Testing the No-Hair Theorem with Gravitational-Wave Observations

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October 23, 2007
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based on "Observable Properties of Orbits in Exact Bumpy Spacetimes", arxiv:0708.0628,
by Jonathan R. Gair, Chao Li, and Ilya Mandel

What is the “no-hair theorem”?

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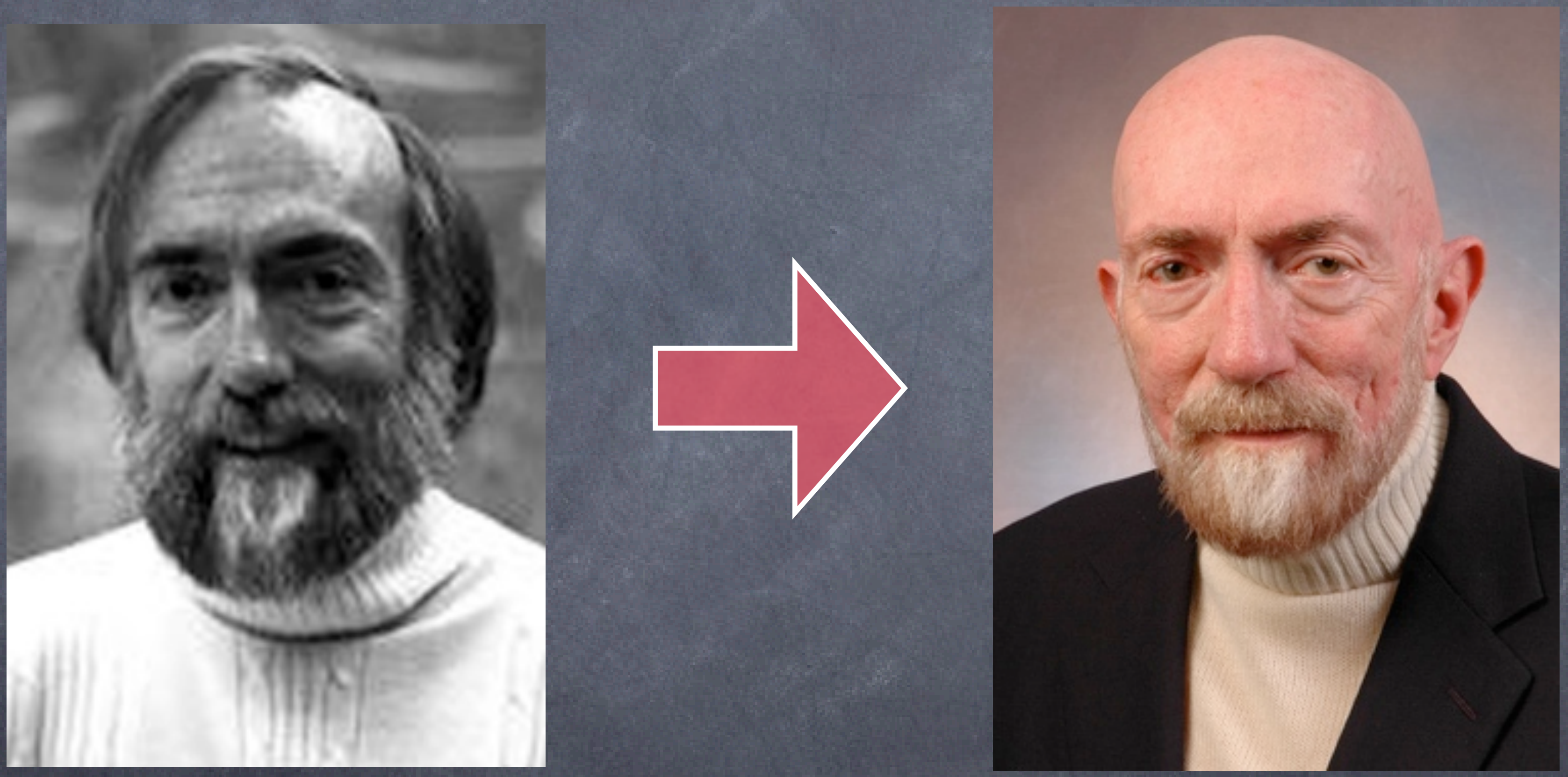
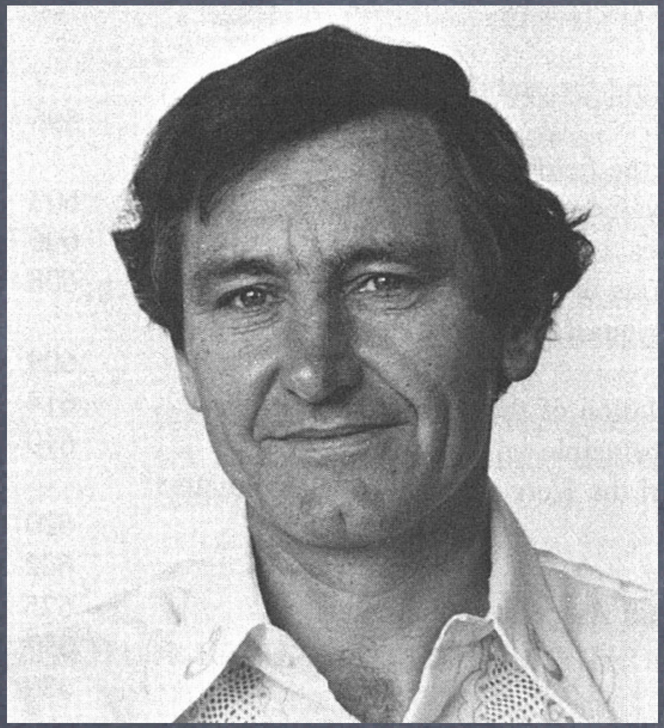


idea taken from Daniel Shaddock

What is the "no-hair theorem"?



Stationary, vacuum, asymptotically flat spacetimes in which the singularity is fully enclosed by a horizon with no closed timelike curves outside the horizon are described by the Kerr metric

The no-hair theorem in English

- "Black holes have no hair" means that all higher-order mass and current multipole moments are uniquely determined by the black hole mass and spin
- Conversely, an object with hair is one for which $M_n + iS_n \neq M(ia)^n$
- The "no-hair theorem" is a mathematical statement, so the title is a bit of a misnomer...

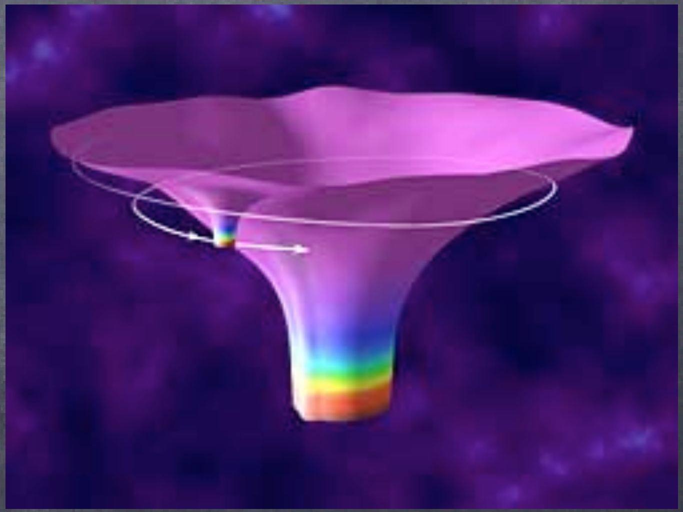
New title: do supermassive black holes have hair?

- Supermassive black holes (SMBHs) are assumed to exist in the cores of most galaxies...
- But are they really black holes?
- Could they be boson stars, or naked singularities, or...?
- Need to measure 3 multipole moments to test "Kerrness", 4 to test if an object is a boson star
- Search for exotic massive compact objects, test of cosmic censorship conjecture, null hypothesis test of the no-hair theorem...

How can we measure the SMBH multipole moments?

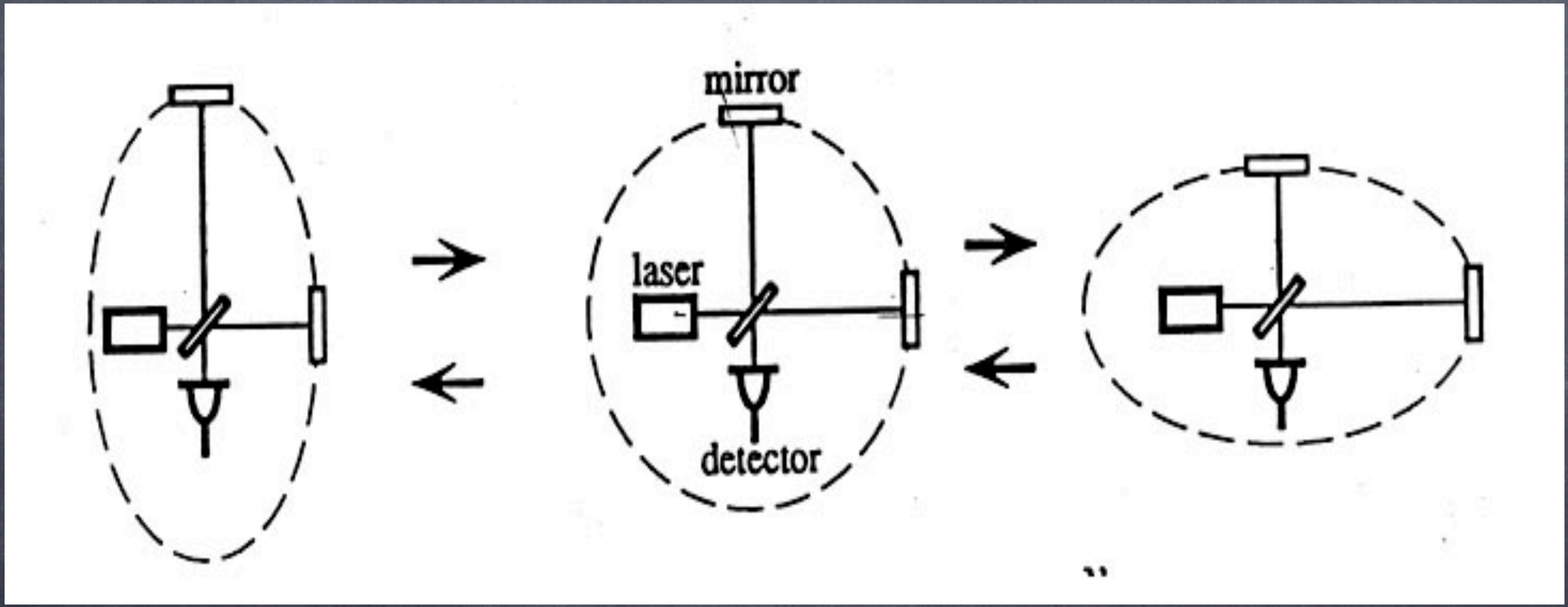
- Extreme-mass-ratio inspirals (EMRIs) are inspirals of stellar-mass compact objects (white dwarfs, neutron stars, stellar-mass black holes) into SMBHs under the influence of radiation reaction from gravitational-wave (GW) emission

- EMRIs are great probes of the strong-field regime (# cycles ~ M/m)



- Information about the spacetime structure and the orbit should be contained in the GWs; how do we access it?

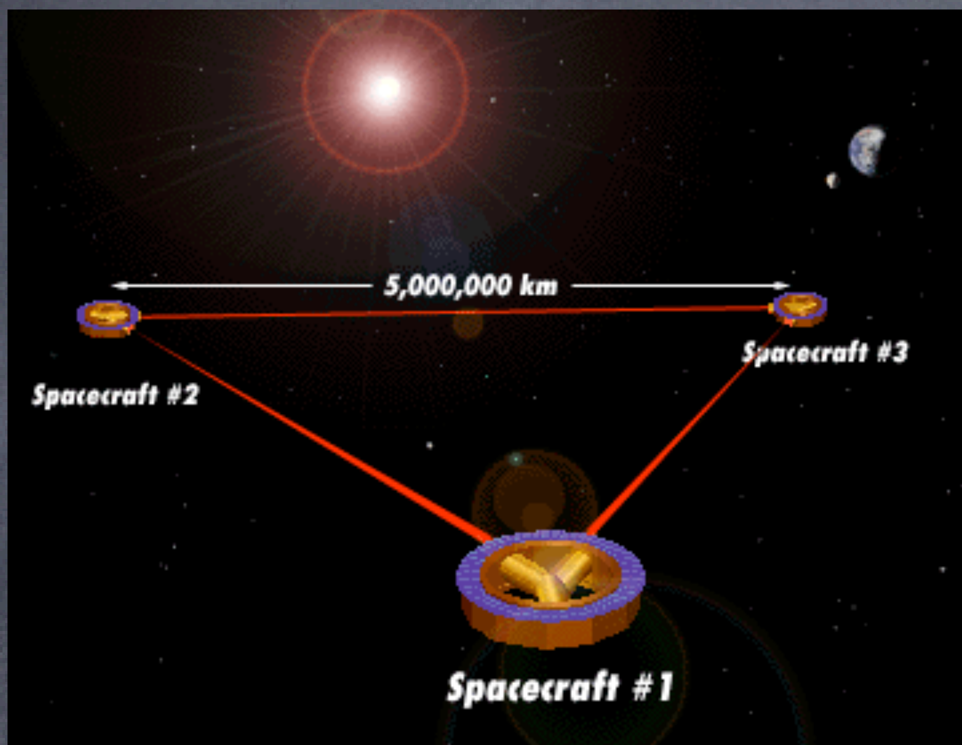
GW detection with Michelson-type interferometers

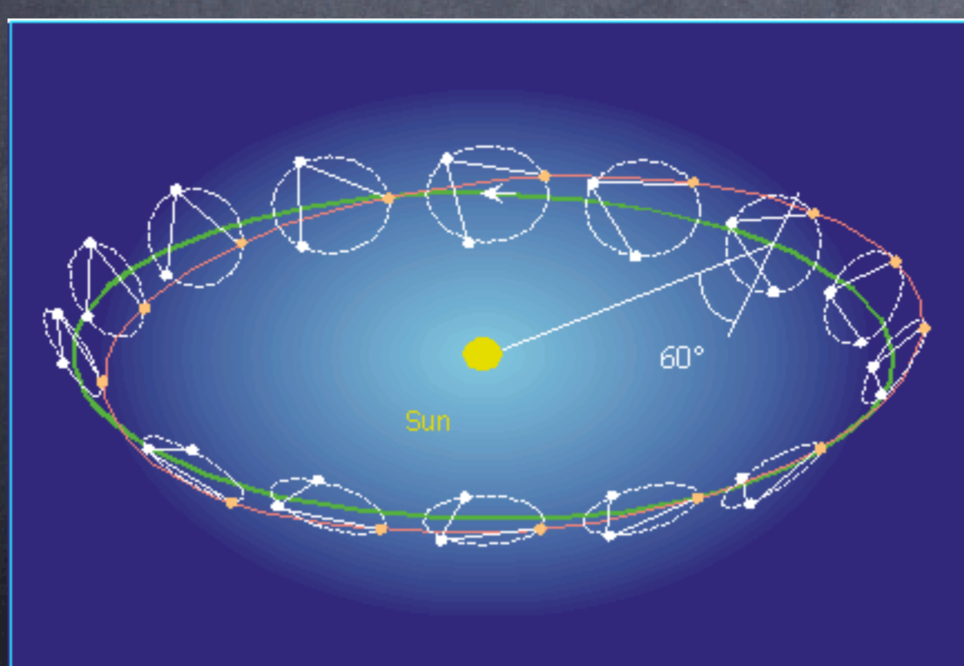


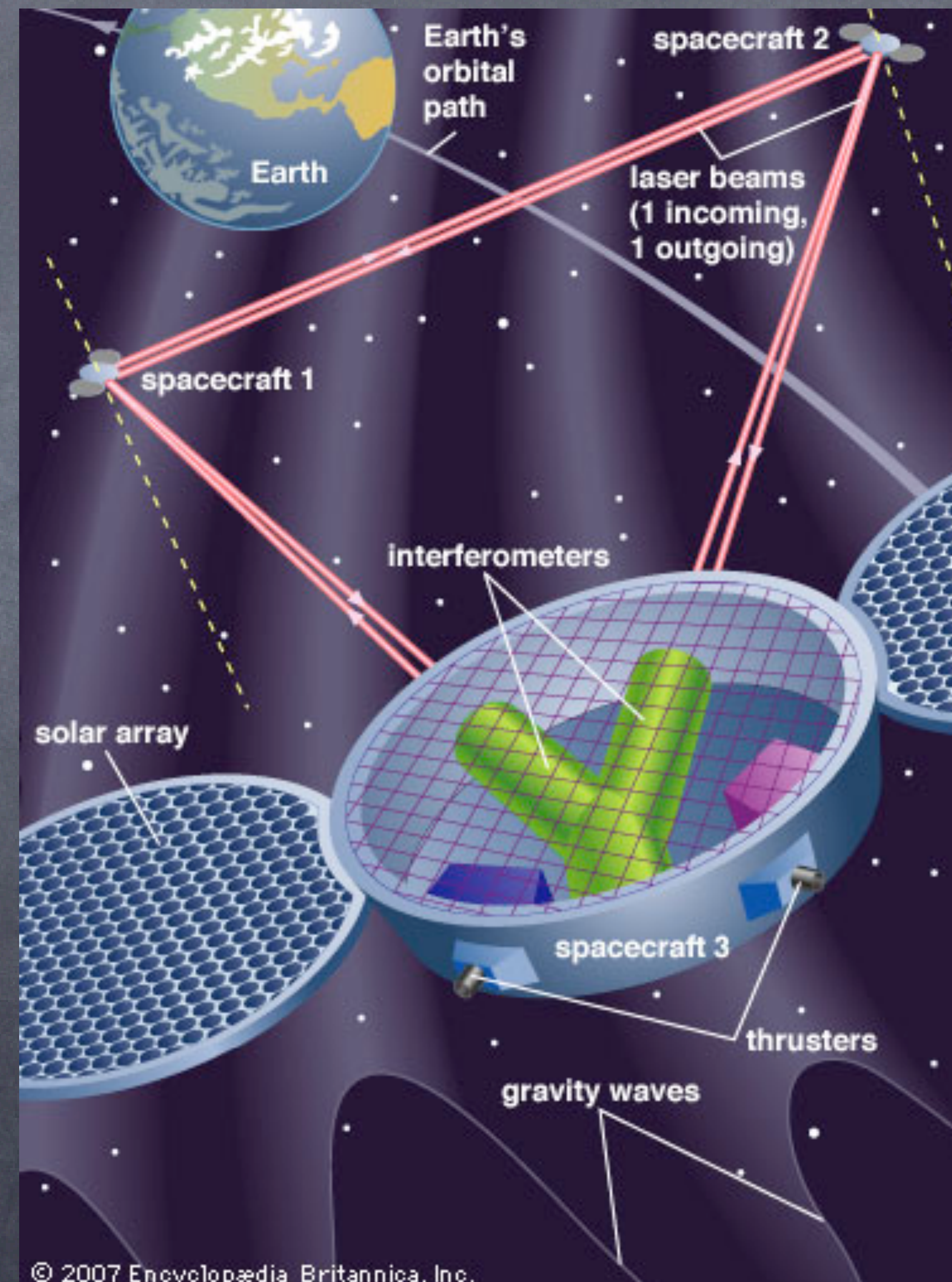
LIGO

- 4 km long arms
- Typical strains $h = \Delta L/L \sim 10^{-21}$–$10^{-22}$
- Needs to measure $\Delta L = hL \sim 10^{-17}$ m
- Peak sensitivity at frequency ~100 Hz
- Sources: stellar-mass binaries (NS, BH)

LISA


5,000,000 km
Spacecraft #2
Spacecraft #3
Spacecraft #1


60°
Sun


Earth's orbital path
Earth
spacecraft 2
laser beams (1 incoming, 1 outgoing)
spacecraft 1
interferometers
solar array
spacecraft 3
thrusters
gravity waves
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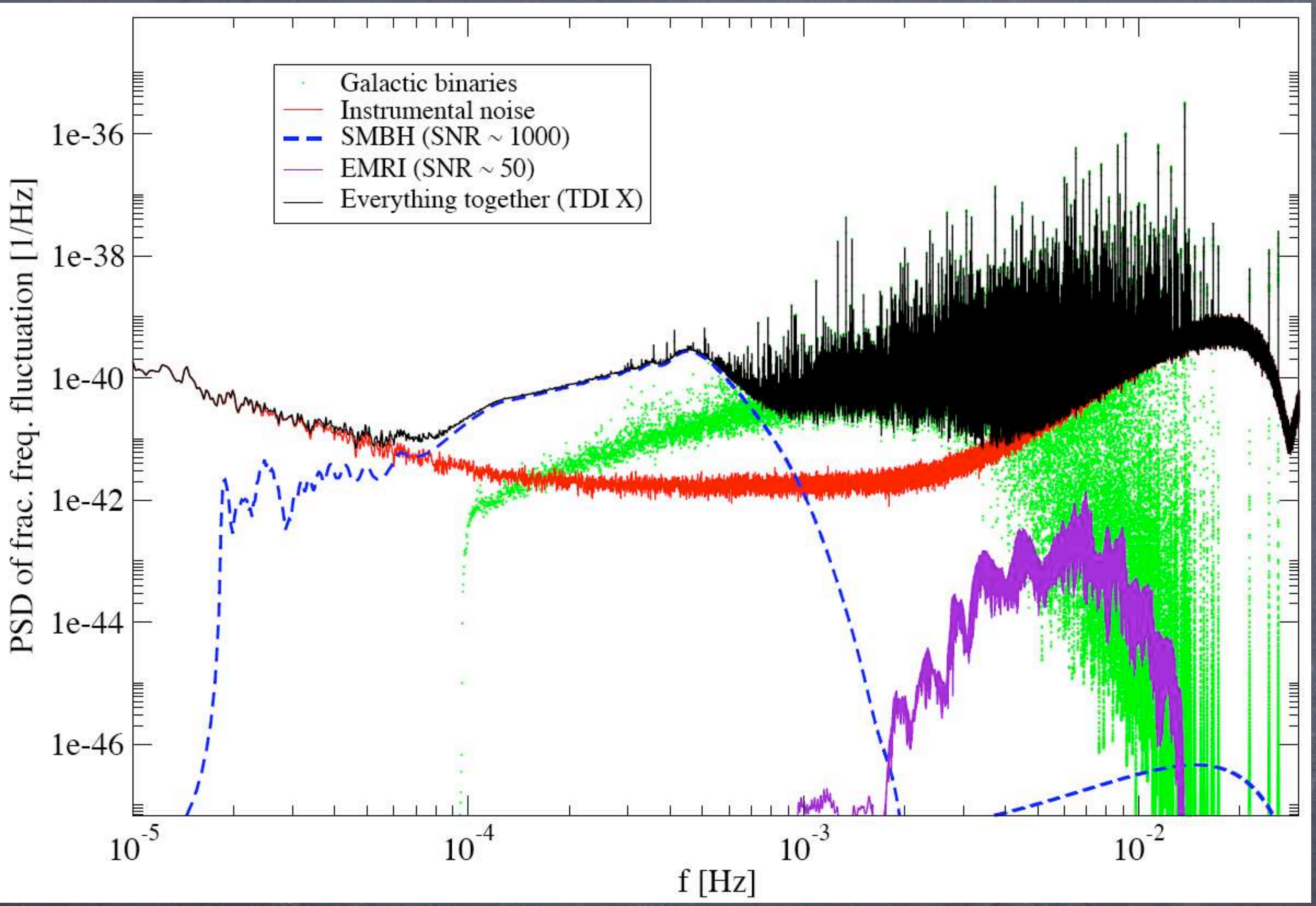
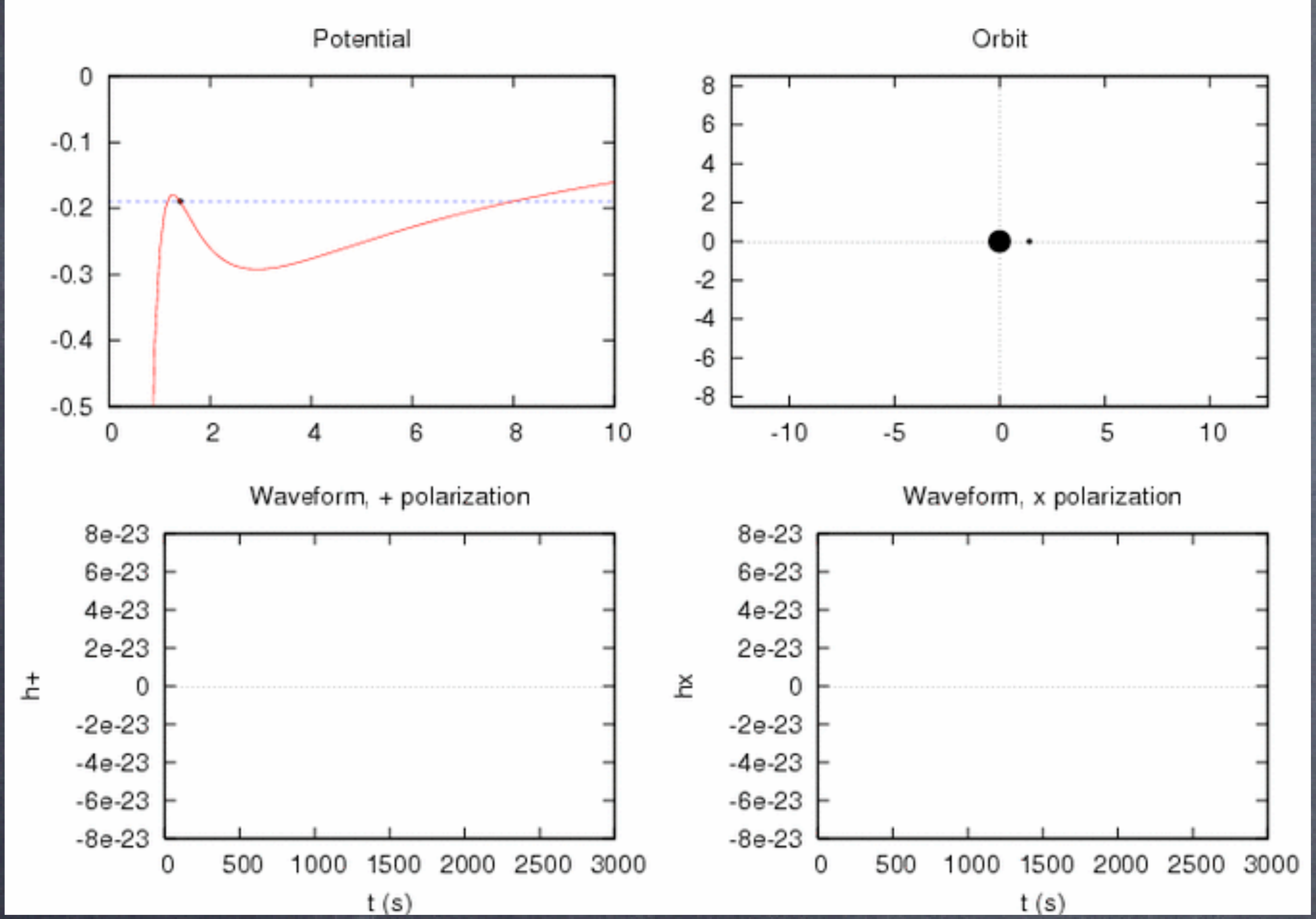
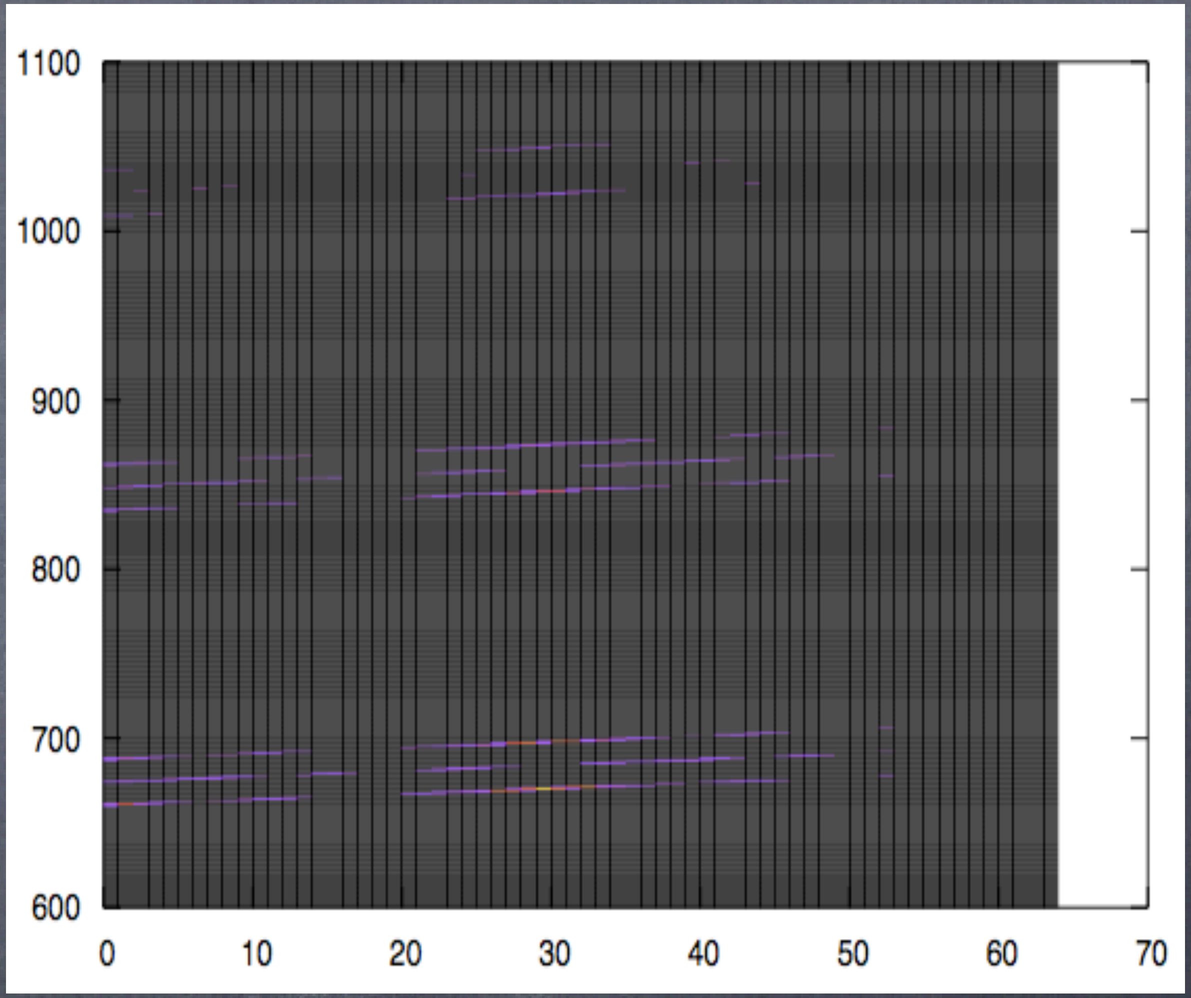
LISA



- Peak sensitivity at frequency ~ 1 mHz
- Typical sources: SMBH binaries, galactic WD binaries, EMRIs

EMRI sights and sounds





Jonathan Gair

EMRI sounds

Observing deviations from Kerr with EMRIs

- LISA can detect tens to thousands of EMRIs
- Ryan's theorem [1995]: GWs from nearly circular, nearly equatorial orbits in stationary, axisymmetric spacetimes encode all of the spacetime multipole moments... in principle
- Can we extend this theorem? Are there obvious observable imprints of an anomalous, non-Kerr quadrupole moment (a "bumpy" spacetime)?
- Are energy E, angular momentum L_z and Carter constant Q conserved in a bumpy spacetime?

Geodesics in bumpy spacetimes

- Use Manko-Novikov bumpy spacetime

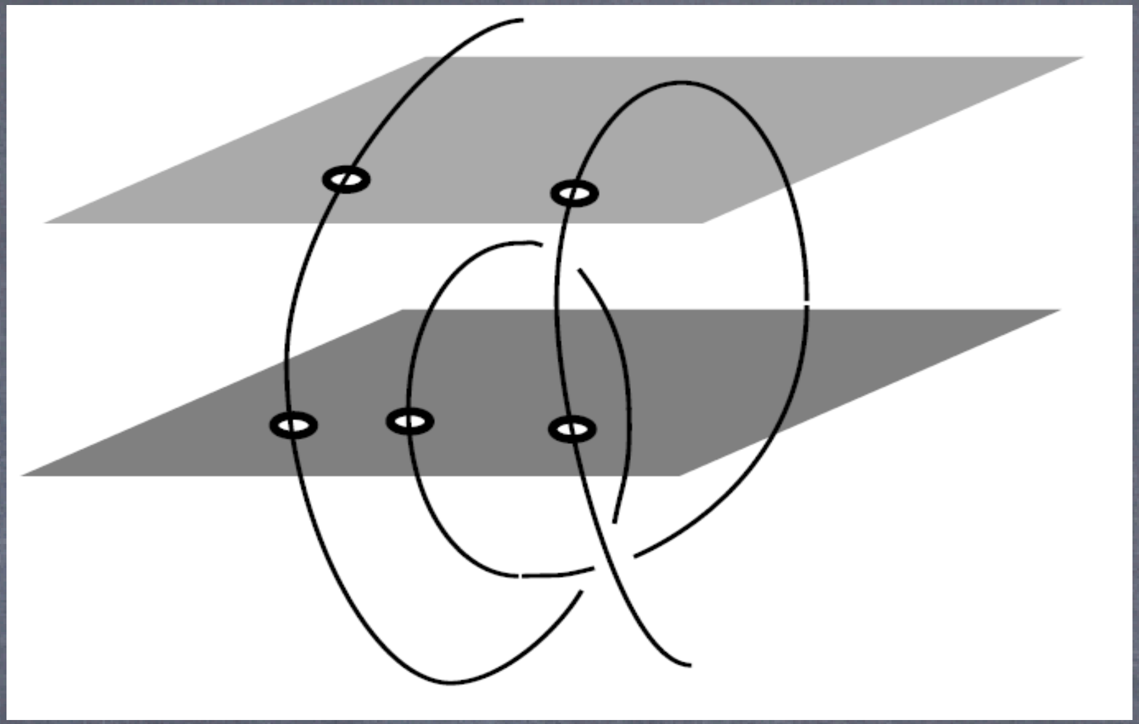
$$ds^2 = -f(\rho,z)\,\left(dt - \omega(\rho,z)\,d\phi\right)^2 + \frac{1}{f(\rho,z)}\left[e^{2\gamma(\rho,z)}\,\left(d\rho^2 + dz^2\right) + \rho^2\,d\phi^2\right]$$

- C code - geodesic equations:

$$\frac{\partial^2 x^\alpha}{\partial \tau^2} = -\Gamma^\alpha_{\beta\gamma}\frac{\partial x^\beta}{\partial \tau}\frac{\partial x^\gamma}{\partial \tau}$$

- Check conservation of E, L_z, 4-velocity norm
- Equations might not separate as in Kerr
- Is there a full set of integrals of motion?

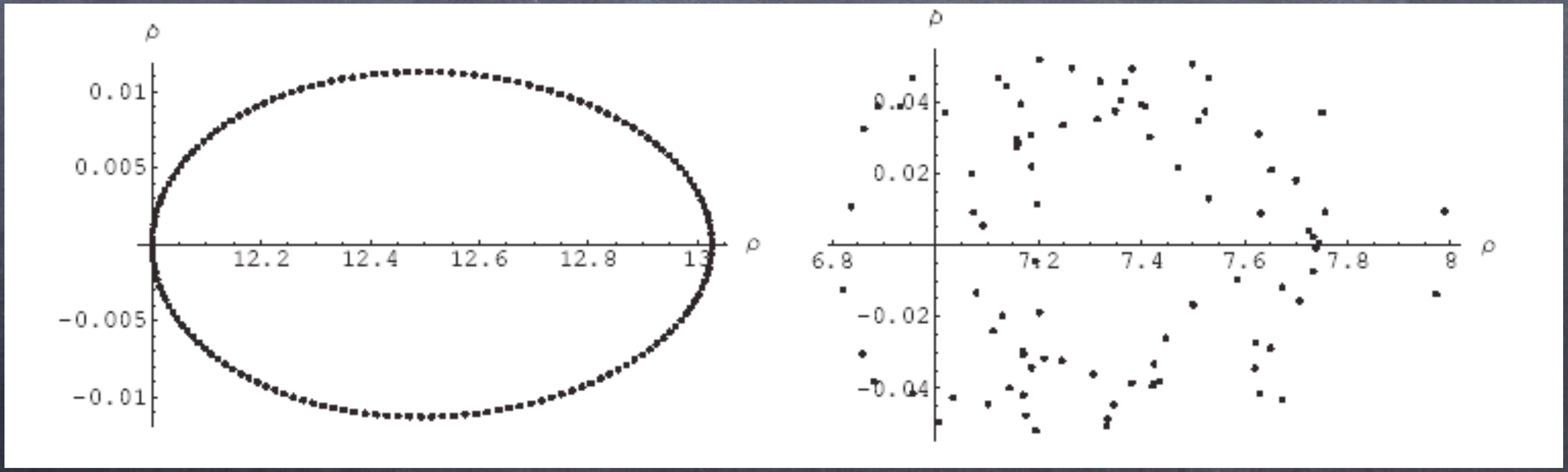
Poincare maps



- Check if spacetime has a full set of integrals of motion
- Plot $d\rho/dt$ vs. ρ for $z=z_0$ crossings
- Phase space plots should be closed curves for all z_0 iff there is a third isolating integral

Poincare maps for motion in Newtonian potential with hexadecapole moment

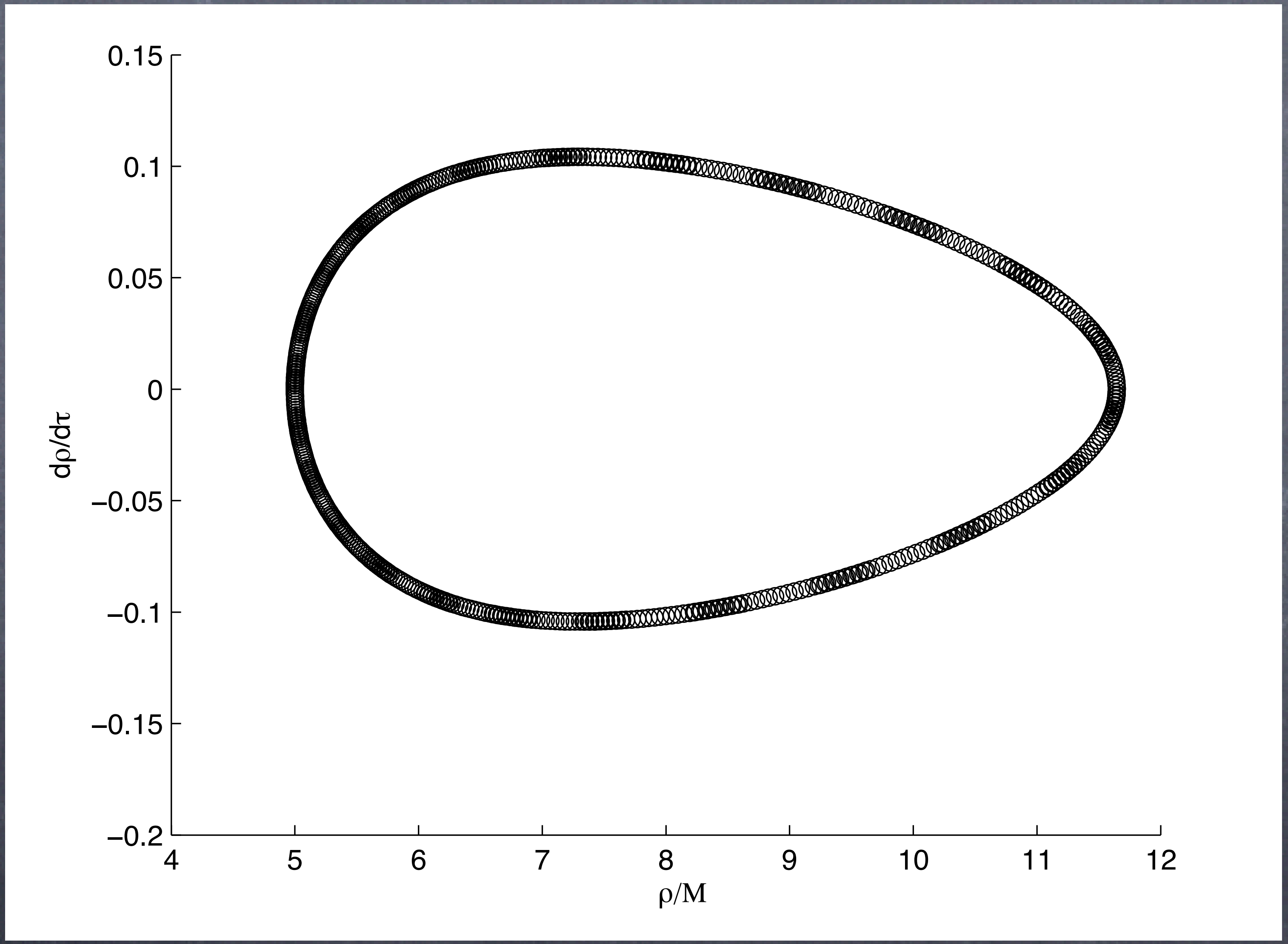
$$V(r,\theta) = -\frac{M_0}{r} + \frac{M_2}{r^3} P_2(\cos\theta) + \frac{M_4}{r^5} P_4(\cos\theta)$$



M_2=10 M_0; M_4=400 M_0

Li

Poincare map in a bumpy spacetime



$E=0.95$, $L_z=-3$, $a/M=0.9$, $q=0.95$

Allowed regions for bound orbits

Effective potential $(\dot{\rho}^2 + \dot{z}^2) = V(E, L_z, \rho, z)$

z

ρ

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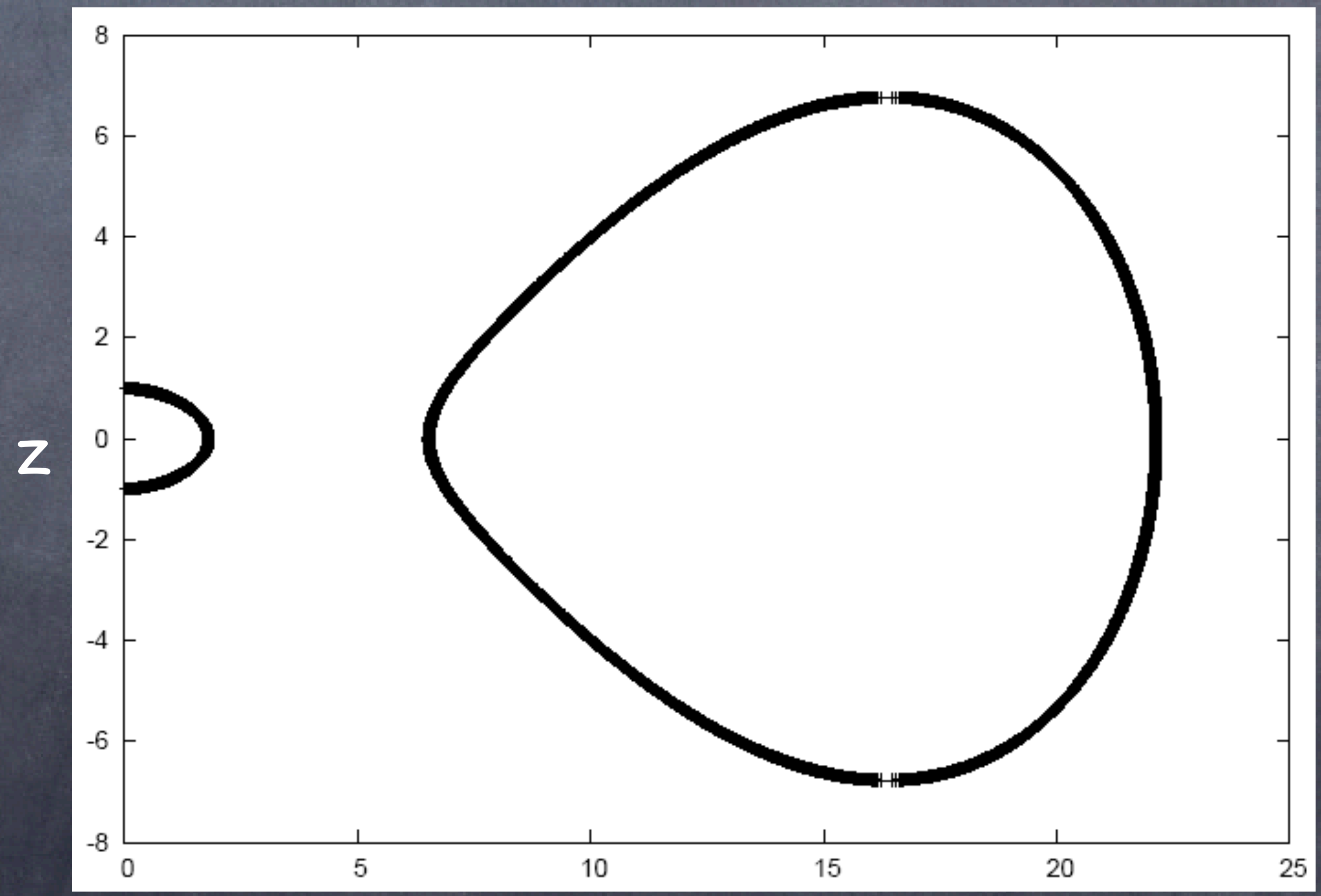

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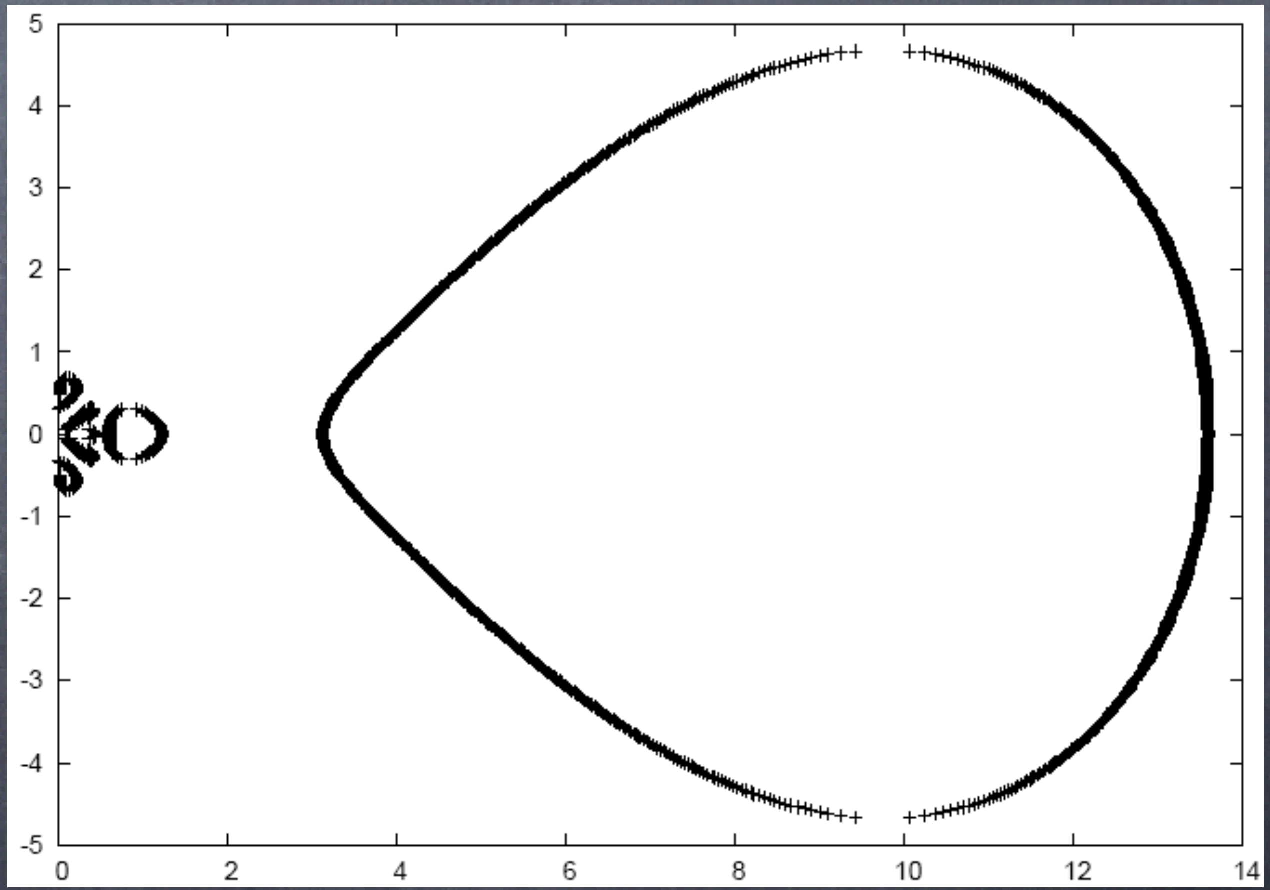

ρ

ρ

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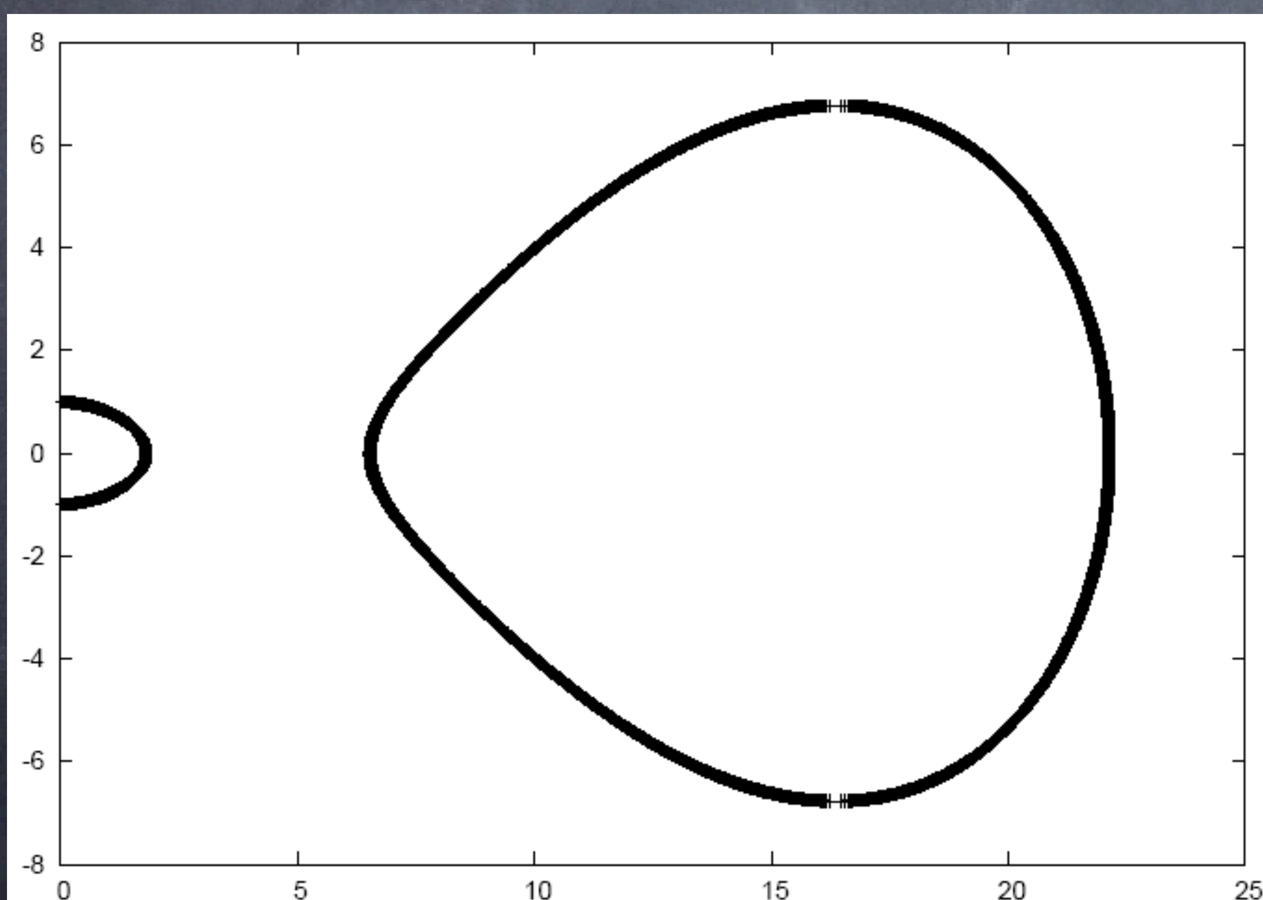
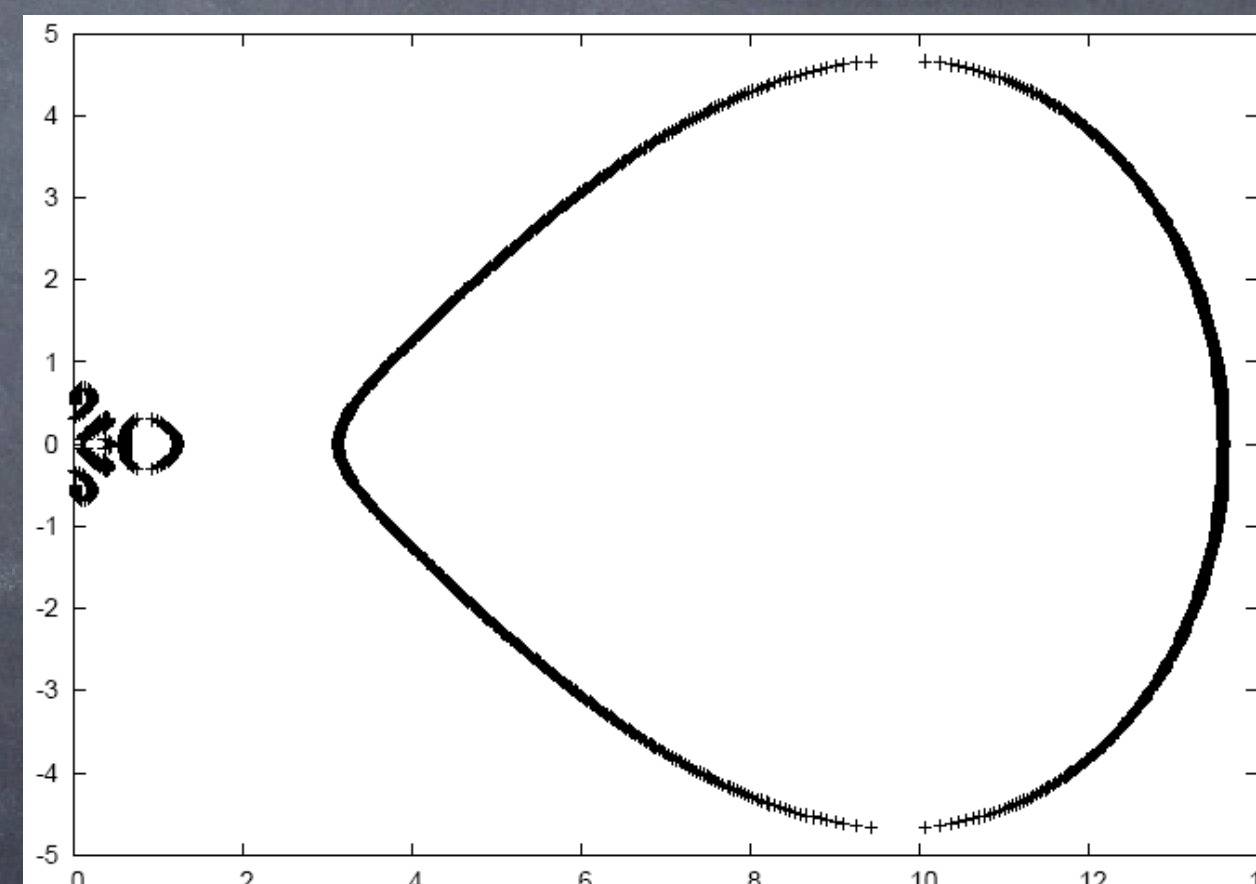
z 

ρ

z 

ρ

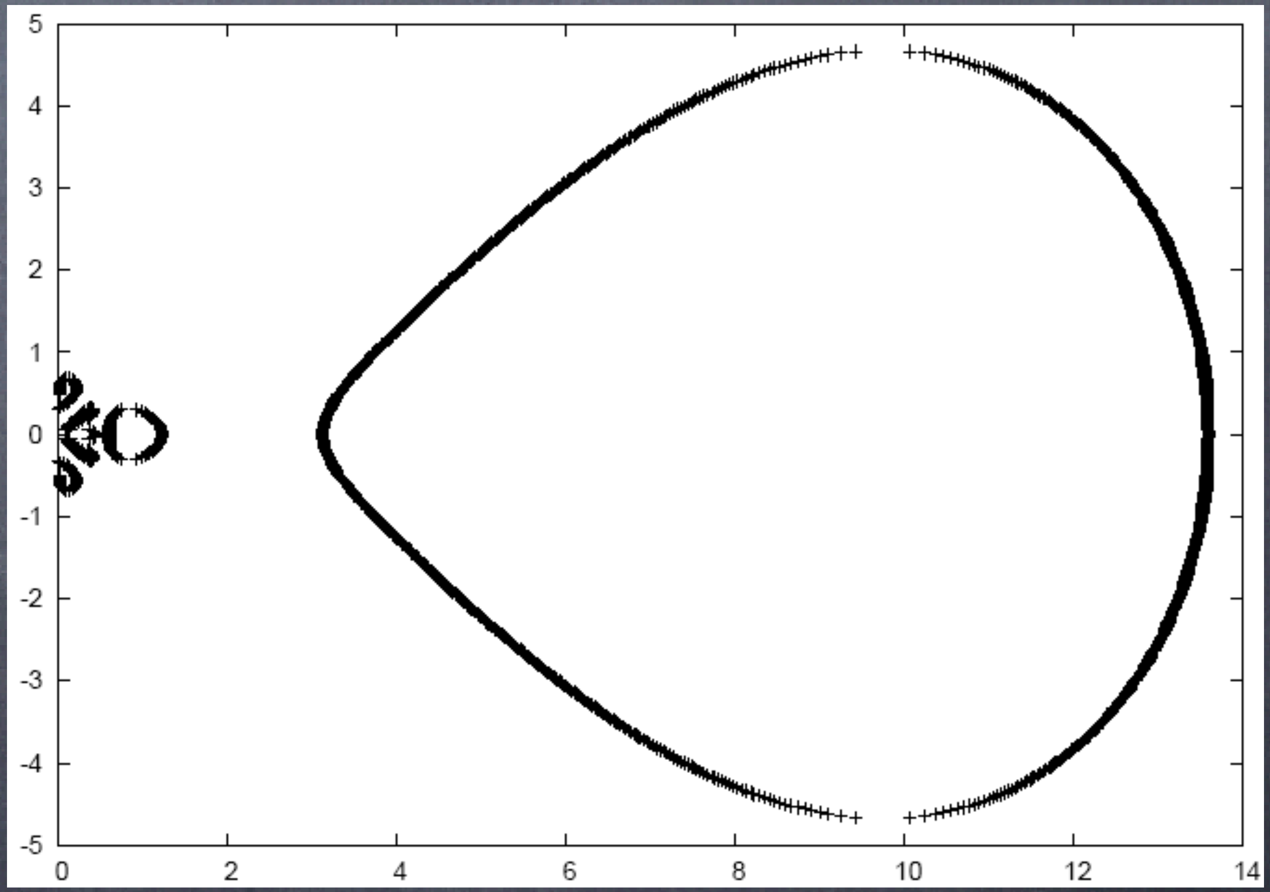
$E=0.95$, $L_z=-3$, $a/M=0.9$, $q=0.95$

Allowed regions for bound orbits

Effective potential $(\dot{\rho}^2 + \dot{z}^2) = V(E, L_z, \rho, z)$

z

z

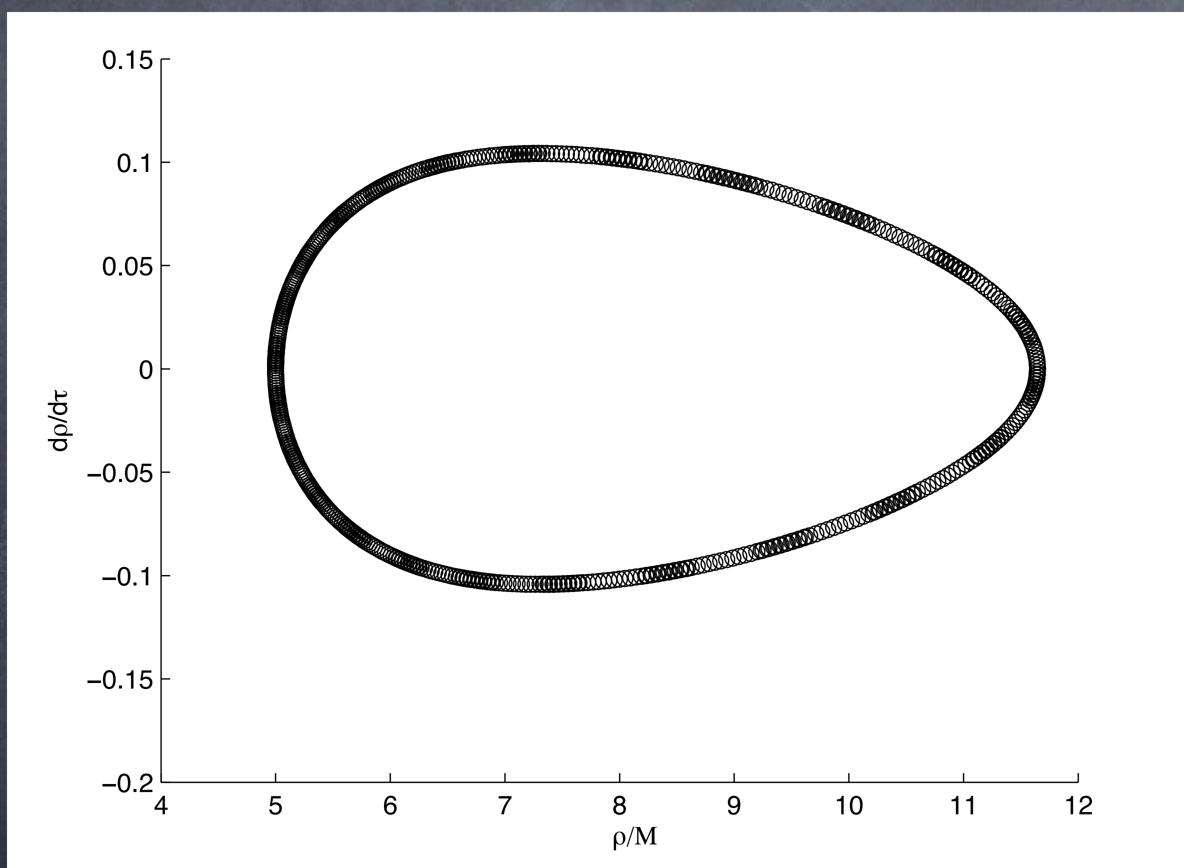


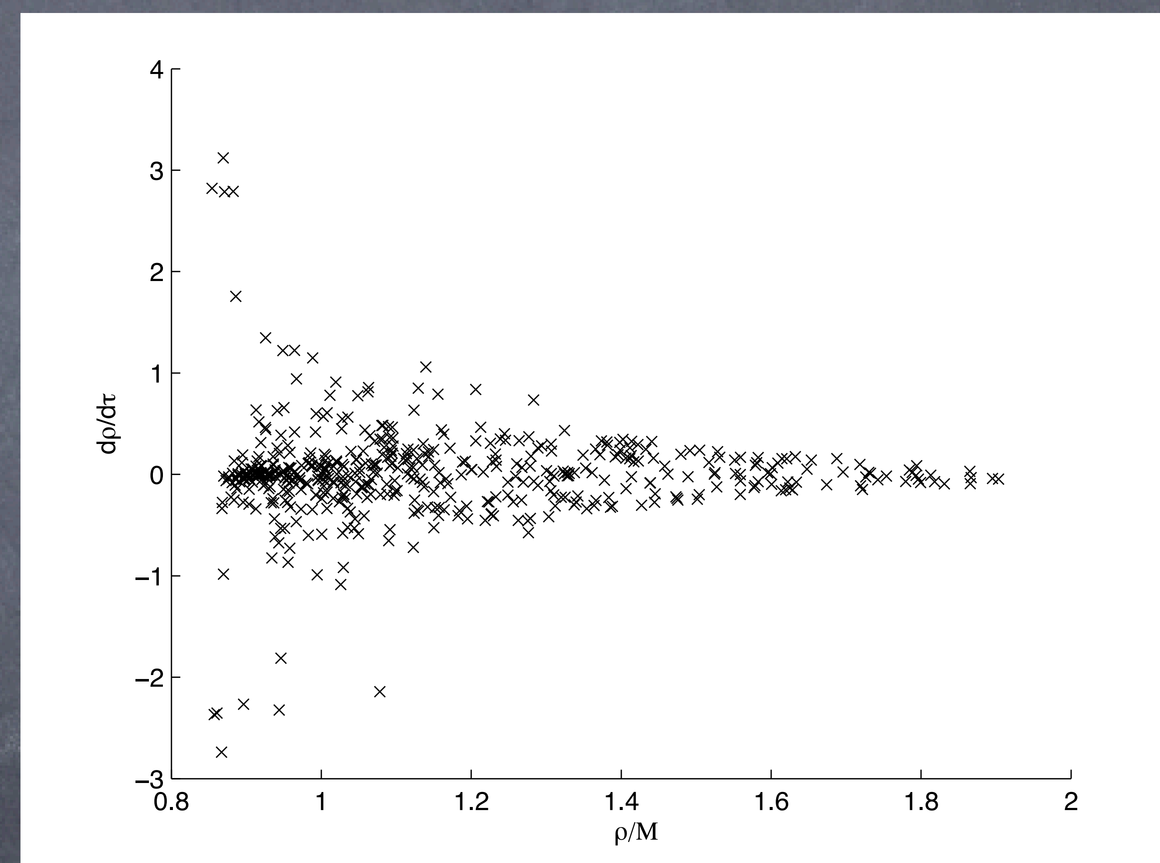
ρ

ρ

$E=0.95$, $L_z=-3$, $a/M=0.9$, $q=0.95$

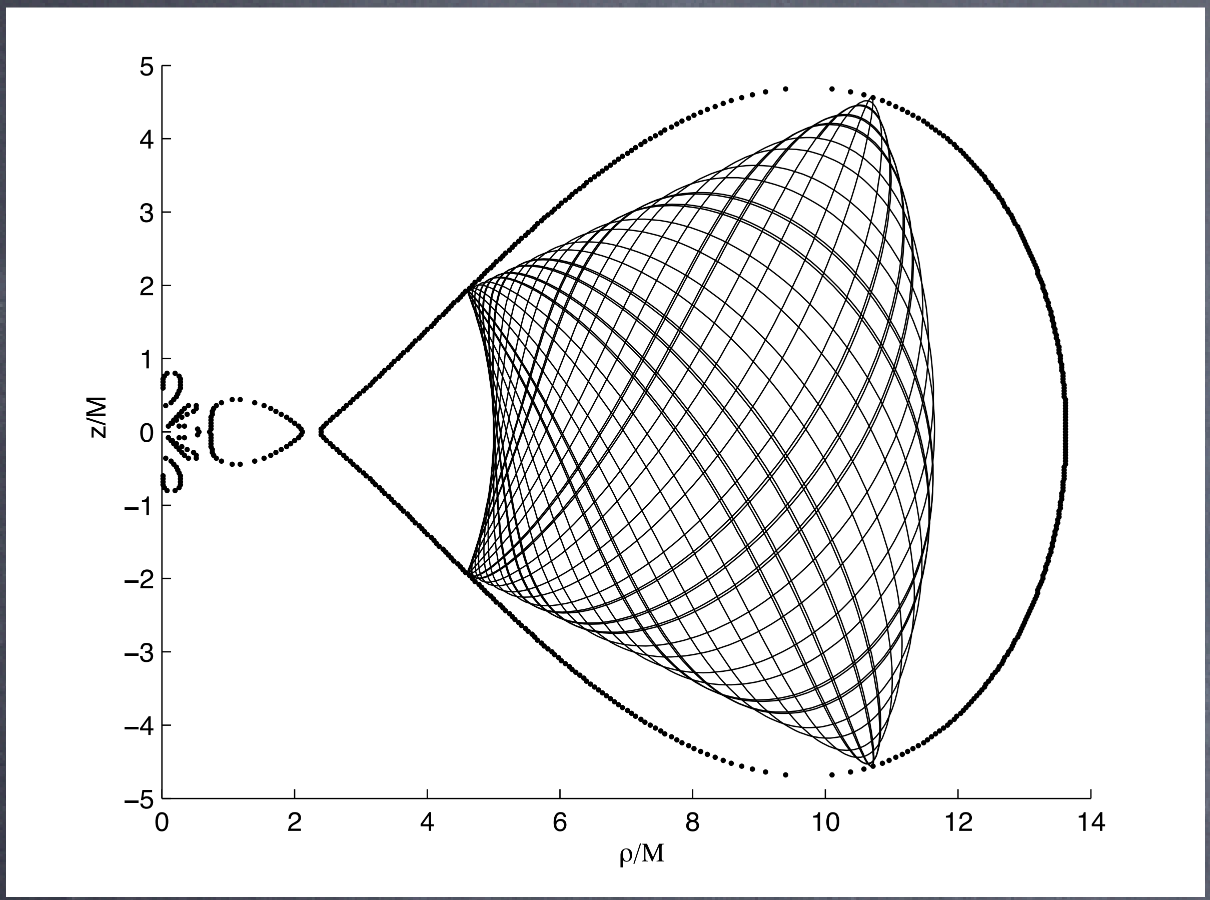
Poincare map in a bumpy spacetime, second look

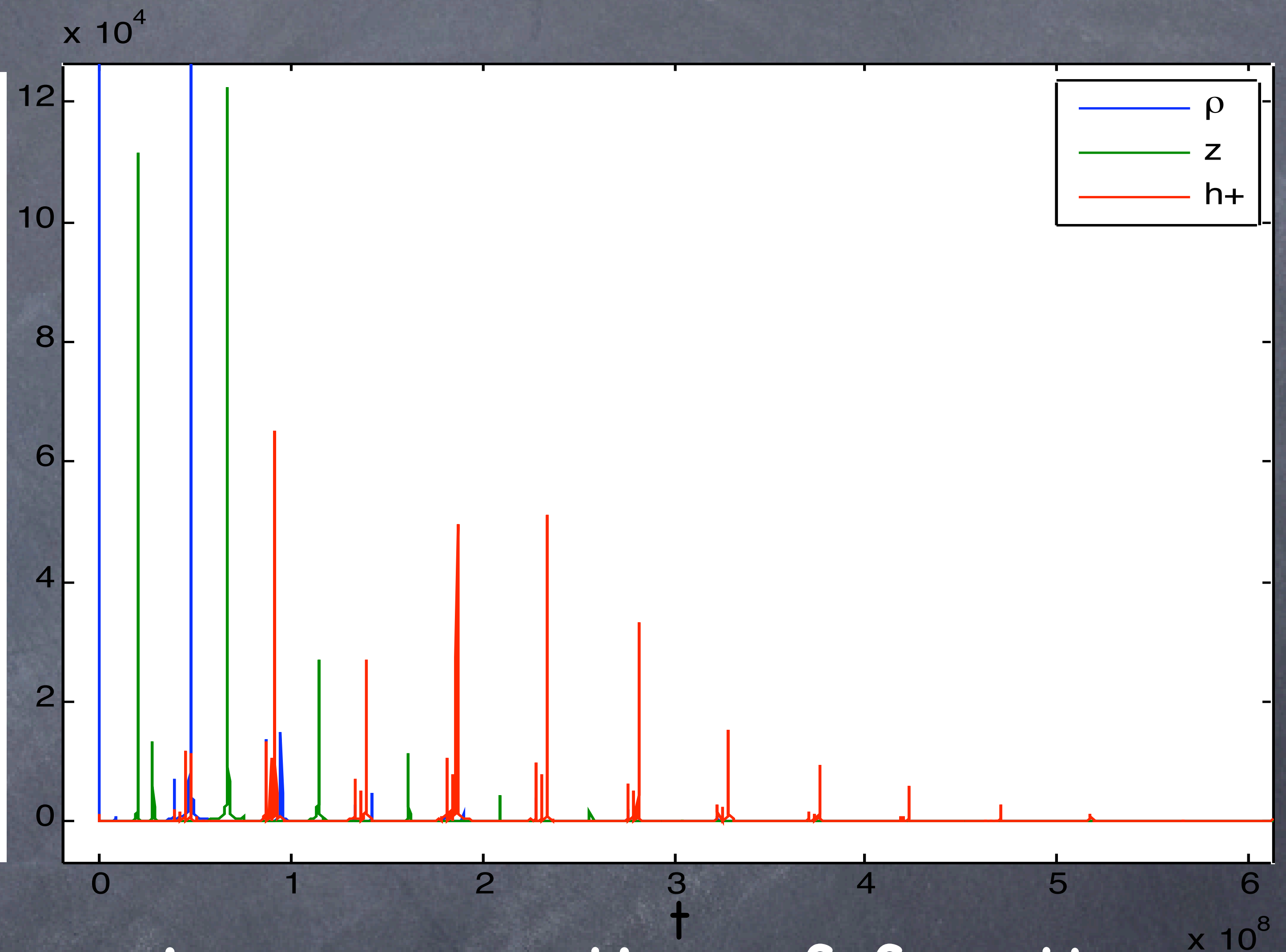




E=0.95, L_z=-3, a/M=0.9, q=0.95

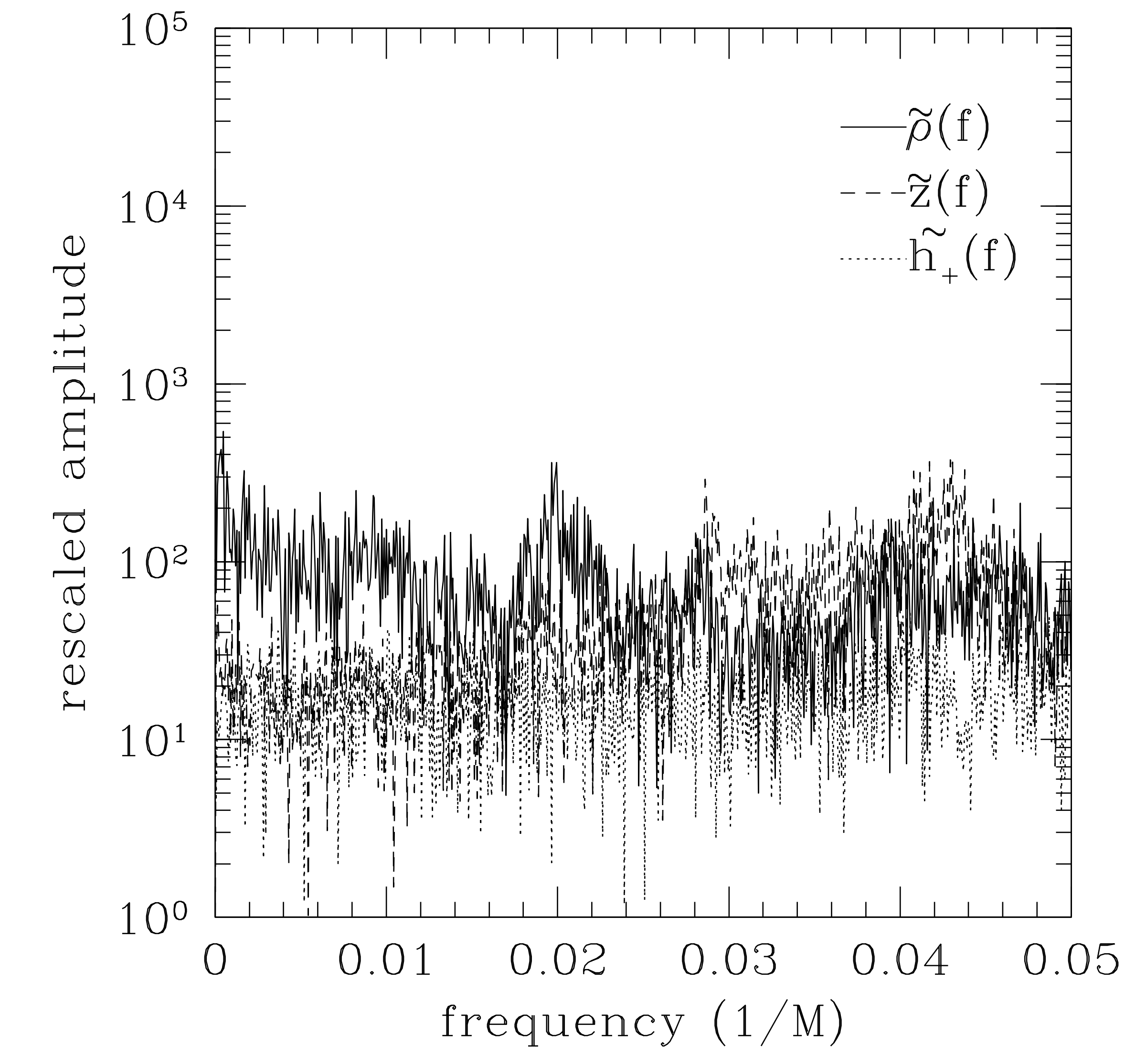
Regular outer region





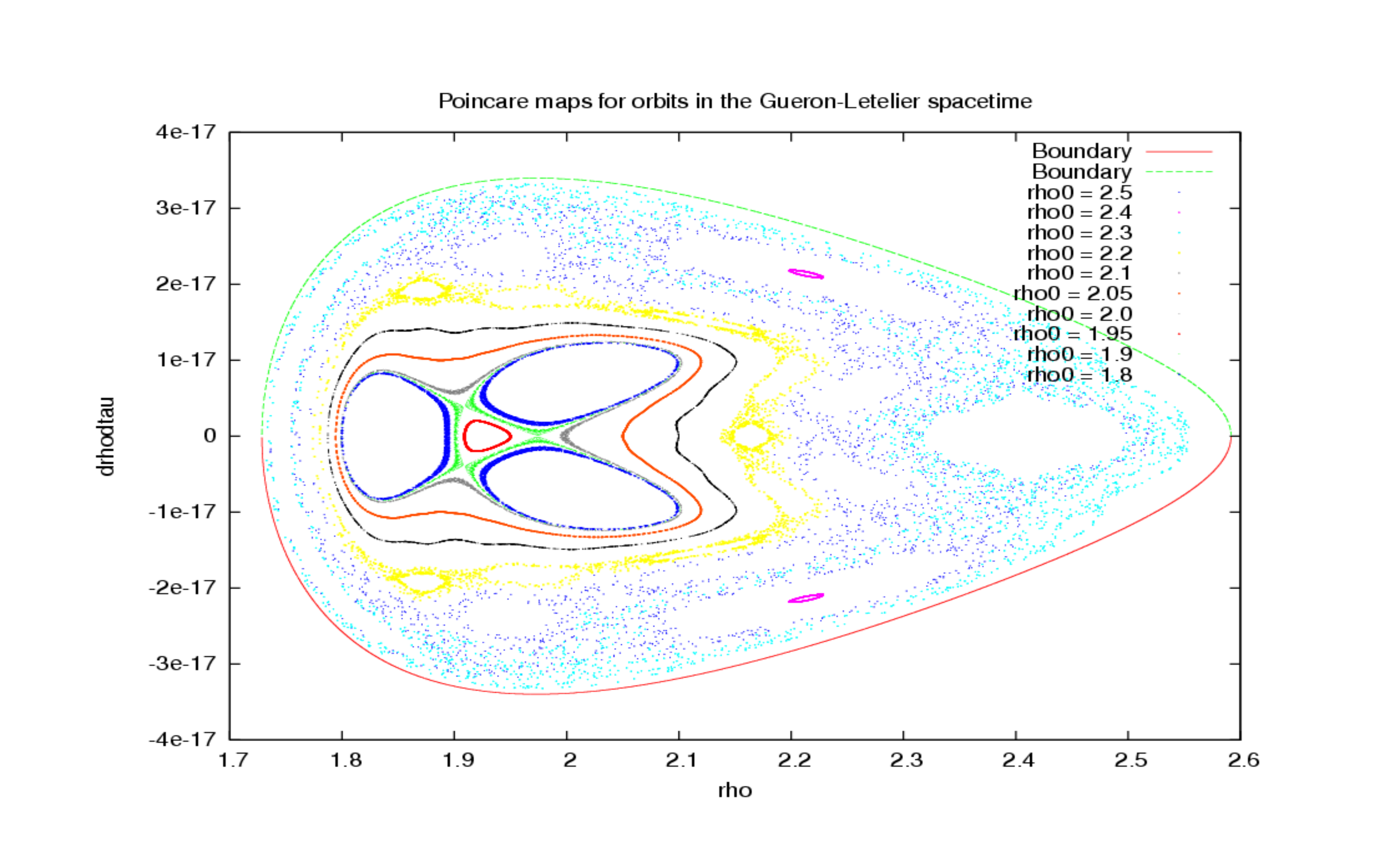
- Regular motion in outer region, suggestive of fourth-degree invariant
- Both ρ and z motion consist of harmonics of two fundamental frequencies to 10^{-7}

Chaotic inner region

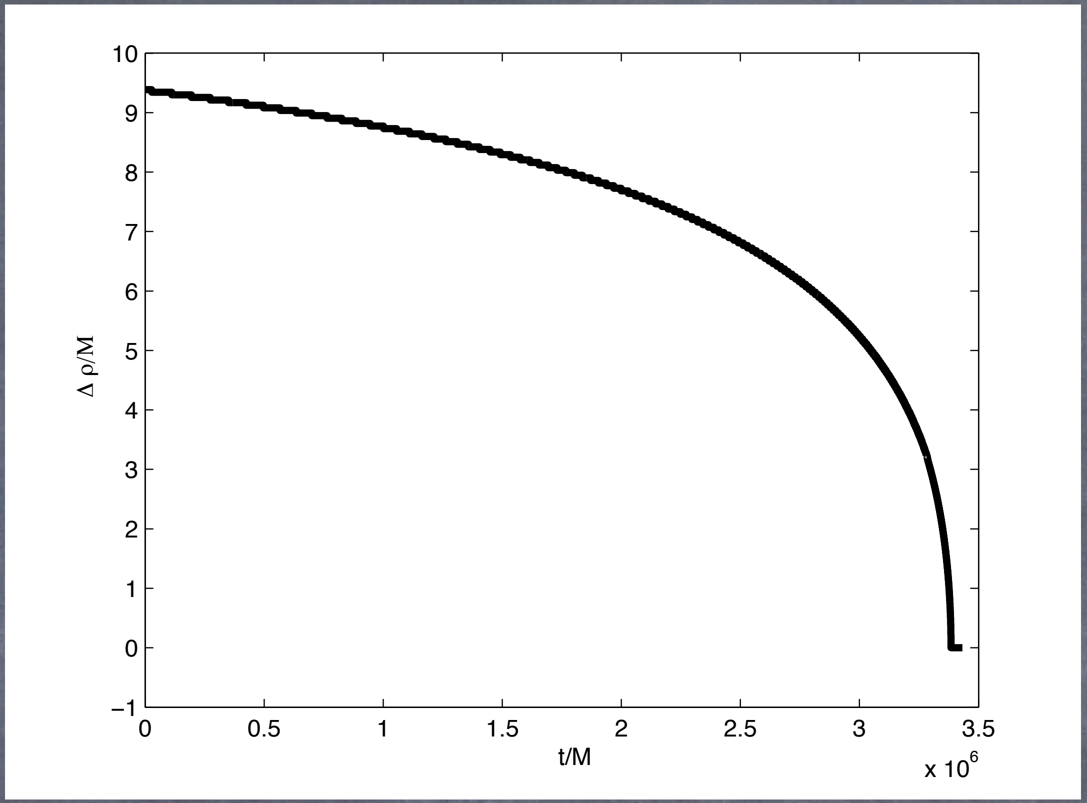


- If motion is chaotic for any initial conditions, it is chaotic for all initial conditions, but an approximate invariant may exist in some cases (invariant tori) [KAM Theorem]

Chaos in Gueron-Letelier spacetime



Is chaos accessible?

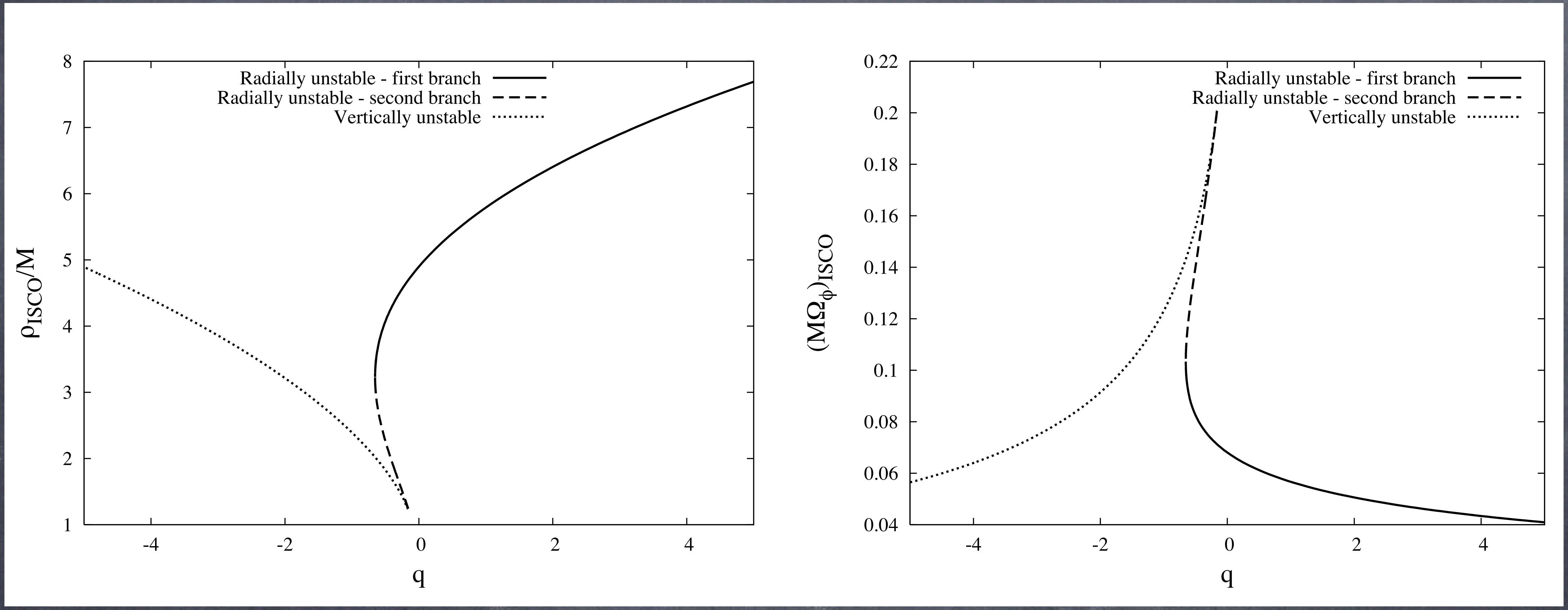


- Inner and outer regions appear to merge under radiation reaction, but never split
- Object starts out in outer, regular region; once the two regions are fully merged, motion is regular (but odd things may happen when the neck is narrow...)

Other observable signatures of bumpiness

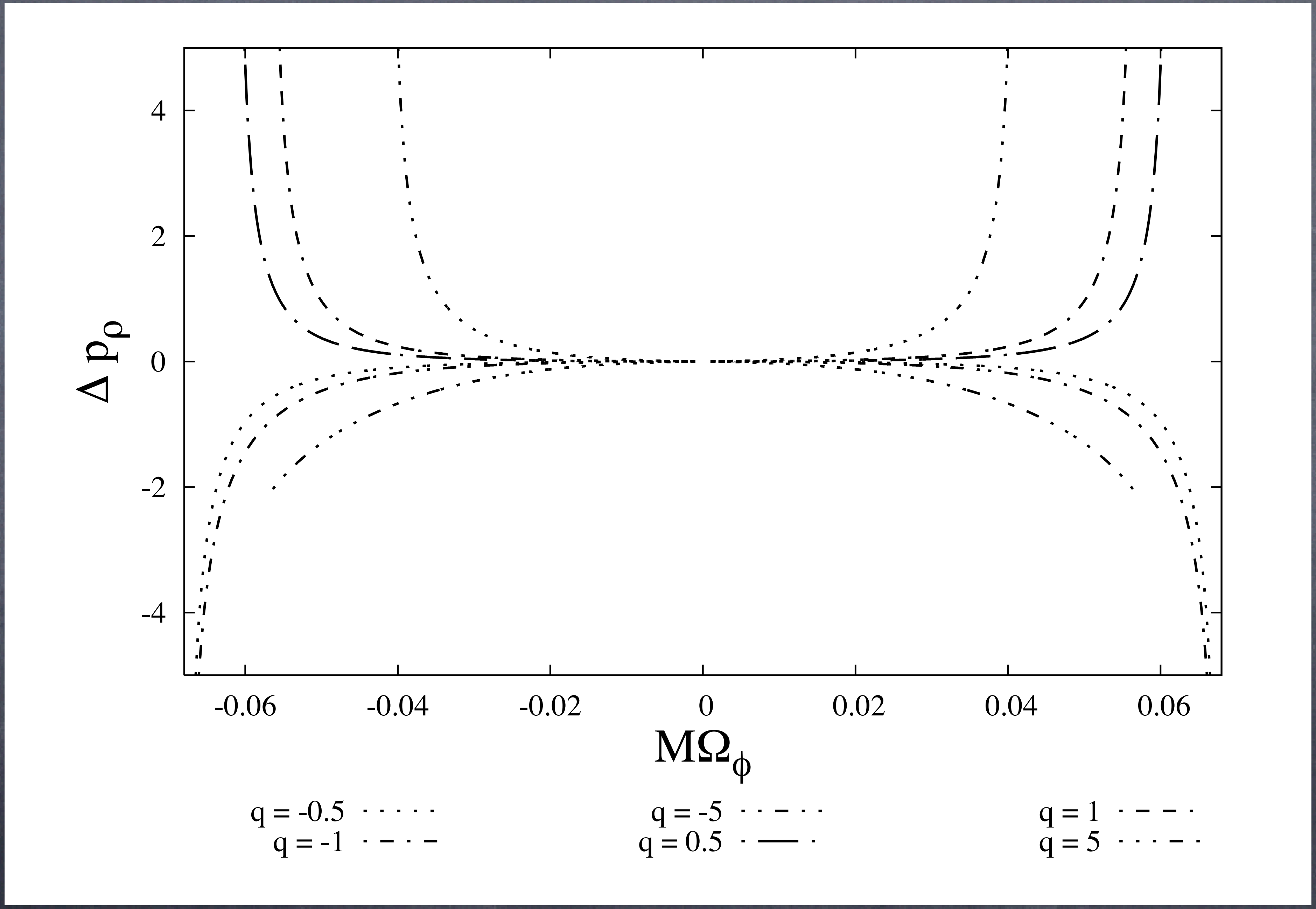
- If the orbits are indeed multi-periodic, then the spacetime "bumpiness" should be observable via:
  1. three fundamental frequencies of gravitational waves
  2. harmonic structure of the waves (relative frequencies and phases of harmonics)
  3. evolution of these with time over inspiral
- Further study required to properly analyze inspiral

Location of innermost stable circular orbit (ISCO)

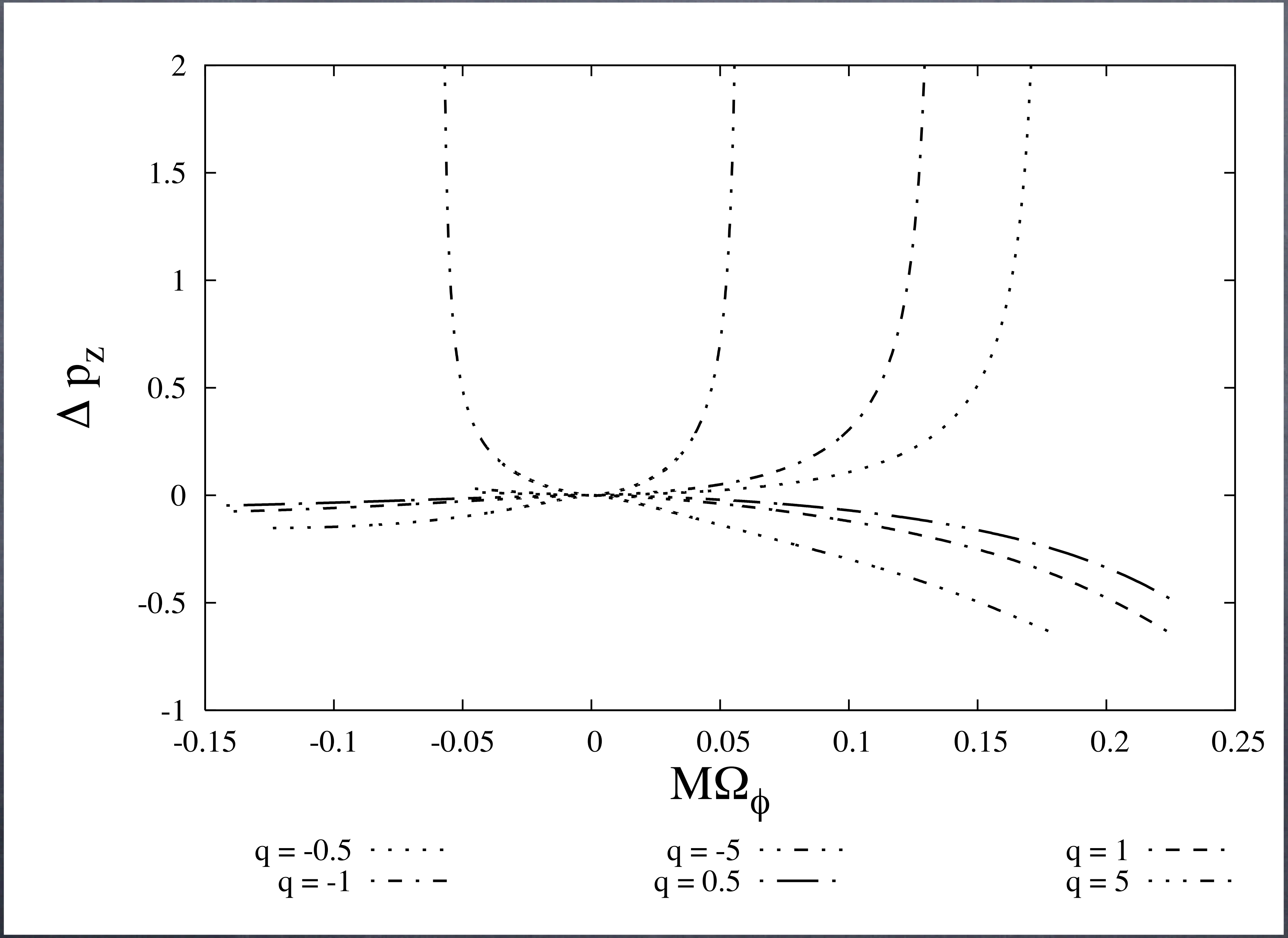


- The ISCO frequency (and hence plunge frequency) depends on the value of the spacetime quadrupole moment

Periapsis precession



Orbital-plane precession



Summary

- Gravitational waves from EMRIs should make it possible to test whether the central body [SMBH] is a Kerr black hole
- Chaos in a non-Kerr spacetime would be an obvious smoking gun, but chaotic regions are probably not accessible
- Location of ISCO, periapsis precession, and orbital-plane precession are possible observables indicating bumpiness
- Frequency evolution over inspiral would be another observable, but more work is required

Do I really believe that SMBHs are not black holes?

- I don't know. But it's dangerous to assume that one will see only what one expects to see. We should be prepared to test our assumptions.
- Every time a new part of the electromagentic spectrum was accessed (radio-astronomy, X-rays, etc.), something unexpected was seen. Gravitational waves are a new window to the universe: expect to see the unexpected!