Black Hole Spins following Minor Mergers

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Distribution of black hole spins following a sequence of minor mergers

- A compact object (CO) is captured by a more massive black hole (BH)
- Radiation reaction leads to CO inspiral
- CO angular momentum and mass are added to the black hole's
- If a BH grows exclusively through a sequence of minor mergers, what will be its spin?

Spin evolution via minor mergers

- Only orbital angular momentum of compact object at LSO is relevant, not its spin
- Inclination is ~constant over inspiral and isotropically distributed

$$\cos\iota = \frac{L_z}{\sqrt{L_z^2+Q}}$$

- Total angular momentum for small χ is

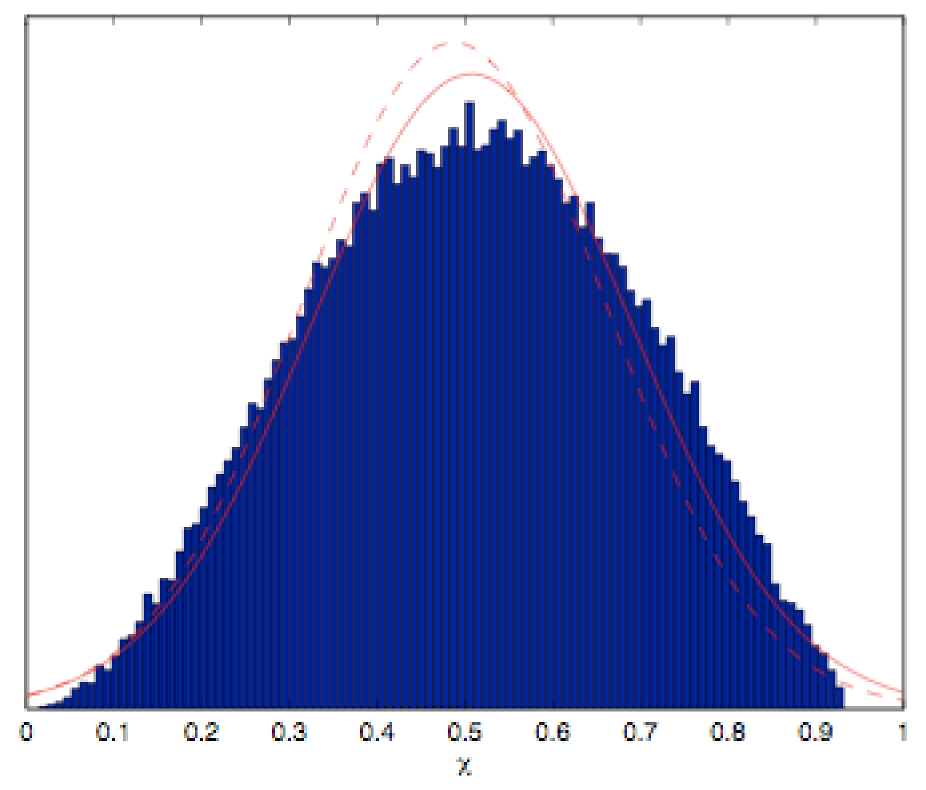
$$\sqrt{L_z^2+Q} \approx Mm\sqrt{12}\left[1-\frac{1}{2}\left(\frac{2}{3}\right)^{3/2}\chi\cos\iota\right]$$

- Minor merger of mass m object with mass M Kerr black hole changes spin parameter $\chi=S/M^2$ to

$$\chi' \approx \frac{1}{(M+m)^2}\sqrt{(\chi M^2+L_z)^2+Q}$$

Monte Carlo simulations of spin distribution

- Evolution from M=5m to M=10m, starting with χ=0.1 and χ=0.9 (see also [Miller, 2002])



Fokker-Planck equation for the evolution of the spin distribution

$$\frac{\partial}{\partial t} f(x,t) = -\frac{\partial}{\partial x}\left[\mu(x,t) f(x,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left[\sigma^2(x,t) f(x,t)\right]$$

- [Hughes & Blandford, 2004] derived a 3-d Fokker-Planck equation but did not solve it
- We parametrize mass by a “time” parameter t=M/m; when $\chi t \gg 1$,

$$\mu(\chi,t) = \frac{\langle d\chi\rangle}{dt} = \frac{\chi}{t}\left(-2 - \frac{4\sqrt{2}}{9}\right) + \frac{4}{\chi t^2},$$

$$\sigma^2(\chi,t) = \frac{\langle (d\chi)^2\rangle}{dt} = \frac{4}{t^2}\left(1 + \frac{4\sqrt{2}\chi^2}{9} - \chi^2\right)$$

- The Fokker-Planck equation still looks complicated…

$$\frac{\partial}{\partial t} f(\chi,t) = -\frac{\partial}{\partial \chi}\left[\frac{\chi}{t}\left(-2 - \frac{4\sqrt{2}}{9} + \frac{4}{\chi^2 t}\right) f(\chi,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial \chi^2}\left[\frac{4}{t^2}\left(1 + \frac{4\sqrt{2}\chi^2}{9} - \chi^2\right) f(\chi,t)\right]$$

Analytical approximation to evolution of probability distribution for the black hole spin

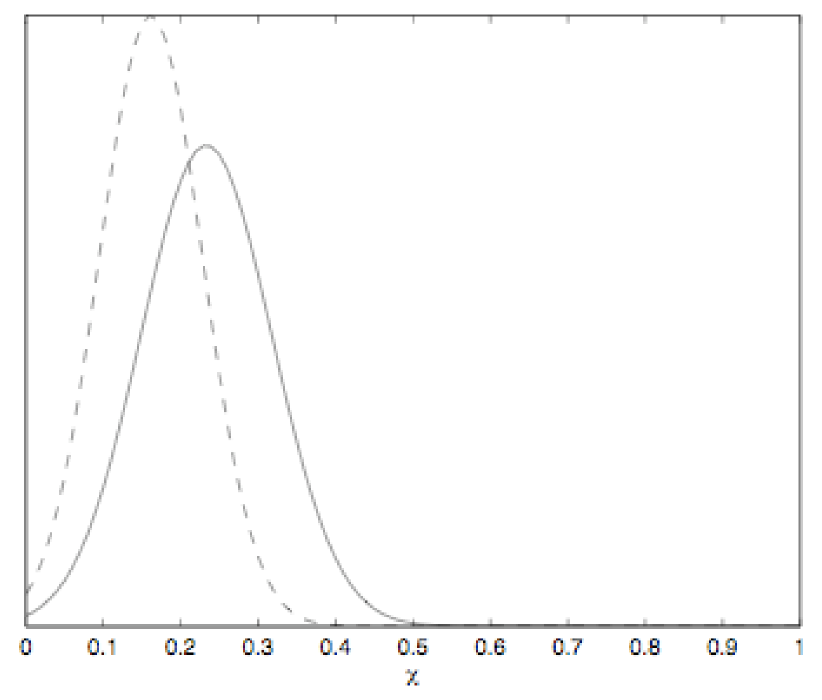
- Probability distribution is roughly Gaussian, so consider evolution of mean and standard deviation
- If $\chi^2 t >> 1$, then $\bar{\chi} \approx \bar{\chi}_0 \left(\frac{t}{t_0}\right)^a \approx \bar{\chi}_0 \left(\frac{M_0}{M}\right)^{2.63}$ where $a \equiv -2 - 4\sqrt{2}/9 \approx -2.63$.
- The random walk is damped, since retrograde orbits have a greater angular momentum at LSO, otherwise exponent would be -2; [Hughes & Blandford found an exponent of -2.4]
- As long as $\chi t >> 1$, mean spin tends toward $\bar{\chi} \to \sqrt{\frac{4}{-at}} \approx \sqrt{\frac{1.5}{t}}$
- As long as $\chi t >> 1$, spin standard deviation tends toward

$$\sigma \to \sqrt{\frac{2(1+b\bar{\chi}^2)}{-at}},$$

where $b \equiv 4\sqrt{2}/9 - 1$.

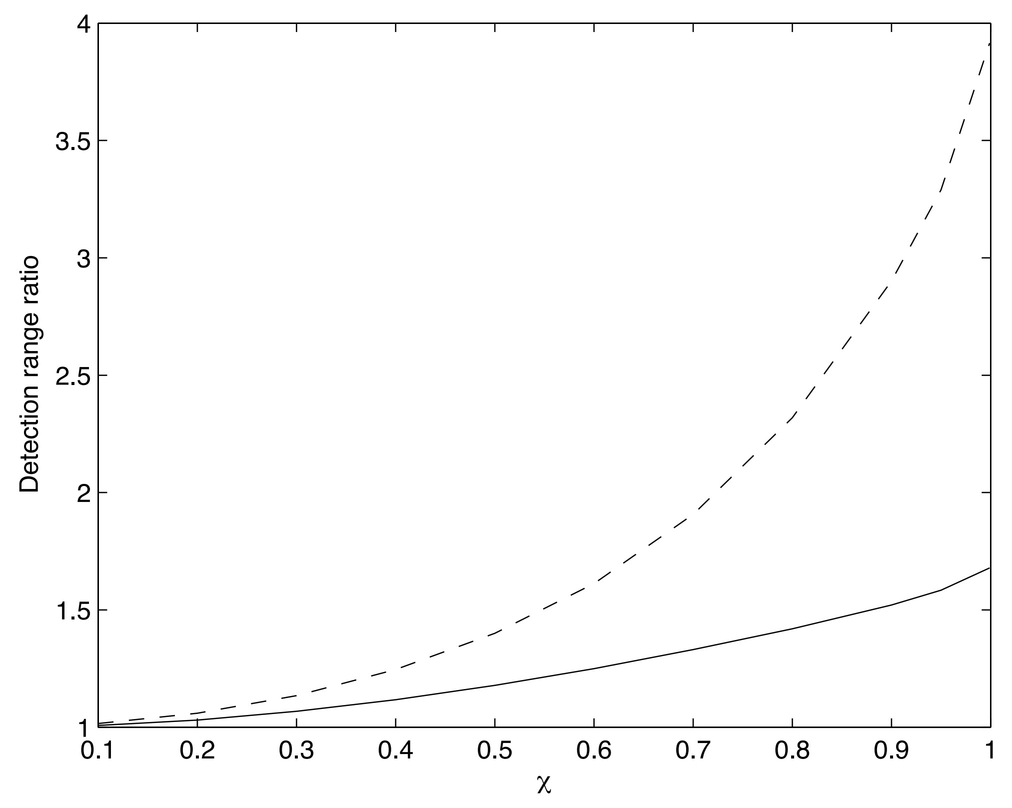
Spin distribution for intermediate mass black holes

- Evolution from t=M/m=50 to t=100 (e.g., from M=70 to M=140 solar masses via capture of m=1.4 solar-mass NSs)
- If initial χ=0.1, then mean spin at t=100 is 0.162, σ=0.066
- If initial χ=0.9, then mean spin at t=100 is 0.233, σ=0.087



Effect of spin on detection range for LIGO IMRIs

- Prograde inspirals can be seen further (higher LSO frequency)

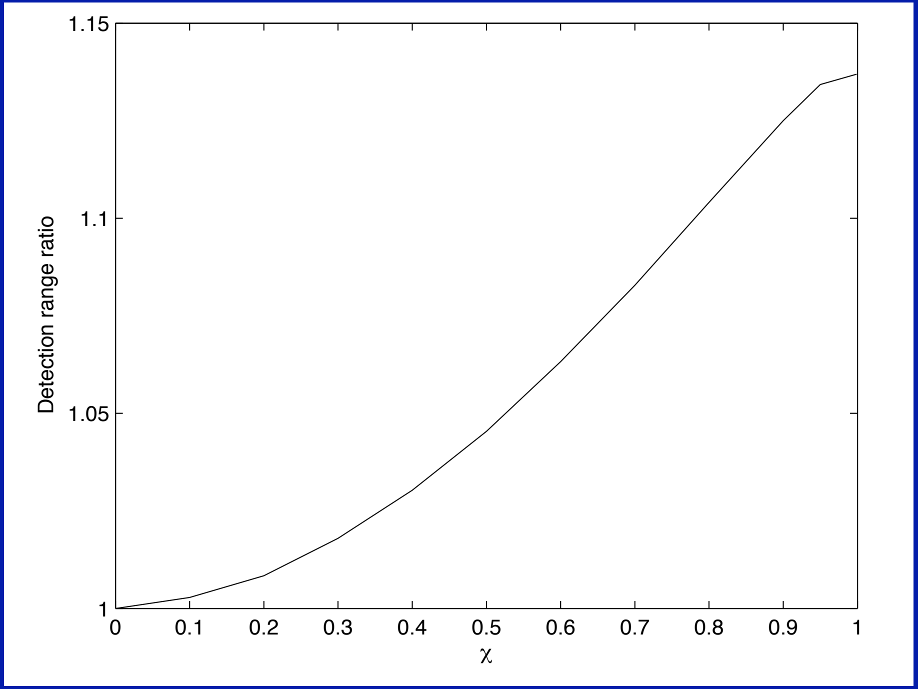


Advanced LIGO:

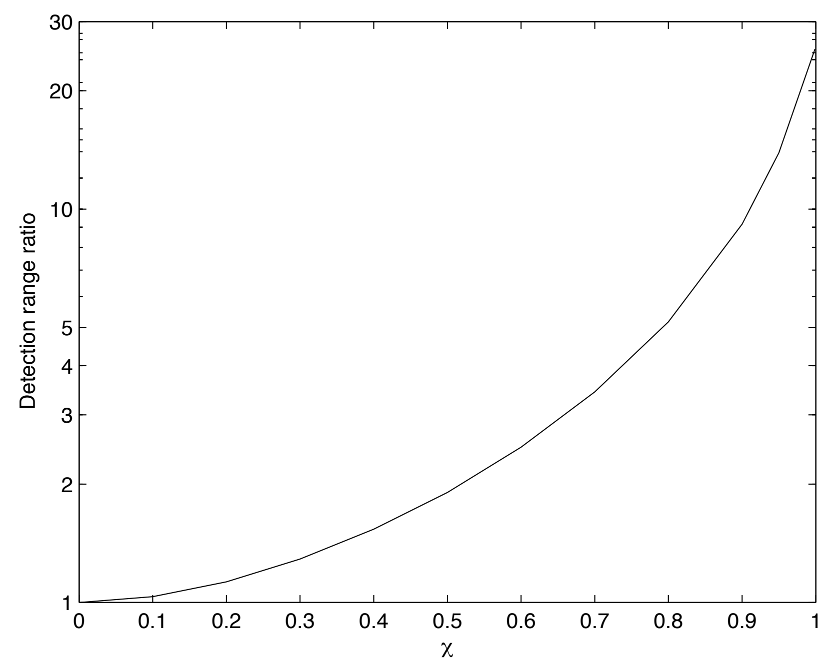
$$\frac{\mathrm{Rate}_{\mathrm{spin}}}{\mathrm{Rate}_{\mathrm{no-spin}}} \sim 1 + 2\chi^2 \left(\frac{M}{100\ M_{\odot}}\right)$$

Effect of spin on detection range for LISA EMRIs

- LISA mission duration limits visible portion of inspiral
- Redshift must be taken into account



$M=10^6\ M_{sun}$, $m=10\ M_{sun}$, SNR=30



$M=10^7\ M_{sun}$, $m=10\ M_{sun}$, SNR=30

Summary

- Found expressions for BH spin distribution following minor mergers
- If intermediate-mass black holes in globular clusters grow via minor mergers, they will have low spins
- Central black hole spins can aid in detection of gravitational waves from intermediate- or extreme-mass-ratio inspirals
- This effect can cause a bias in favor of detecting inspirals into rapidly spinning BHs