IMRIs

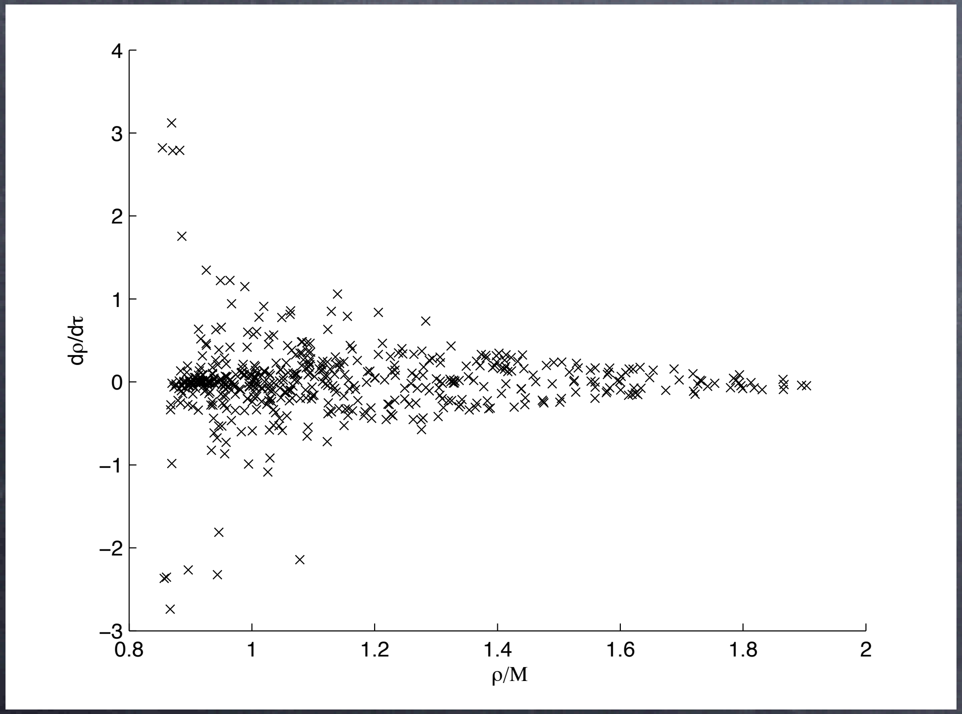


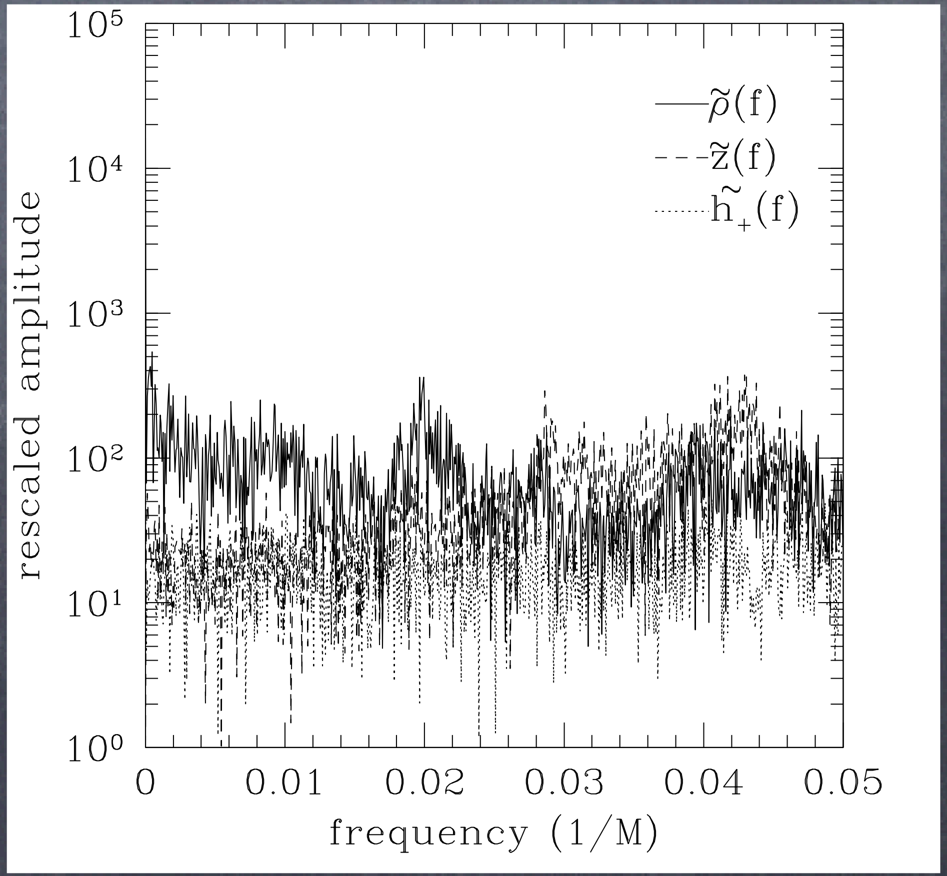
- Are massive "black holes" really hairless?
- Or could they be boson stars, naked singularities, ...?
- Need to measure 3 multipole moments to test "Kerrness", 4 to test if an object is a boson star
- LISA EMRIs into SMBHs will be the best probes of the strong-field regime (#cycles ~ M/m), but Advanced LIGO IMRIs into IMBHs may provide the first interesting test
- Information about the spacetime structure and the orbit should be contained in GWs; how do we access it?

Chaotic motion [Poincare maps]





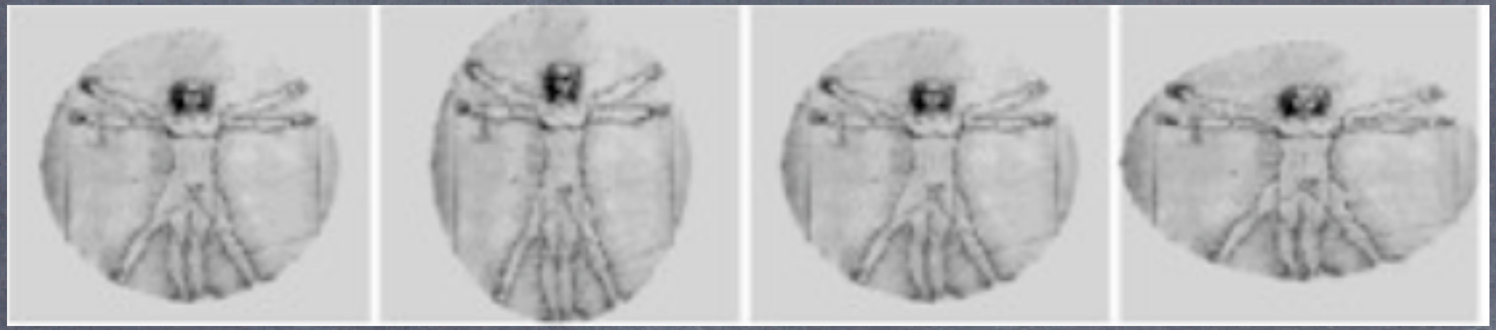




Gair, Li, Mandel. 2008. PRD 77 024035. arXiv:0708.0628

Summary

- The first gravitational-wave detections may happen in a few years
- Advanced LIGO could detect a few IMRIs per year
- Eccentricities will be low, circular waveforms can be used for detection
- Further work is needed to develop a waveform family for detecting IMRI signals
- Gravitational waves from EMRIs should make it possible to test whether the central body [SMBH] is a Kerr black hole
- This is a very exciting time for gravitational-wave astronomy: stay tuned!



New subtitle: do massive black holes have hair?

- Are massive "black holes" really black holes?
- Could they be boson stars, or naked singularities, or...?
- Need to measure 3 multipole moments to test "Kerrness", 4 to test if an object is a boson star
- Search for exotic massive compact objects, test of cosmic censorship conjecture, null hypothesis test of the no-hair theorem...

Summary

- Advanced LIGO could detect a few IMRIs per year
- Eccentricities will be low, circular waveforms can be used for detection (But should we use EMRI waveforms? Hybrid waveforms? ...?)
- Gravitational waves from EMRIs should make it possible to test whether the central body [SMBH] is a Kerr black hole
- Chaos in a non-Kerr spacetime would be an obvious smoking gun, but chaotic regions are probably not accessible
- Location of ISCO, periapsis precession, and orbital-plane precession are possible observables indicating bumpiness
- Frequency evolution over inspiral would be another observable, but more work is required

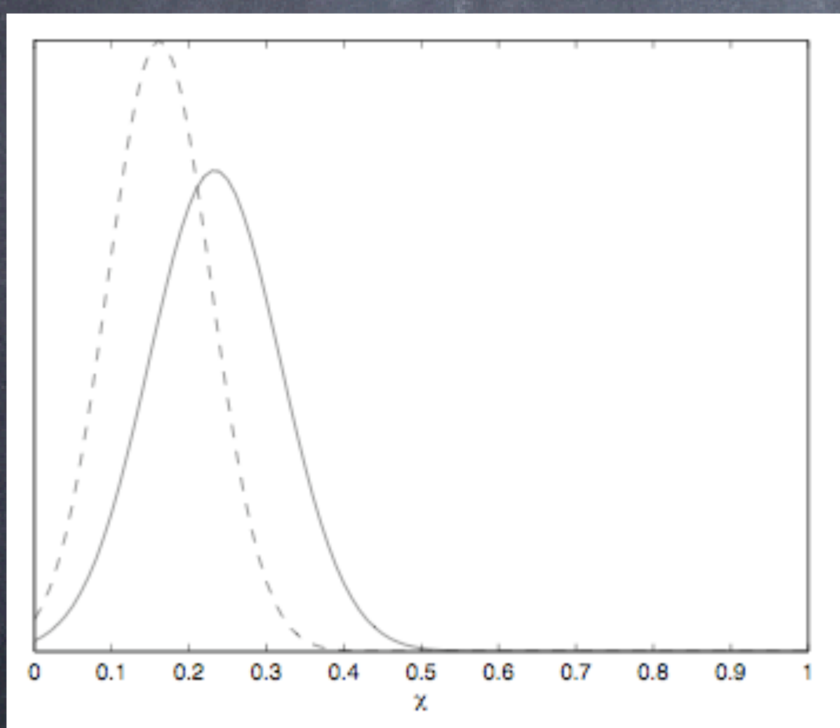
Event rates – upper limit

- Model-independent upper limit
- One core-collapsed globular cluster per Mpc^3
- One suitable IMBH per globular cluster
- IMBH grows from 50 to 350 solar masses by capture of COs in Hubble time
- Advanced LIGO could see IMRIs up to ~1000 Mpc (depending on masses, spin)
- Advanced LIGO may see tens of IMRIs per year (only 1 in 1000 years with Initial LIGO)
- Issues: kicks above 50 km/s eject IMBH; lower rates late in cluster history [e.g. simulations by O'Leary et al. 2006]

Spin and detection range

Evolution of spin distribution via minor mergers

$$\frac{\partial}{\partial t} f(\chi,t) = -\frac{\partial}{\partial \chi}\left[\frac{\chi}{t}\left(-2-\frac{4\sqrt{2}}{9}+\frac{4}{\chi^2 t}\right) f(\chi,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial \chi^2}\left[\frac{4}{t^2}\left(1+\frac{4\sqrt{2}\chi^2}{9}-\chi^2\right) f(\chi,t)\right]$$



t=M/m

$$\bar{\chi} \approx \bar{\chi}_0 \left(\frac{t}{t_0}\right)^{a} \approx \bar{\chi}_0 \left(\frac{M_0}{M}\right)^{2.63}$$

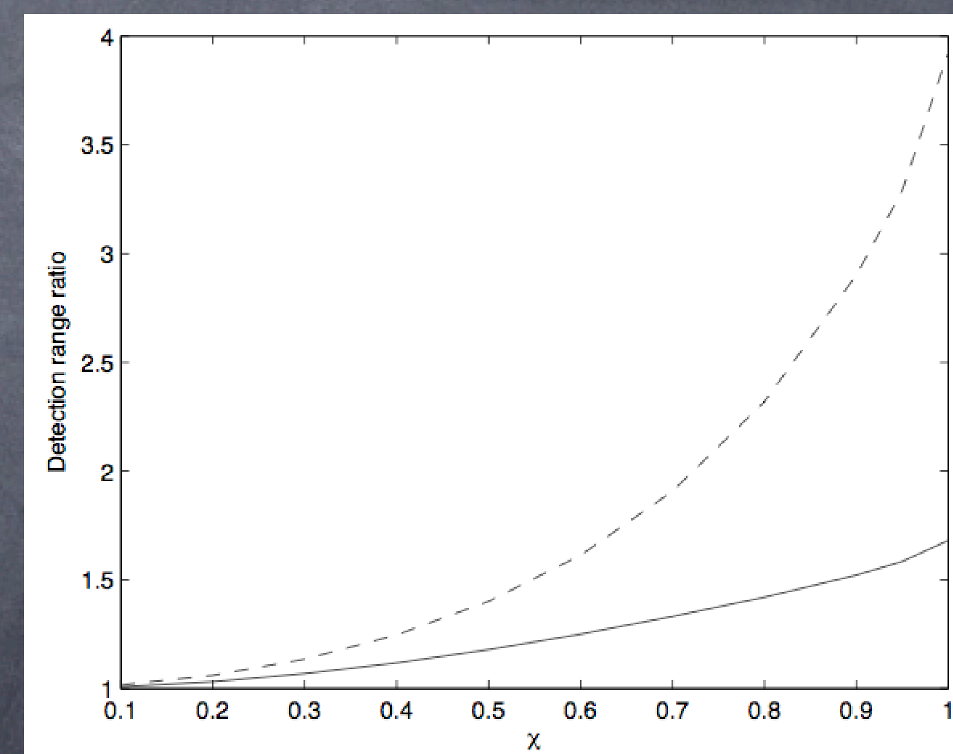
Evolution from t=M/m=50 to t=100 (e.g., from M=70 to M=140 solar masses via capture of m=1.4 solar-mass NSs)

If initial χ=0.1, then mean spin at t=100 is 0.162, σ=0.066

If initial χ=0.9, then mean spin at t=100 is 0.233, σ=0.087

Range increase due to spin

$$\frac{\text{Range}_{\text{spin}}}{\text{Range}_{\text{no-spin}}} \sim 1 + 0.6\chi^2 \left(\frac{M}{100\ M_\odot}\right)$$



Solid line - inspiral into 100 Msun IMBH

Dashed line - inspiral into 200 Msun IMBH

Effect is very pronounced for LISA:

can cause bias in spin estimate

Observing deviations from Kerr with EMRIs

- LISA can detect tens to thousands of EMRIs
- Ryan's theorem [1995]: GWs from nearly circular, nearly equatorial orbits in stationary, axisymmetric spacetimes encode all of the spacetime multipole moments... in principle
- Can we extend this theorem? Are there obvious observable imprints of an anomalous, non-Kerr quadrupole moment (a "bumpy" spacetime)?
- Are energy E, angular momentum L_z and Carter constant Q conserved in a bumpy spacetime?

Geodesics in bumpy spacetimes

- Use Manko-Novikov bumpy spacetime

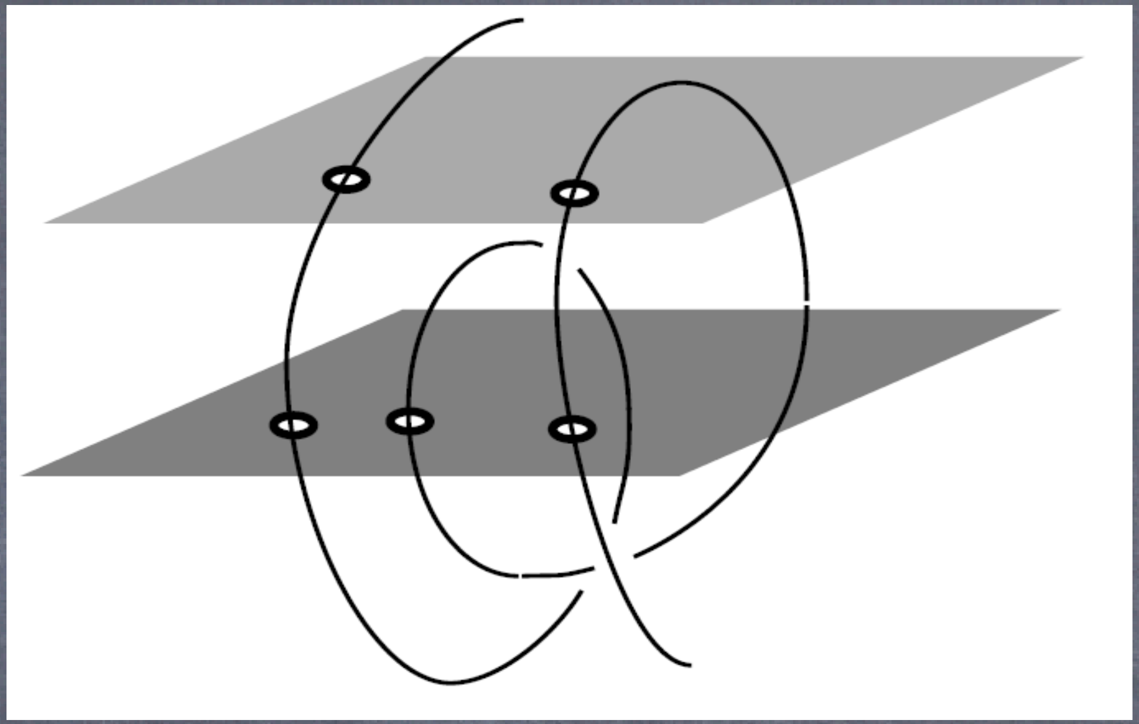
$$\mathrm{d}s^2 = -f(\rho,z)\,\left(\mathrm{d}t - \omega(\rho,z)\,\mathrm{d}\phi\right)^2 + \frac{1}{f(\rho,z)}\left[\mathrm{e}^{2\,\gamma(\rho,z)}\,\left(\mathrm{d}\rho^2 + \mathrm{d}z^2\right) + \rho^2\,\mathrm{d}\phi^2\right]$$

- C code - geodesic equations:

$$\frac{\partial^2 x^\alpha}{\partial \tau^2} = -\Gamma^\alpha_{\beta\gamma}\frac{\partial x^\beta}{\partial \tau}\frac{\partial x^\gamma}{\partial \tau}$$

- Check conservation of E, L_z, 4-velocity norm
- Equations might not separate as in Kerr
- Is there a full set of integrals of motion?

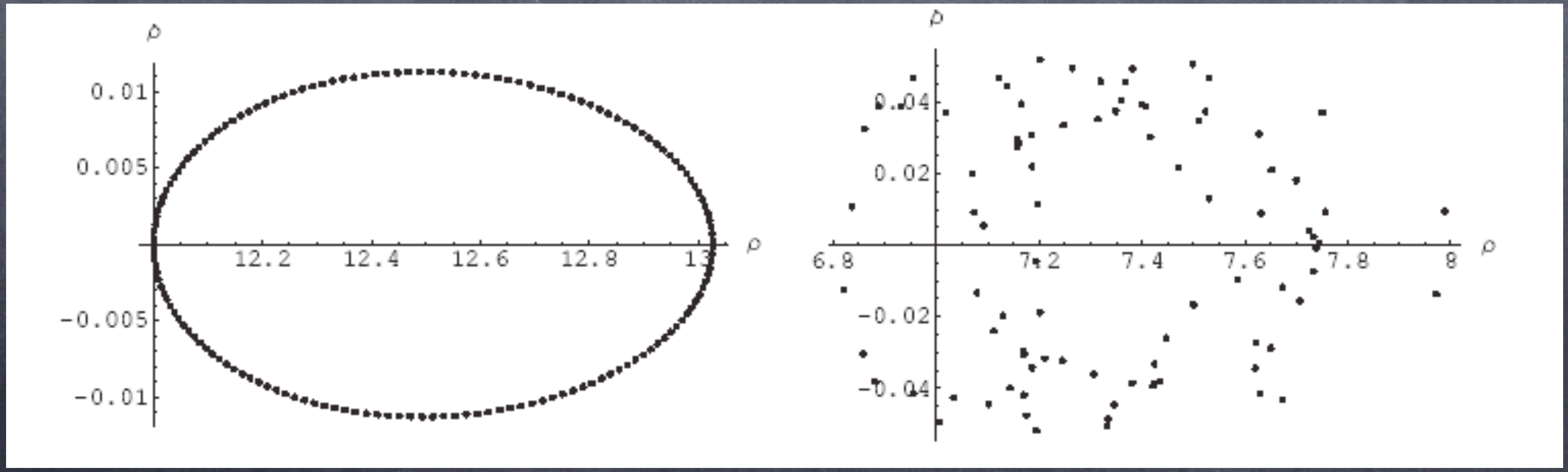
Poincare maps



- Check if spacetime has a full set of integrals of motion
- Plot $d\rho/dt$ vs. ρ for $z=z_0$ crossings
- Phase space plots should be closed curves for all z_0 iff there is a third isolating integral

Poincare maps for motion in Newtonian potential with hexadecapole moment

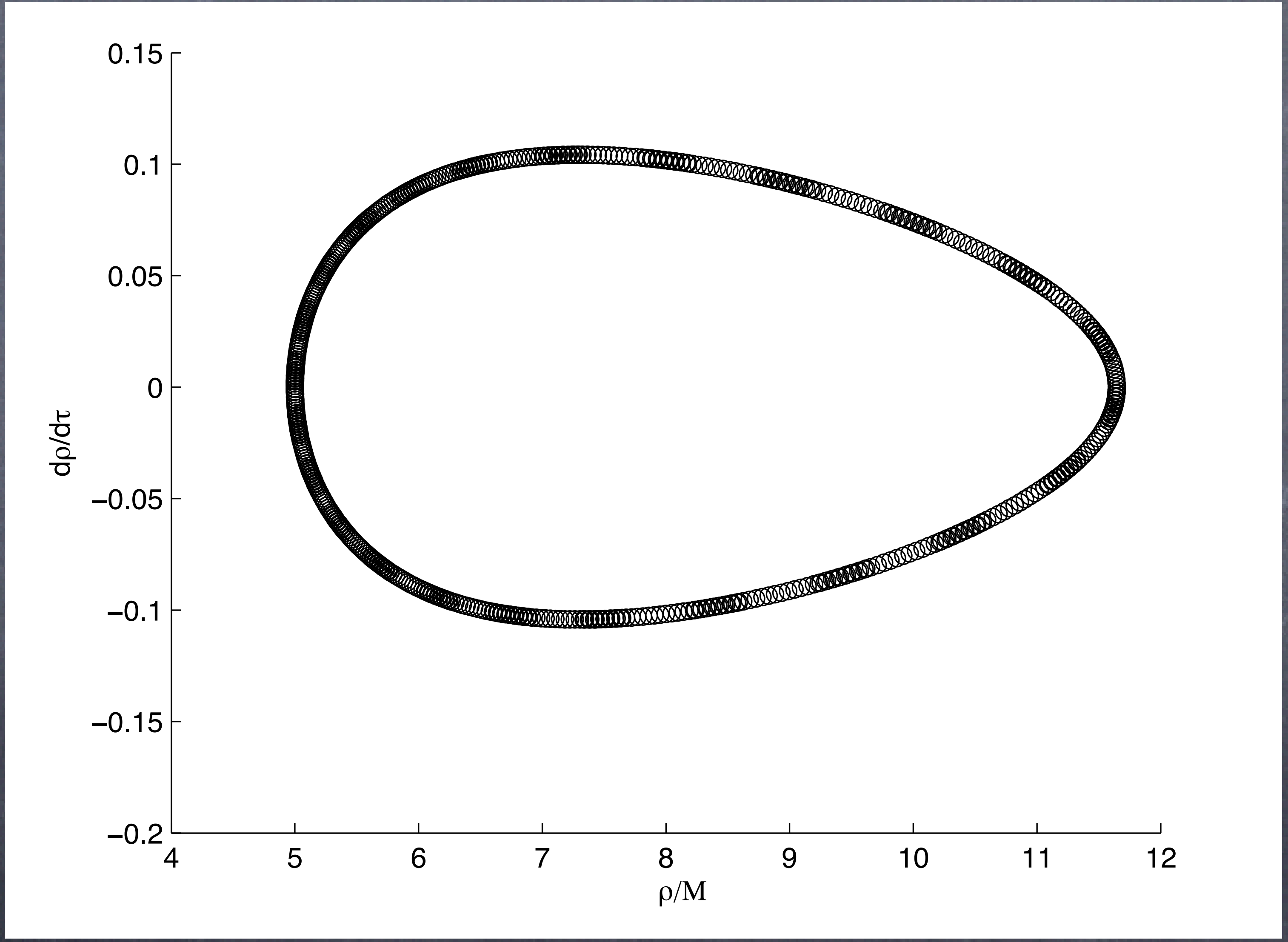
$$V(r,\theta) = -\frac{M_0}{r} + \frac{M_2}{r^3}P_2(\cos\theta) + \frac{M_4}{r^5}P_4(\cos\theta)$$



M_2=10 M_0; M_4=400 M_0

Li

Poincare map in a bumpy spacetime



$E=0.95$, $L_z=-3$, $a/M=0.9$, $q=0.95$

Allowed regions for bound orbits

Effective potential $(\dot{\rho}^2 + \dot{z}^2) = V(E, L_z, \rho, z)$

z

ρ

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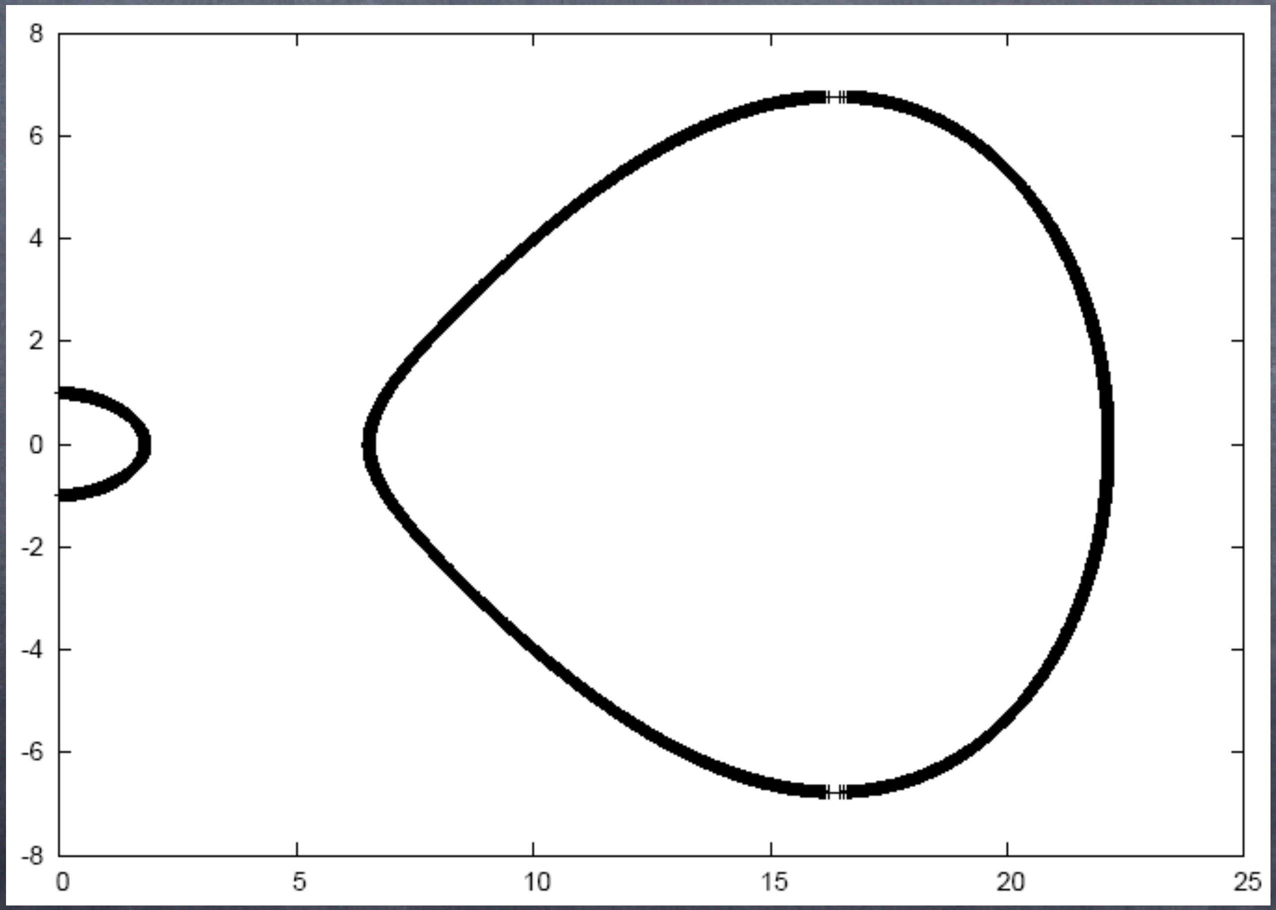

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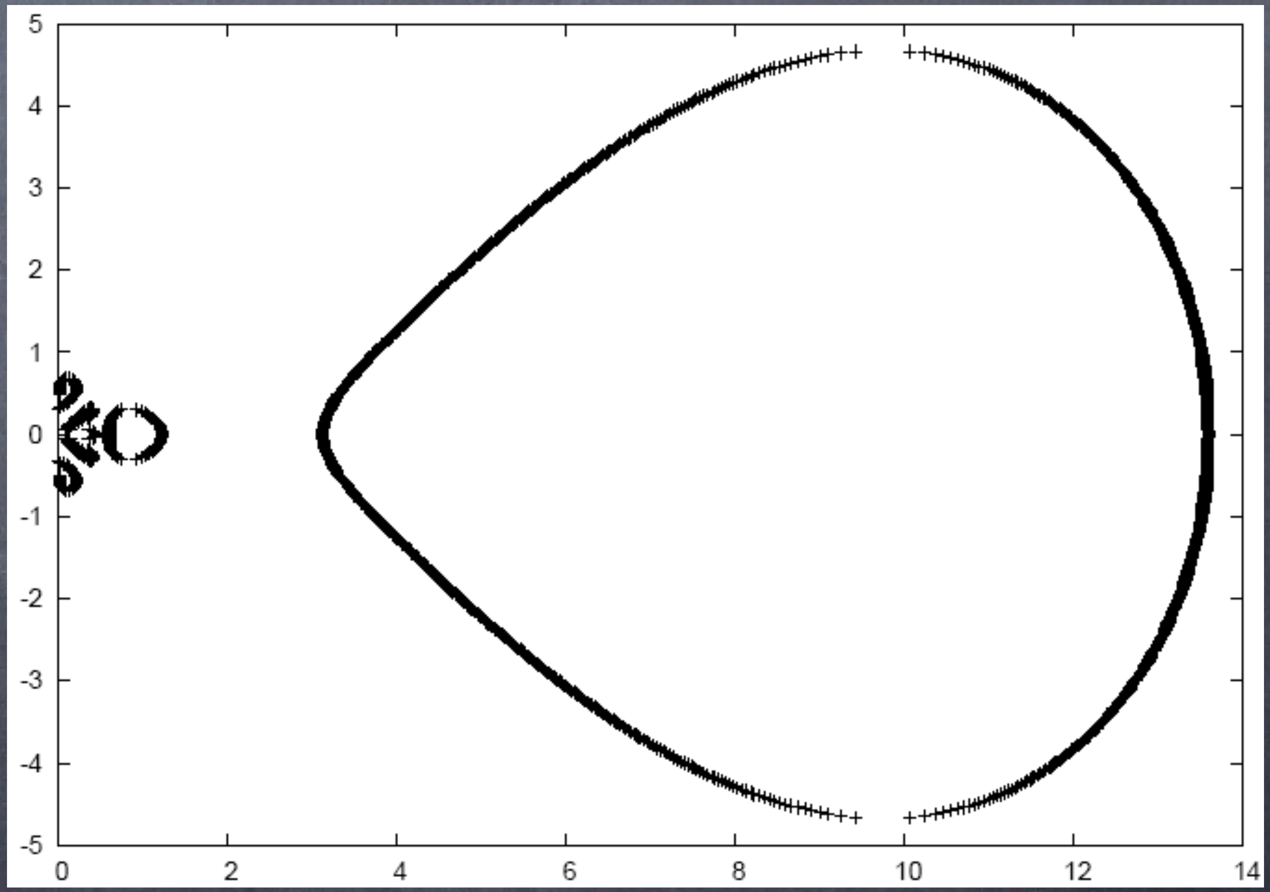
z 

ρ ρ

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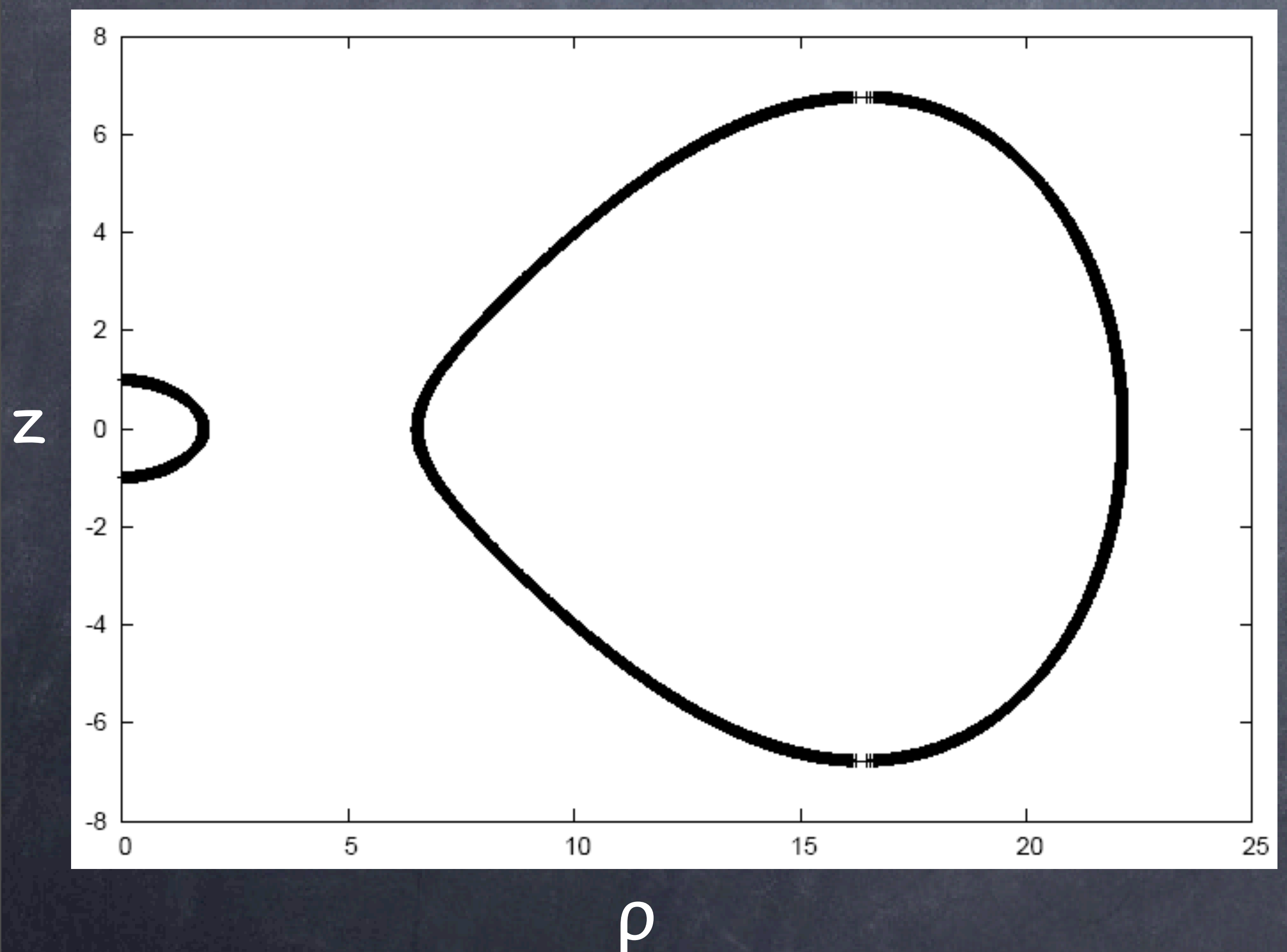

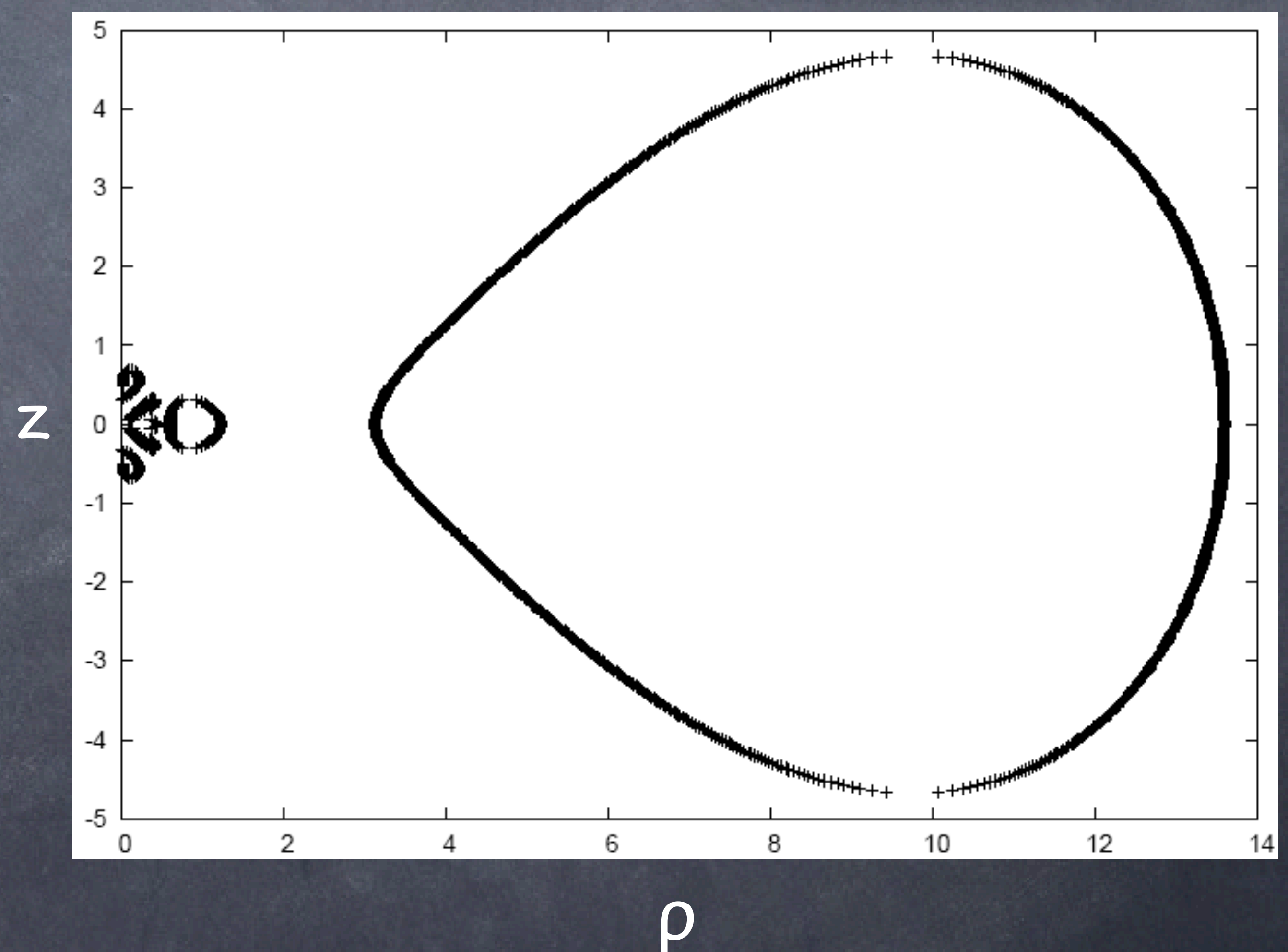

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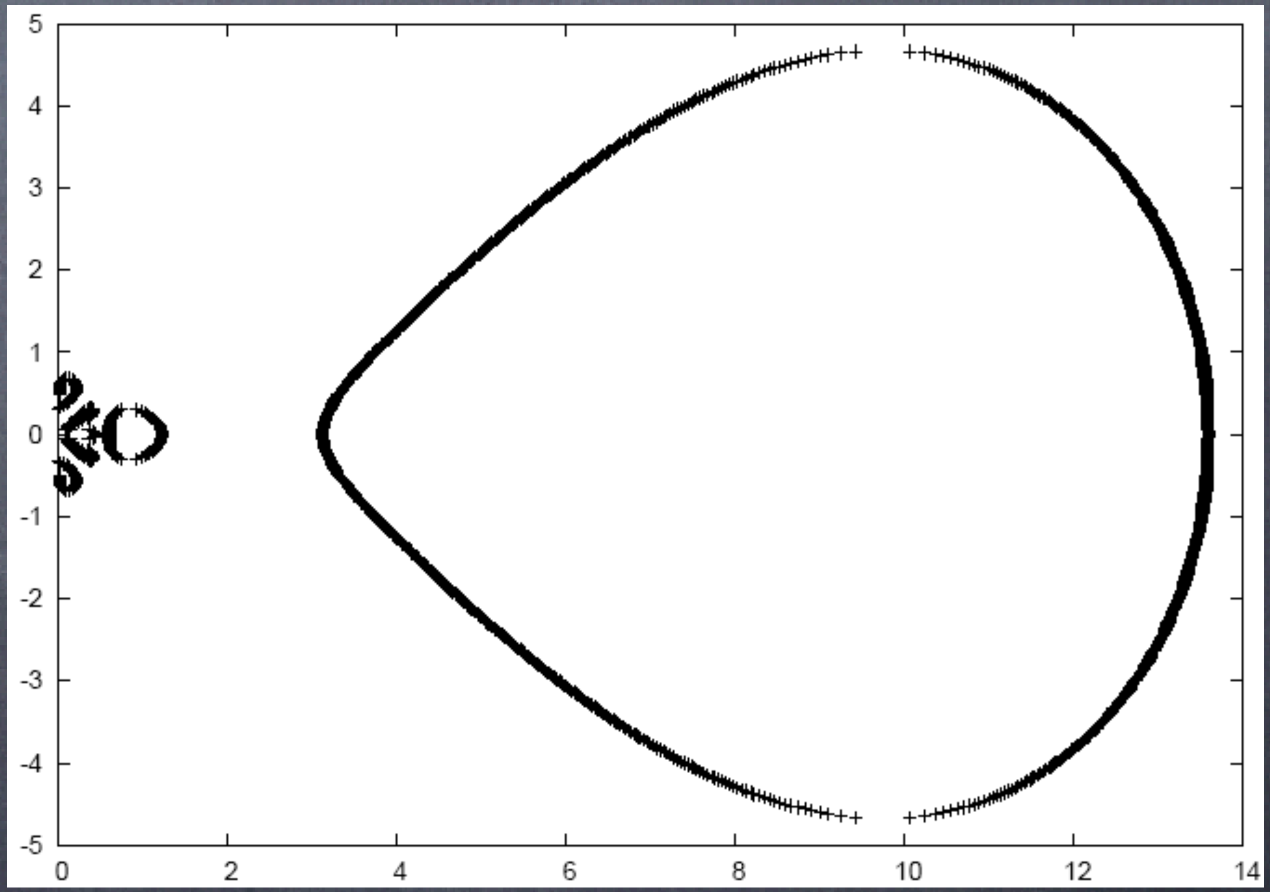
z

z

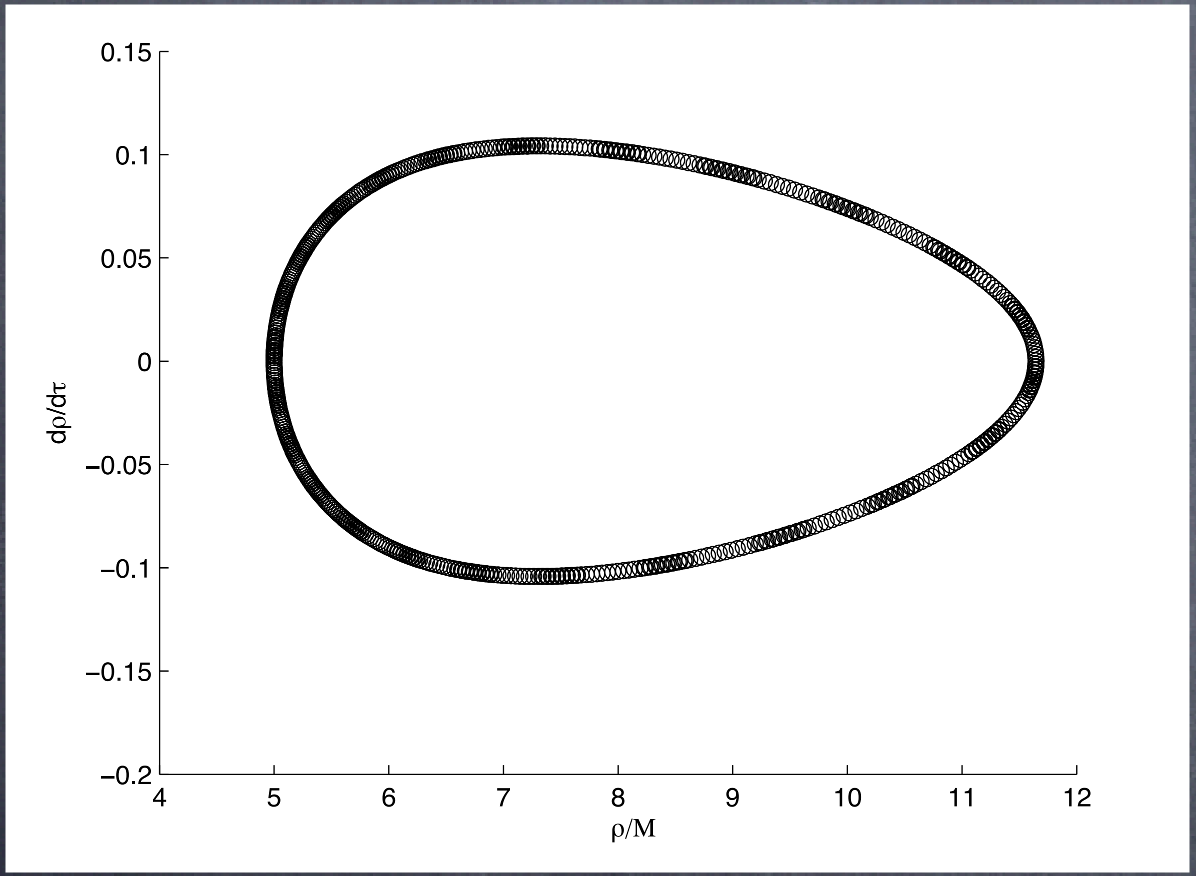


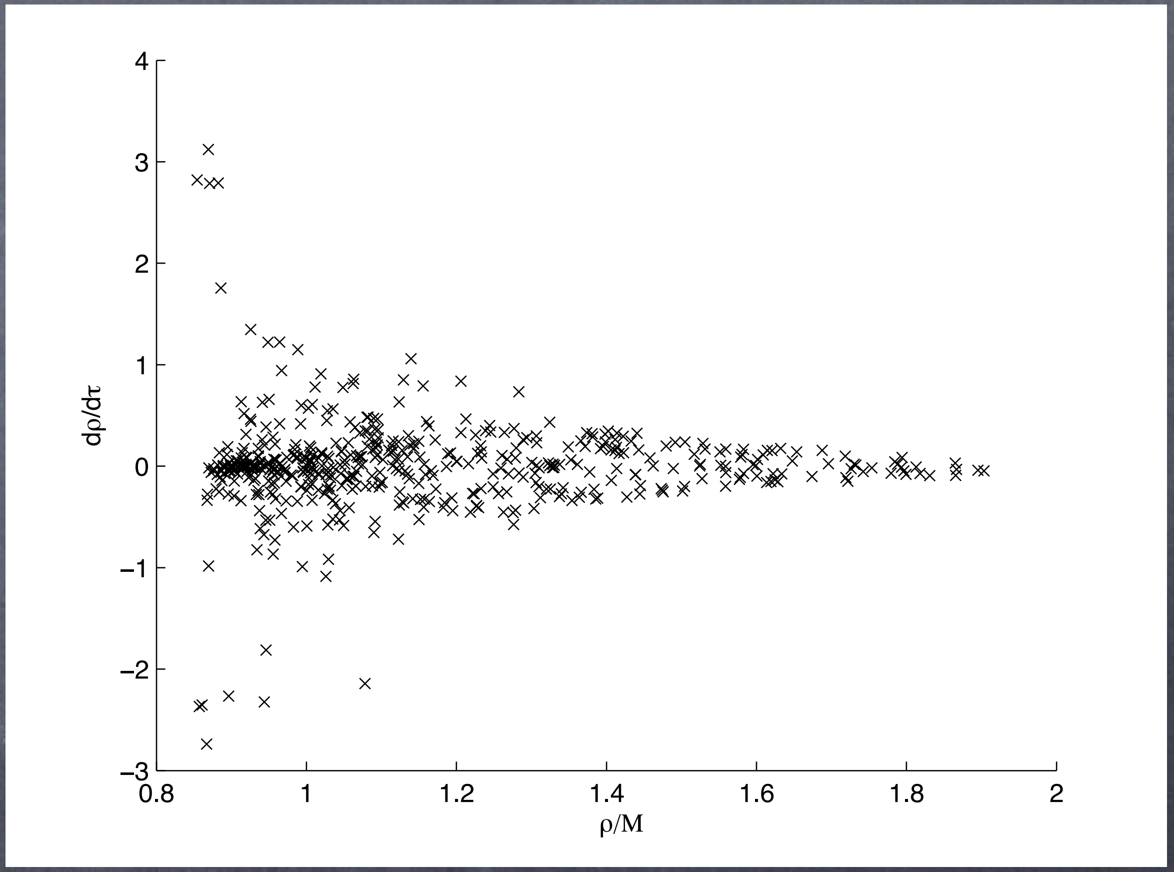
ρ

ρ

$E=0.95$, $L_z=-3$, $a/M=0.9$, $q=0.95$

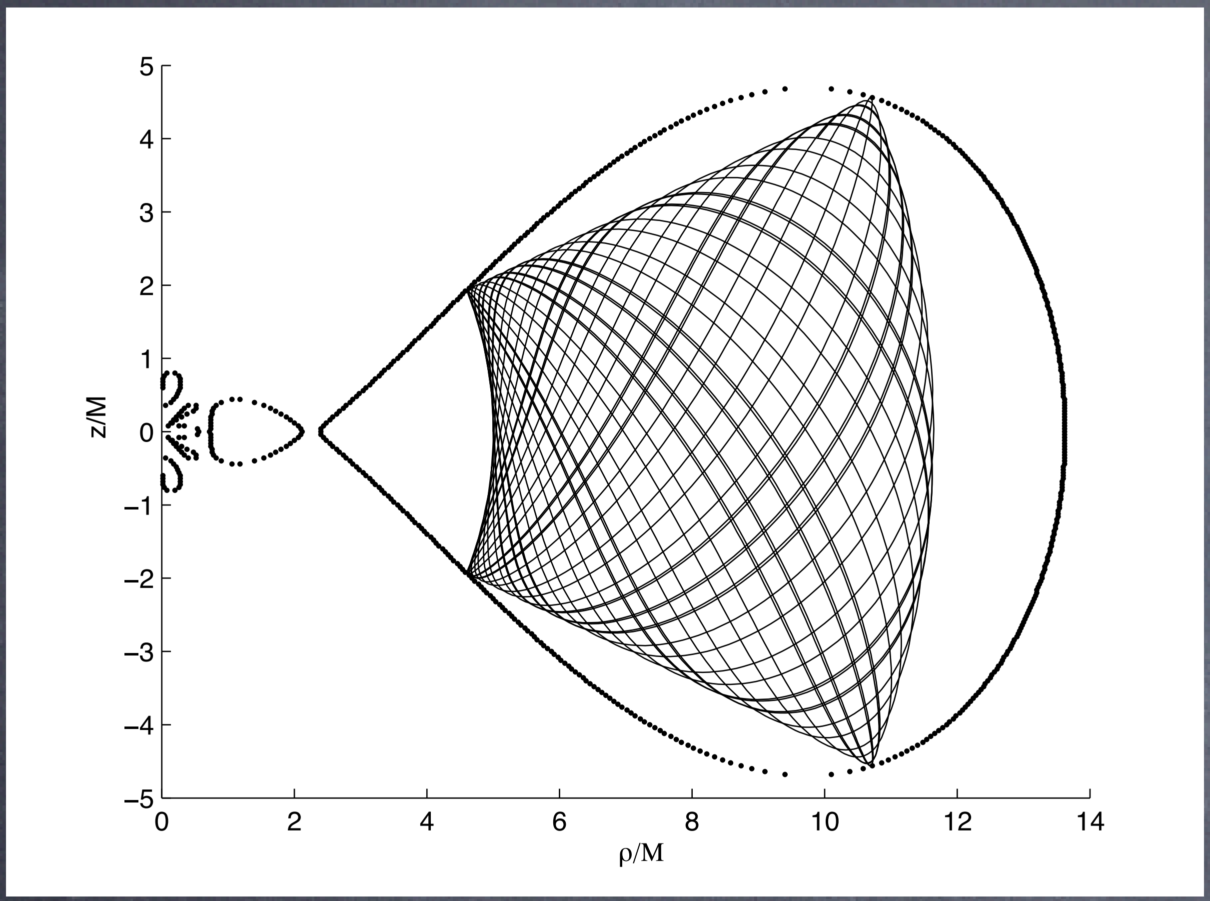
Poincare map in a bumpy spacetime, second look

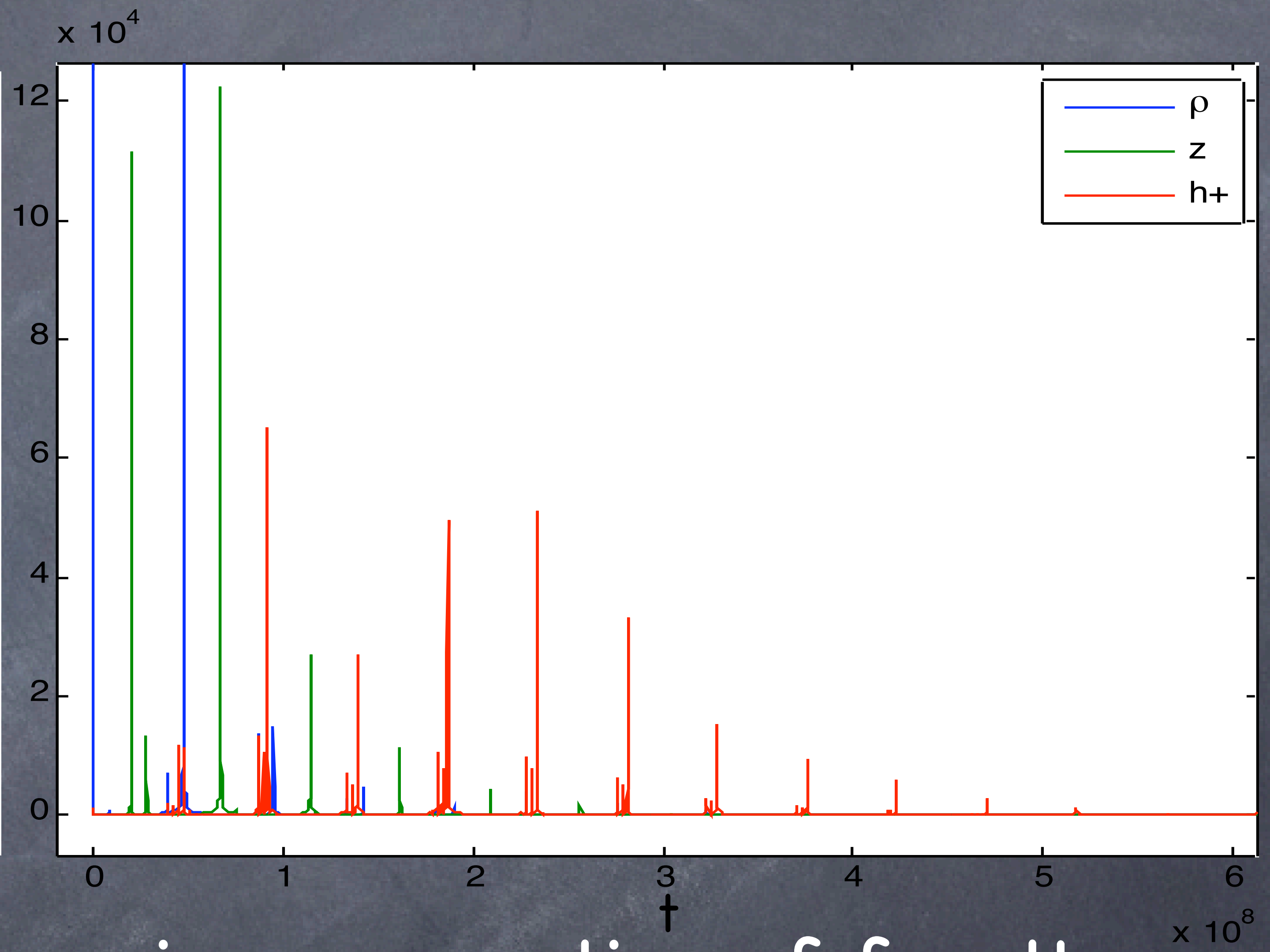




$E=0.95$, $L_z=-3$, $a/M=0.9$, $q=0.95$

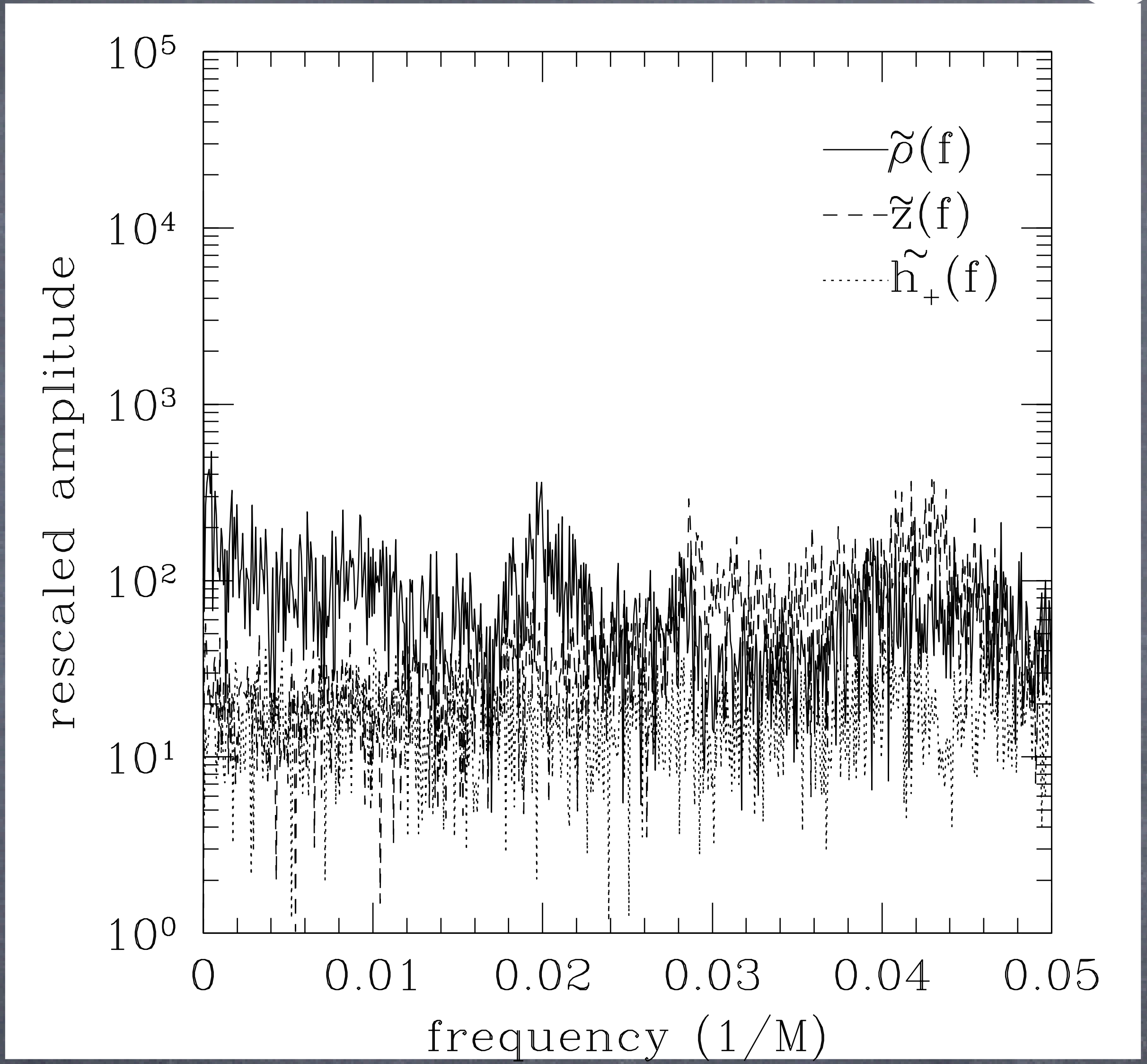
Regular outer region





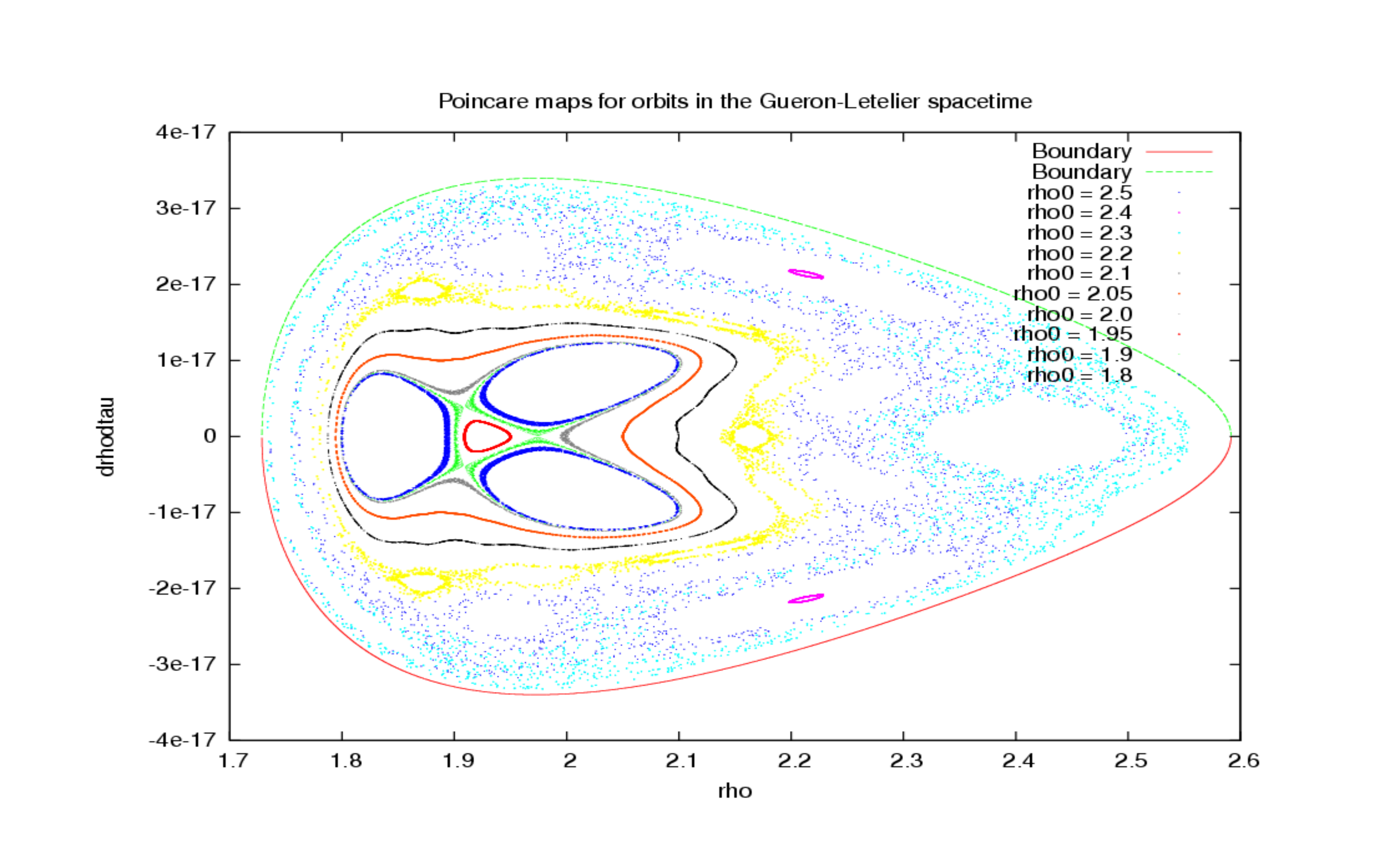
- Regular motion in outer region, suggestive of fourth-degree invariant
- Both ρ and z motion consist of harmonics of two fundamental frequencies to 10^{-7}

Chaotic inner region

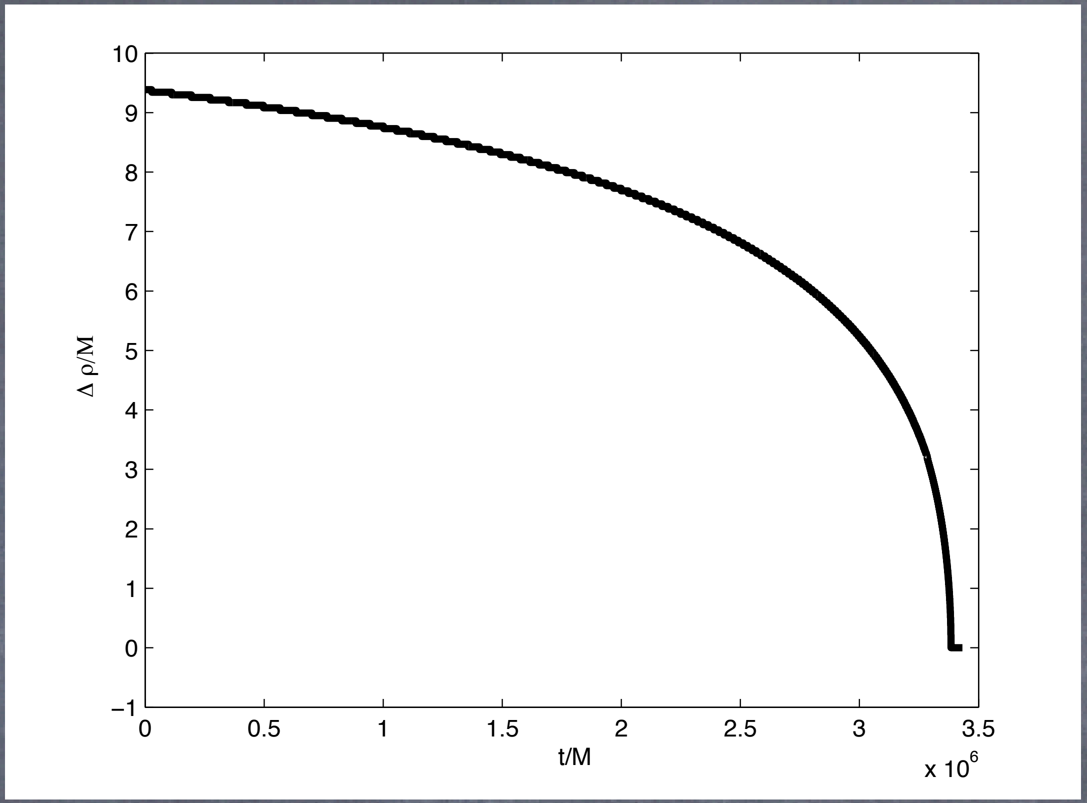


- If motion is chaotic for any initial conditions, it is chaotic for all initial conditions, but an approximate invariant may exist in some cases (invariant tori) [KAM Theorem]

Chaos in Gueron-Letelier spacetime



Is chaos accessible?

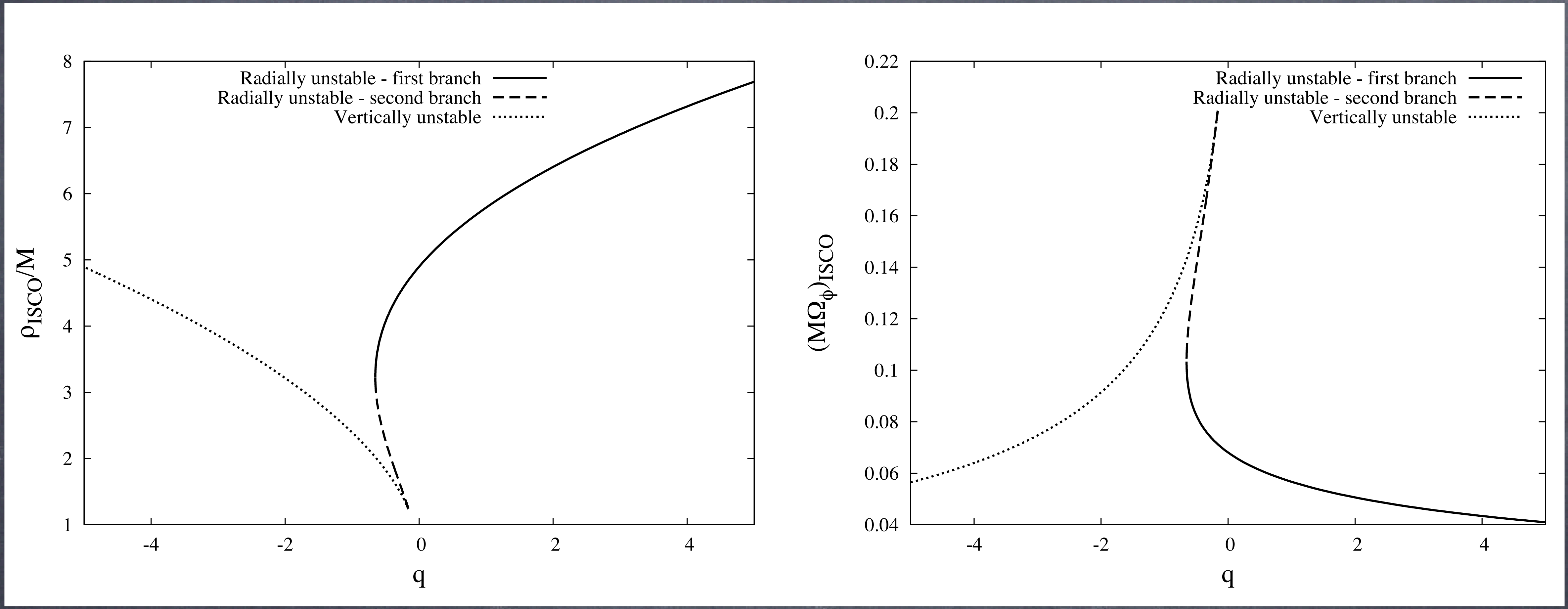


- Inner and outer regions appear to merge under radiation reaction, but never split
- Object starts out in outer, regular region; once the two regions are fully merged, motion is regular (but odd things may happen when the neck is narrow...)

Other observable signatures of bumpiness

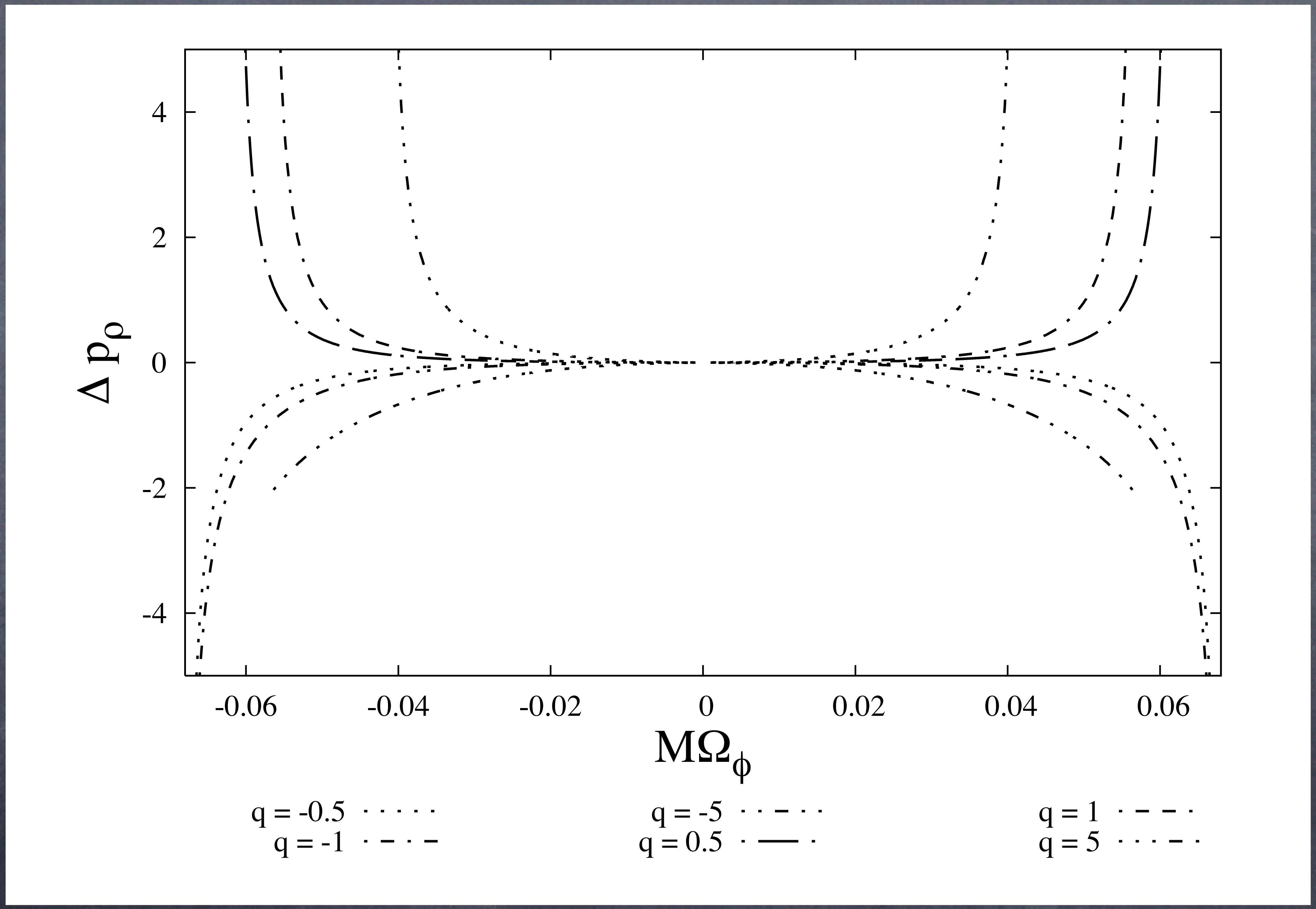
- If the orbits are indeed multi-periodic, then the spacetime “bumpiness” should be observable via:
  1. three fundamental frequencies of gravitational waves
  2. harmonic structure of the waves (relative frequencies and phases of harmonics)
  3. evolution of these with time over inspiral
- Further study required to properly analyze inspiral

Location of innermost stable circular orbit (ISCO)

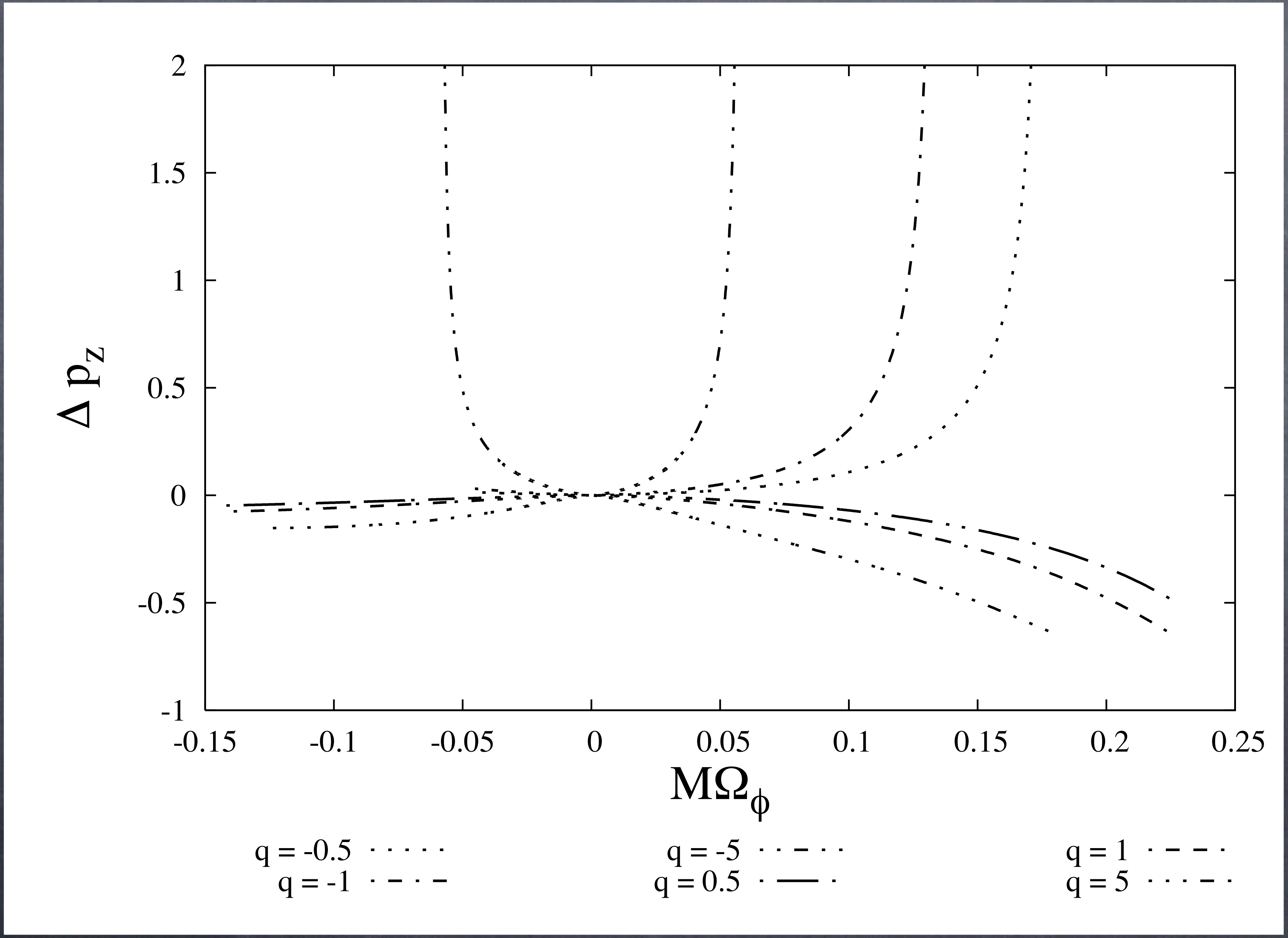


- The ISCO frequency (and hence plunge frequency) depends on the value of the spacetime quadrupole moment

Periapsis precession



Orbital-plane precession



Summary

- Advanced LIGO could detect a few IMRIs per year
- Eccentricities will be low, circular waveforms can be used for detection (But should we use EMRI waveforms? Hybrid waveforms? ...?
- Gravitational waves from EMRIs should make it possible to test whether the central body [SMBH] is a Kerr black hole
- Chaos in a non-Kerr spacetime would be an obvious smoking gun, but chaotic regions are probably not accessible
- Location of ISCO, periapsis precession, and orbital-plane precession are possible observables indicating bumpiness
- Frequency evolution over inspiral would be another observable, but more work is required

Do I really believe that IMBHs exist and MBHs are not black holes?

- I don't know. But it's dangerous to assume that one will see only what one expects to see. We should be prepared to test our assumptions.

- Every time a new part of the electromagentic spectrum was accessed (radio-astronomy, X-rays, etc.), something unexpected was seen. Gravitational waves are a new window to the universe: expect to see the unexpected!