Time-Frequency Analysis of Extreme-Mass-Ratio Inspiral Signals:

Mock LISA Data Challenge, Round 2

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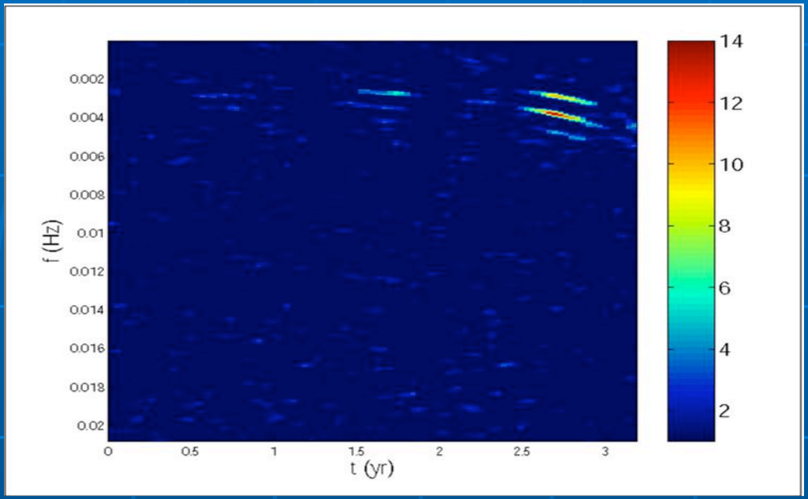
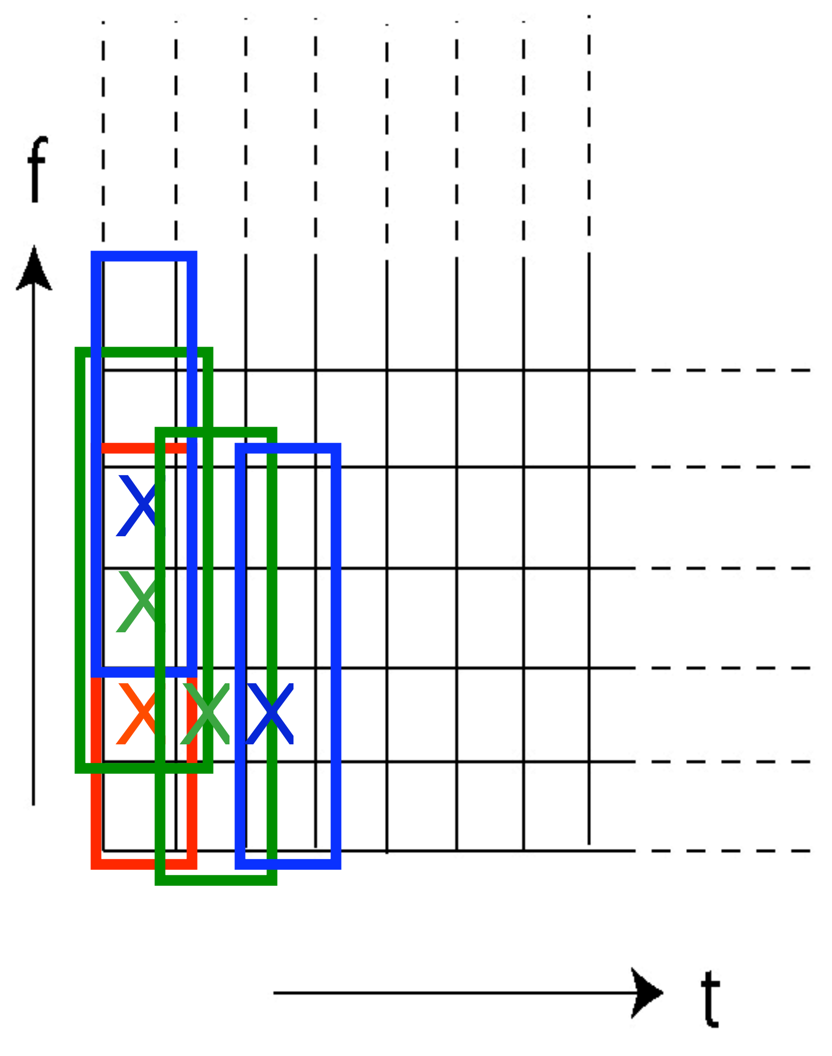
Challenges of EMRI Data Analysis

- LISA may detect tens to thousands of extreme-mass-ratio inspirals (EMRIs) of compact stellar-mass objects into massive (~10^6 solar mass) black holes per year (rate prediction is subject to astrophysical uncertainties).
- EMRI signals are weak but long-lived. This suggests that they should be extracted using matched filtering. However, due to size of signal parameter space (17 parameters), a fully coherent-matched filtering requires too many templates (10^{40}). Therefore, coherent grid-based matched-filtering is computationally impossible.
- Exploring the whole parameter space in a Markov-Chain Monte Carlo (MCMC) search may also be intractable, unless a better parameter estimate is obtained first.
- Time-frequency techniques have been proposed as a solution [Wen & Gair, 2005; Gair & Wen, 2005]
- Searches in time-frequency spectrograms can quickly identify possible EMRI tracks (if the signal-to-noise ratio is sufficient), sometimes by eye; subsequent parameter extraction can serve to limit the region of parameter space for a grid-based search or as a seed for MCMC.

Track search in a time-frequency spectrogram



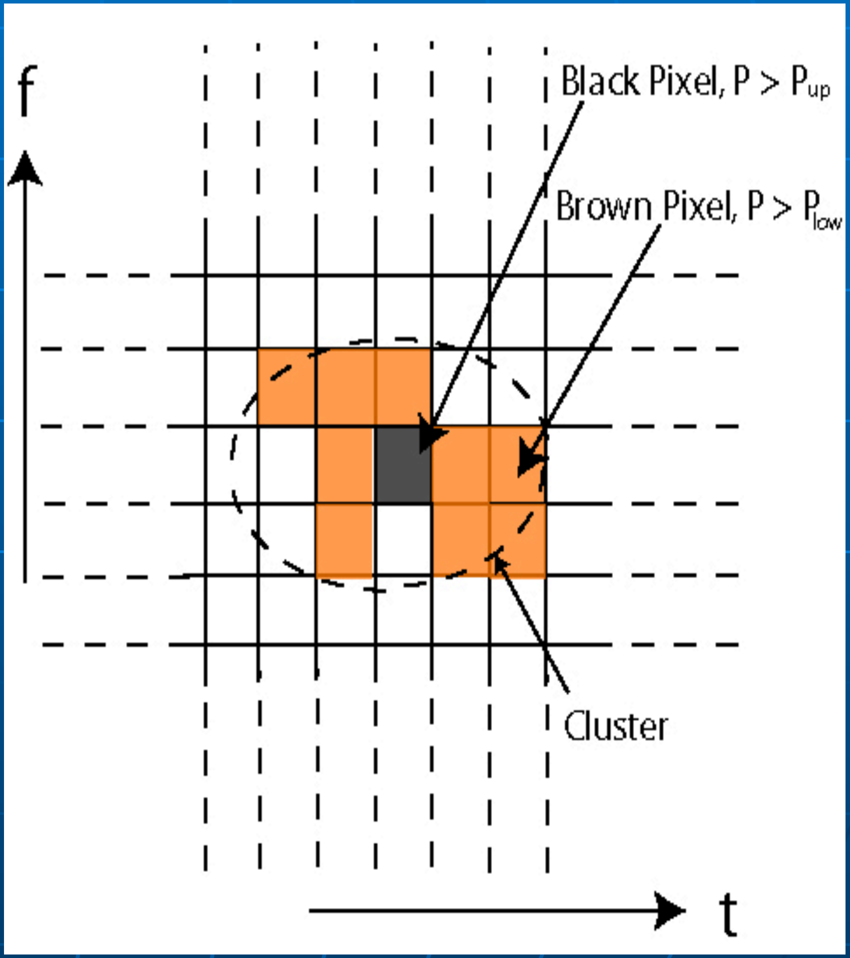


- Simplest method is to look for regions of excess power
  - Construct binned spectrograms using rectangular bins of 2^n x 2^m.
  - Search for bright pixels.
- Can detect typical EMRIs out to 2Gpc (Wen & Gair) under ideal assumptions.
- Cannot handle source confusion or

Hierarchical Algorithm for Clusters and Ridges (HACR)

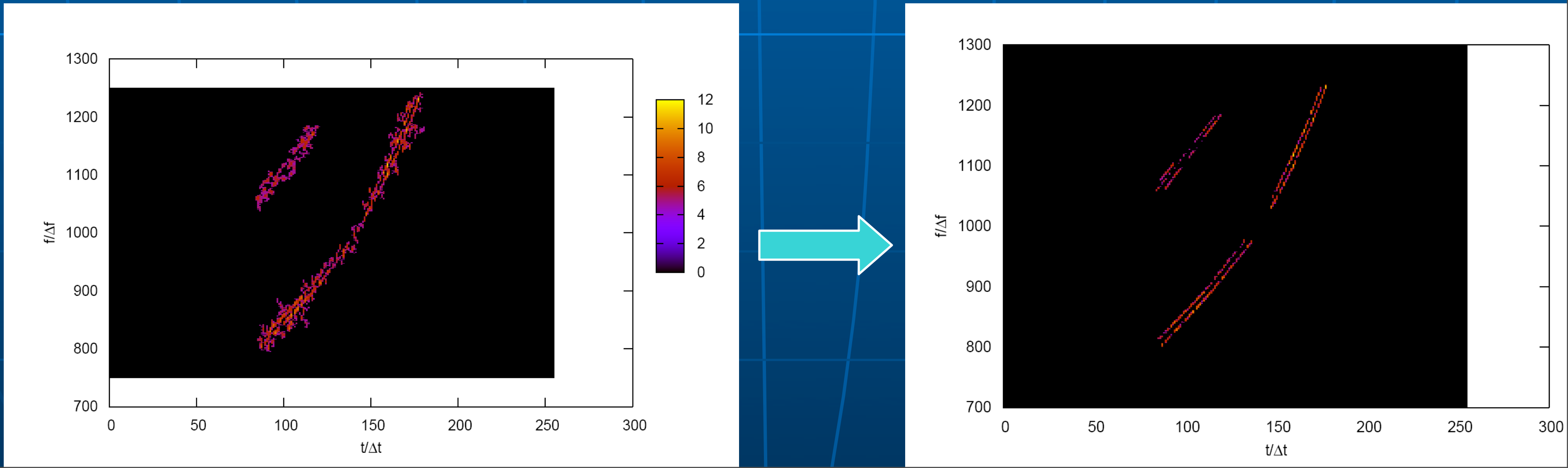
- HACR is an improvement on excess-power search [Gair & Jones, 2006]

1. Search for 'black pixels', i.e., pixels with power greater than an upper threshold, P_{up}.
2. Count the number of 'brown pixels' (i.e., pixels with power greater than a lower threshold, Plow) in a contiguous group (a cluster) connected to each black pixel.
3. If a cluster has more than a threshold number of pixels, Np, it constitutes a detection.



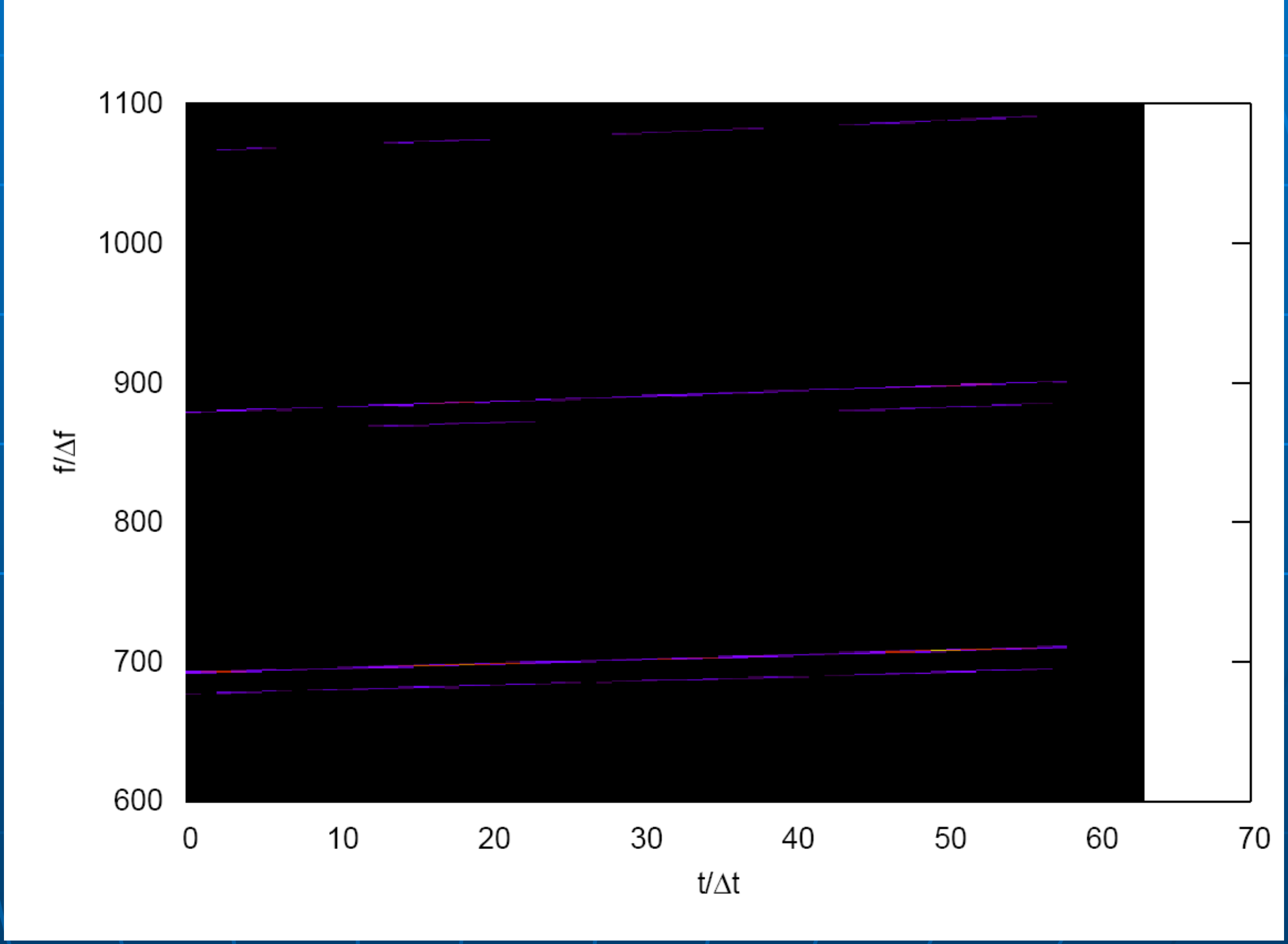
Prescription for time-frequency search

- Tune HACR thresholds via training set data to get a 0.1 false alarm probability per LISA mission
- Bandpass and whiten (using expected instrumental noise) the A, E, and T data streams
- FFT segments 2^{16} data points long (~11 days); obtain a spectrogram with 64x32768 pixels
- Search summed A+E spectrogram with HACR
- Clean clusters with piecewise linear filters and percolation
- Estimate parameters from detected tracks



Information contained in a spectrogram

- A typical set of tracks consists of harmonics of the azimuthal orbital frequency, corrected by the periapsis precession, with sidebands due to orbital-plane precession around the black-hole spin axis.
- Tracks extracted Challenge 1.3.1 data are shown at right.
- Waveforms are based on [Barack & Cutler, 2004]

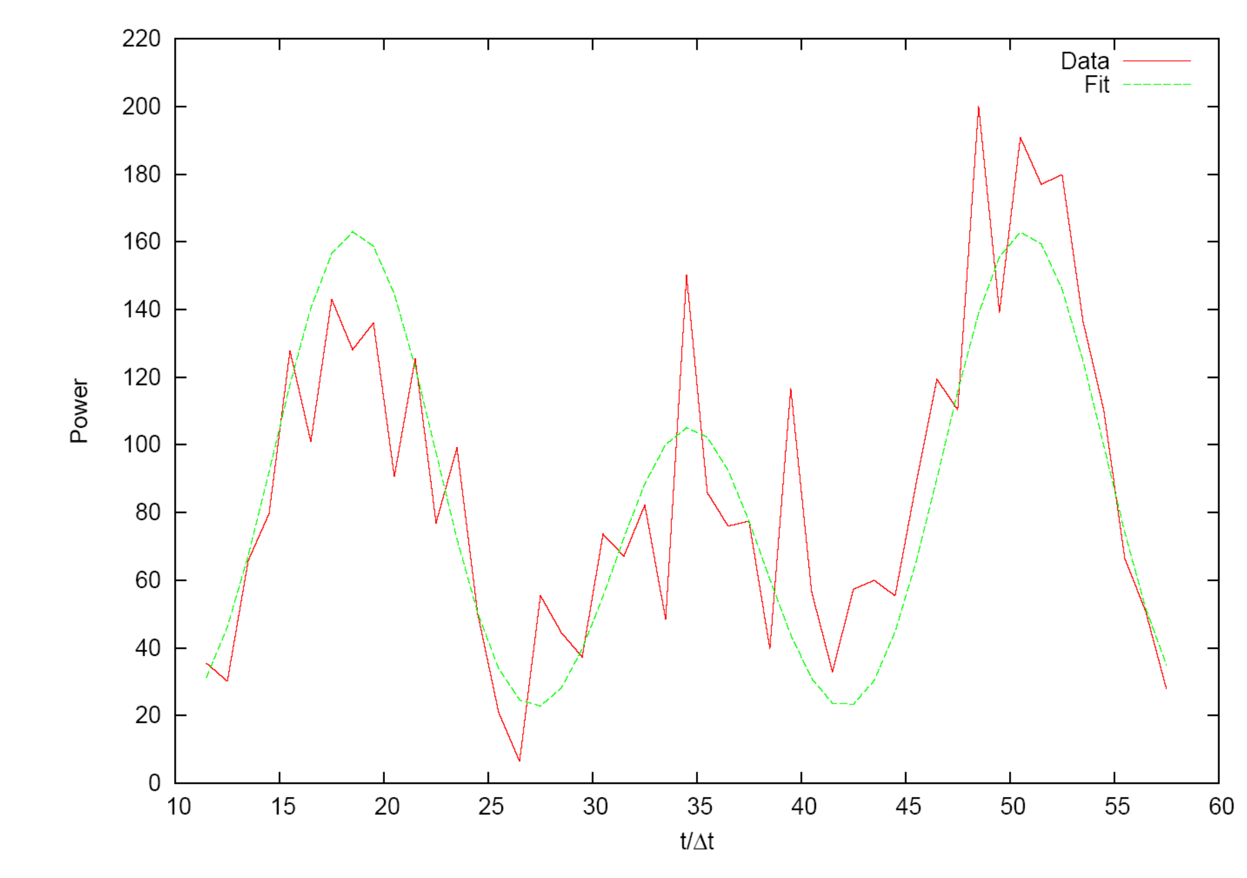


Information contained in a spectrogram

- Information is contained in the following spectrogram features:
  - Separation between main harmonics -> azimuthal frequency
  - Absolute frequency of given harmonic -> periapsis precession frequency
  - Separation of sidebands -> orbital precession frequency
  - Plunge frequency
  - Frequency evolution in time (linear and, possibly, quadratic)
- From these, we can estimate the following parameters:
  - Azimuthal frequency
  - Eccentricity
  - Black-hole spin
  - Orbital plane inclination
  - Mass of massive black hole
  - Mass of inspiraling object
- Relative power in different tracks could be used to improve estimates of eccentricity and detect orientation of black-hole spin, θ_K and φ_K; we did not incorporate this information yet.
- Time-frequency techniques are not sensitive to the phase angles

Sky position estimation

- Estimate sky position by fitting the signal amplitude variation along the strongest harmonic to the approximate form derived from [Cutler, 1998].
- Results for Training Set 1.3.2 are encouraging: $\theta_s=1.78$ (actual $\theta_s=1.83$), $\varphi_s=3.64$ (actual $\varphi_s=3.62$) gave the best fit



$$\mathcal{M}(t) \propto (F_I^+)^2 + (F_I^\times)^2 + (F_{II}^+)^2 + (F_{II}^\times)^2 \propto 1 + 6\cos^2\theta + \cos^4\theta$$

$$\text{where } \cos\theta = \frac{1}{2}\cos\theta_S - \frac{\sqrt{3}}{2}\sin\theta_S\cos\left(\frac{2\pi t}{T} - \phi_S\right)$$

Results Summary

- We detected n=2,3,4 harmonics and sidebands for Challenges 1.3.1 and 1.3.2; n=2,3 harmonics and at least one sideband for Challenges 1.3.3 and 1.3.4; and could not detect a signal for Challenge 1.3.5.
- There are near-degeneracies between some of the EMRI parameters, and our estimates were not perfect; however, we could detect some parameter combinations (e.g., those corresponding to the periapsis precession frequency and orbital-plane precession frequency) quite accurately, which can be useful for constraining parameters
- We can constrain the source sky position if the signal is sufficiently strong
- Further improvements that we intend to develop include:
  - Optimizing track search techniques
  - Adding signals from A and E channels coherently by using sky location estimates
  - Fully utilizing all track information, including relative power in different harmonics
  - Automating all aspects of the search
  - Interfacing the search directly with other search techniques in a multi-stage search