Using EMRIs to probe bumpy black-hole spacetimes

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Outline

- What are bumpy black holes and why are they interesting
- Program of study
- Analysis of geodesics in Manko-Novikov spacetime
  - Loss of third integral of motion?
- Future work and outstanding issues

Bumpy black holes

- What are they?
  - Stationary, axisymmetric spacetimes with anomalous (non-Kerr) multipole moments

$$M_n + iS_n \neq M(ia)^n$$

- Why are they interesting?
  - Models for testing whether compact objects are Kerr black holes
  - Test of cosmic censorship conjecture, search for exotic compact objects, null hypothesis test of the no-hair theorem

Can we see the bumpiness?

- How accurately can we determine the amplitude of an anomalous quadrupole moment from gravitational wave emission?
- Most easily detectable by considering EMRIs
- Ryan claimed $\Delta(Q/M^3)\sim 1.5\%$ for LISA, but he used nearly circular, nearly equatorial orbits
- Glampedakis and Babak encountered "confusion" problem, but they considered equatorial geodesics without inspiral
- Barack and Cutler find $\Delta(Q/M^3)\sim 10^{-4}$ for LISA by including evolution of periastron and orbital precession and inspiral rate due to Q

Program under way

- Understand geodesics in bumpy spacetimes
- Analyze kludged waveforms from geodesics
- Study inspirals and inspiral waveforms in bumpy spacetimes

Bumpy spacetimes

- Decomposition in multipole moments
- Manko-Novikov: exact vacuum stationary axisymmetric solution from Ernst potential

$$ds^2 = -f(\rho, z)\ (dt - \omega(\rho, z)\, d\phi)^2 + \frac{1}{f(\rho, z)}\ \left[e^{2\gamma(\rho, z)}\ (d\rho^2 + dz^2) + \rho^2\, d\phi^2\right]$$

- Collins-Hughes: similar approach, but perturbative and no spin
- Yunes-Gonzalez: perturbed Kerr via Chrzanowski-Ori from Teukolsky function
- Li-Lovelace: similar to above

Computing geodesics

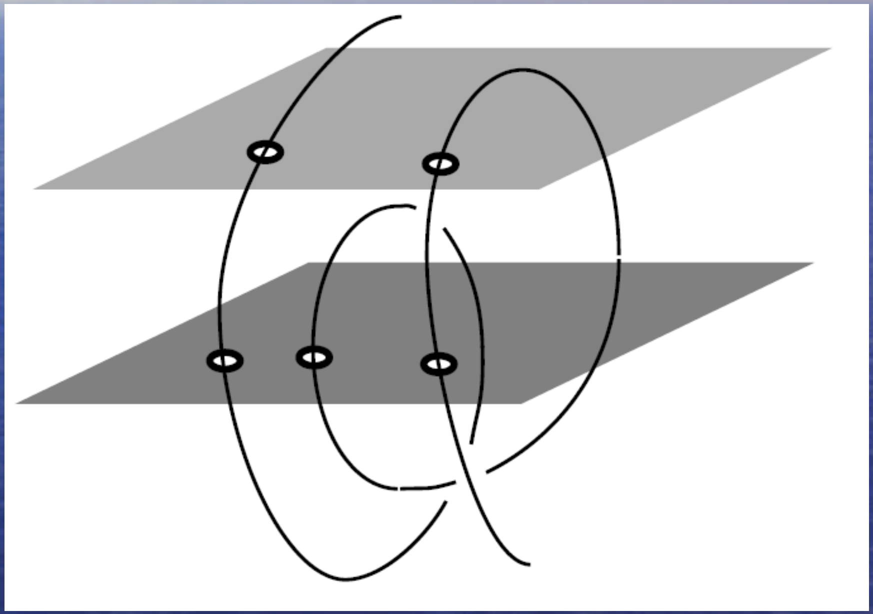
- C code - geodesic equations:

$$\frac{\partial^2 x^\alpha}{\partial \tau^2} = -\Gamma^\alpha_{\beta\gamma} \frac{\partial x^\beta}{\partial \tau} \frac{\partial x^\gamma}{\partial \tau}$$

- Do not explicitly assume conservation of E and L_z in the code
- Check conservation of E, L_z, 4-velocity norm

Poincare maps

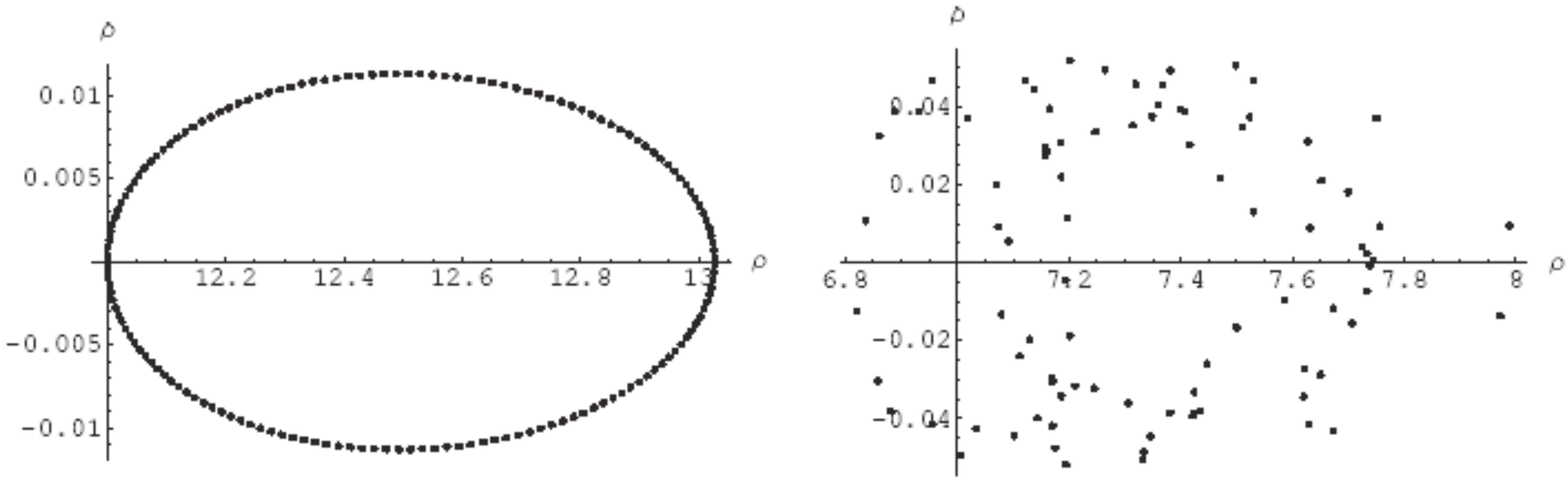
- Poincare maps
  - Plot dρ/dt vs. ρ for $z=z_0$ crossings
  - Phase space plots should be closed curves for all z_0 iff there is a third isolating integral



- Look for disappearance of the third integral of motion

Poincare maps for motion in Newtonian potential with hexadecapole moment

$$V(r,\theta) = -\frac{M_0}{r} + \frac{M_2}{r^3} P_2(\cos\theta) + \frac{M_4}{r^5} P_4(\cos\theta)$$



$M_2 = 10\ M_0$; $M_4 = 400\ M_0$

Courtesy of Chao Li

Allowed regions for bound orbits in Manko-Novikov spacetime

$$\mathrm{d}s^2 = -f(\rho,z)\,(\mathrm{d}t - \omega(\rho,z)\,\mathrm{d}\phi)^2 + \frac{1}{f(\rho,z)}\left[\mathrm{e}^{2\gamma(\rho,z)}\,(\mathrm{d}\rho^2 + \mathrm{d}z^2) + \rho^2\,\mathrm{d}\phi^2\right]$$

$$\left(\dot{\rho}^2 + \dot{z}^2\right) = V(E, L_z, \rho, z)$$

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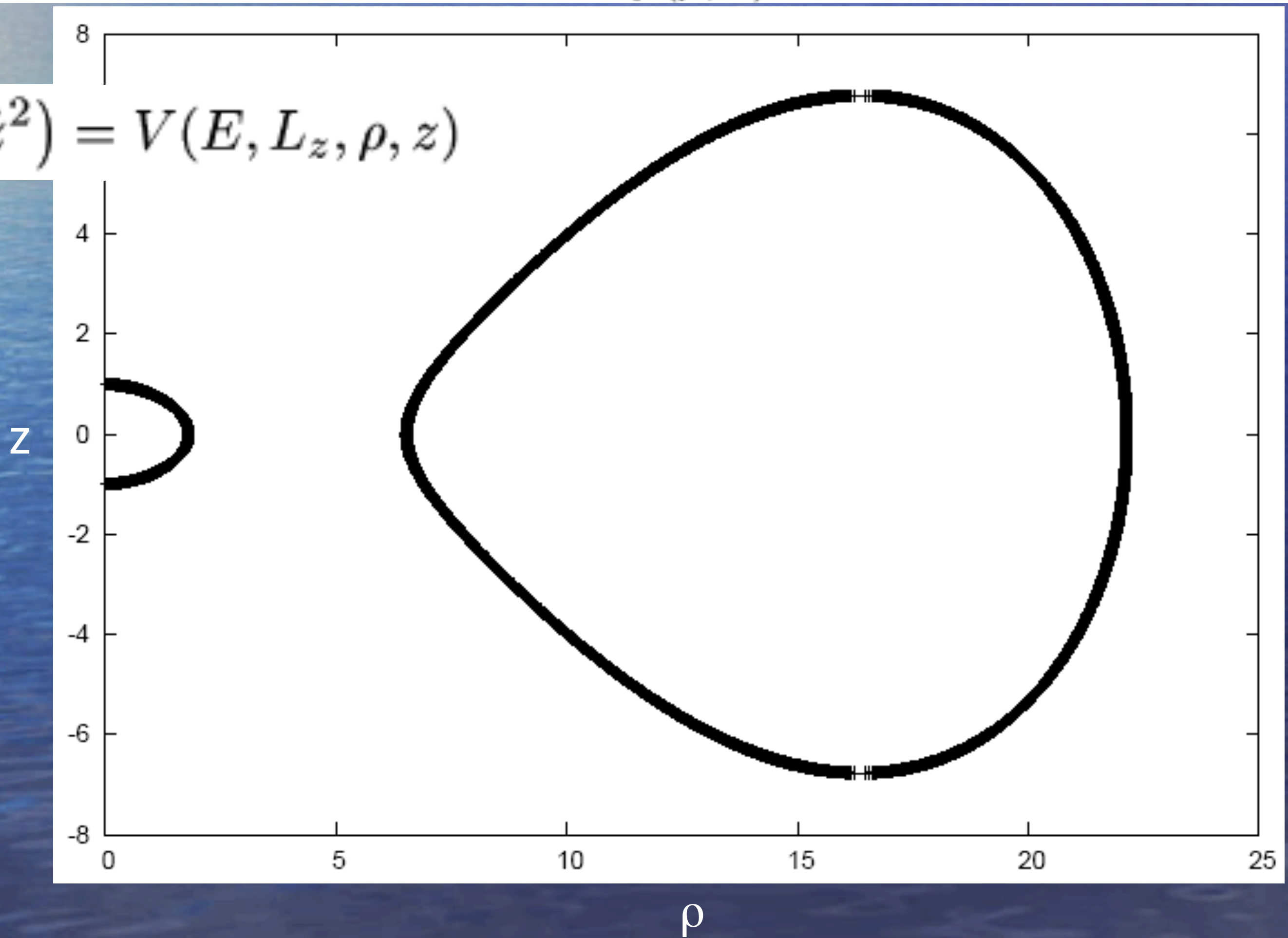
E=0.95
L_z=3



Allowed regions for bound orbits in Manko-Novikov spacetime

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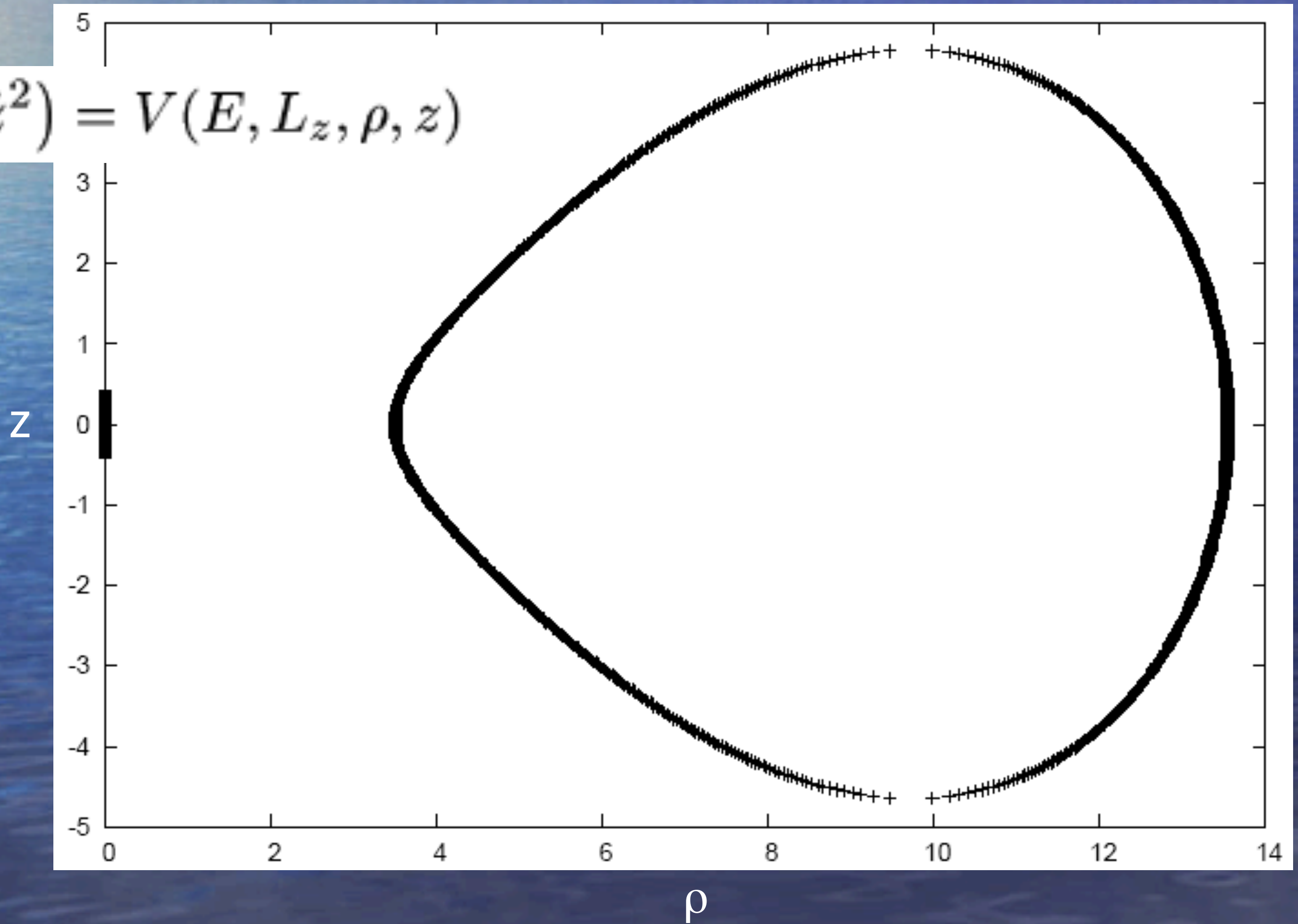

$E=0.95$

$L_z=3$

$a=0.9$

Allowed regions for bound orbits in Manko-Novikov spacetime

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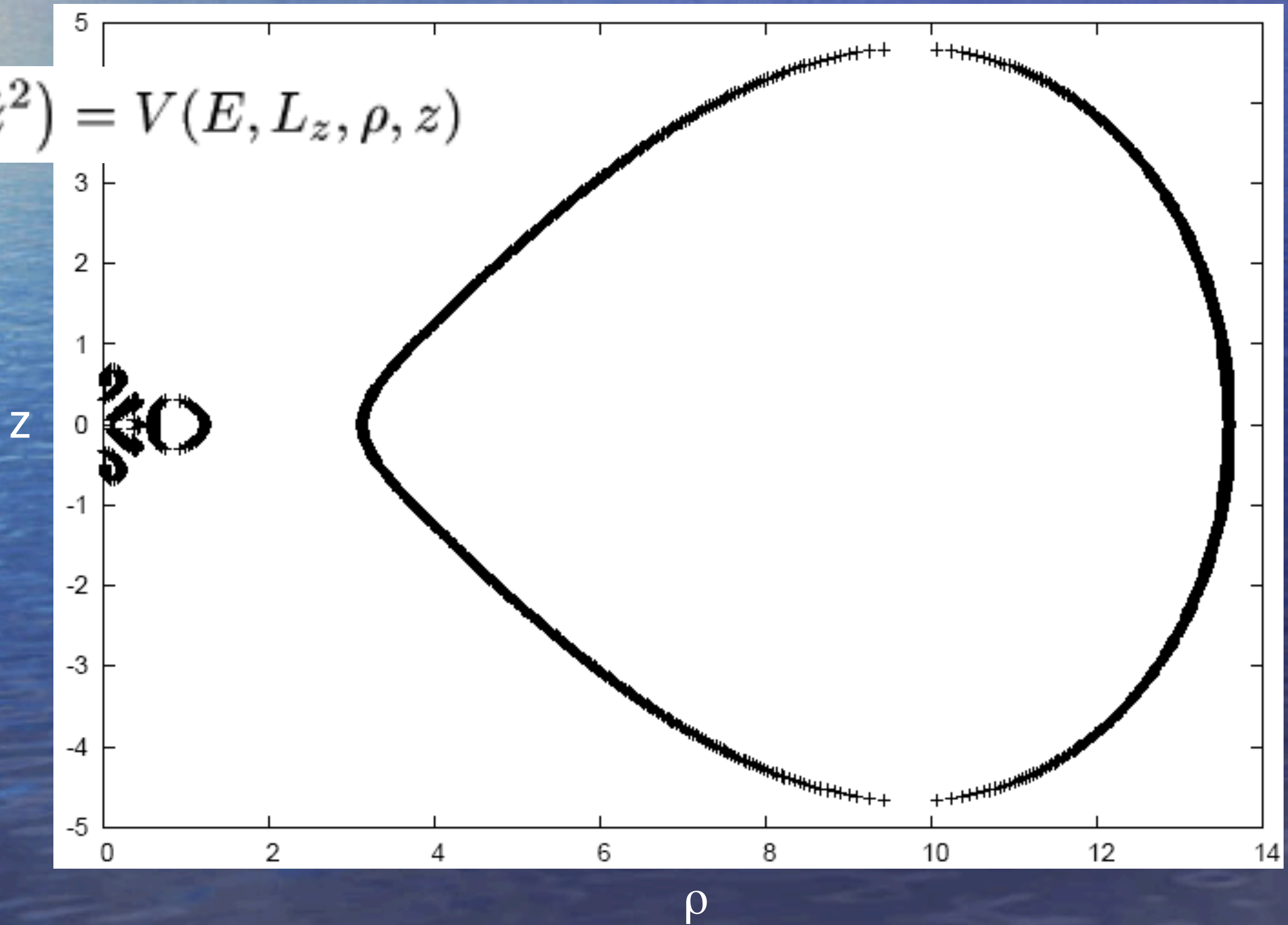
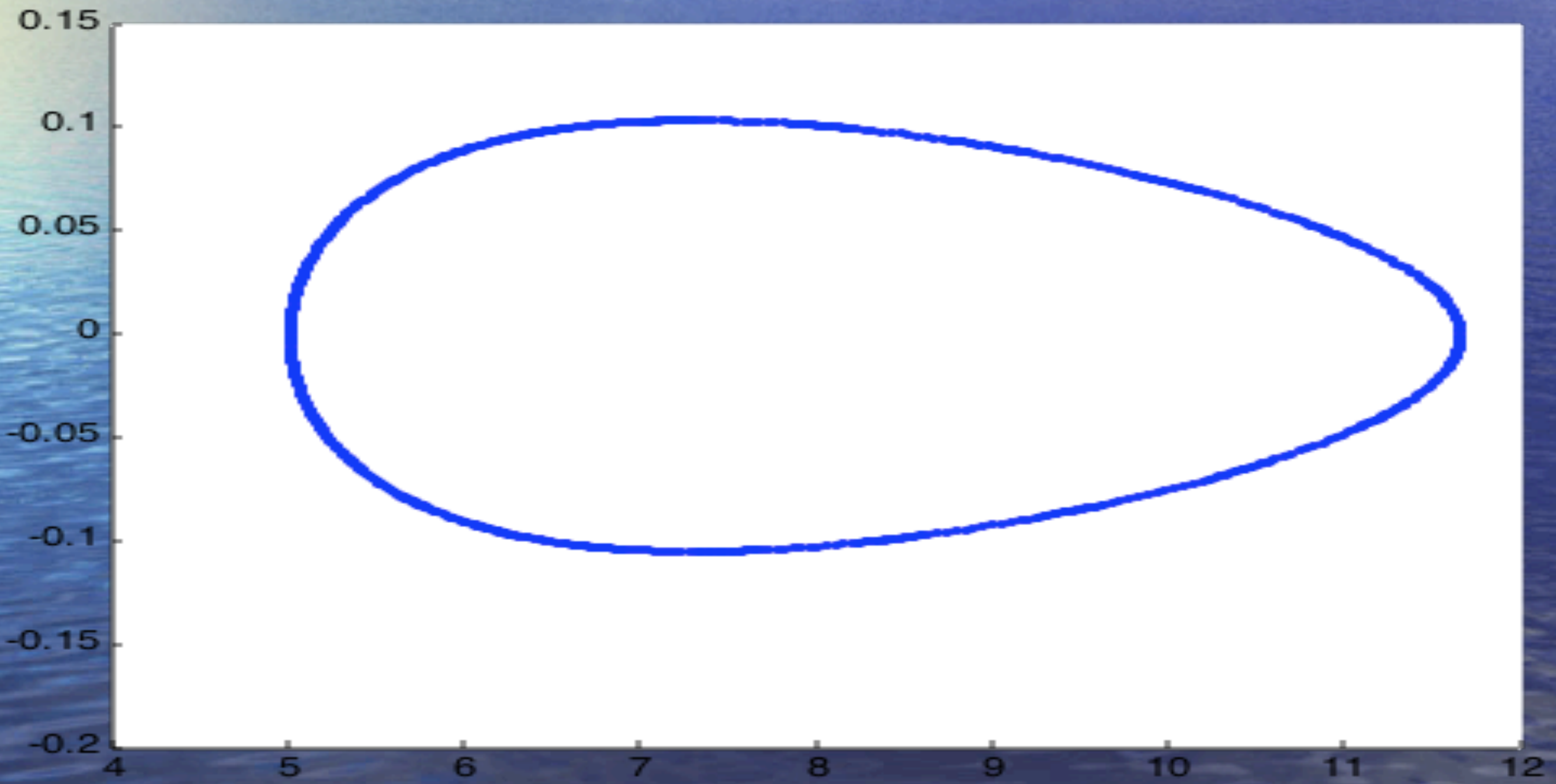
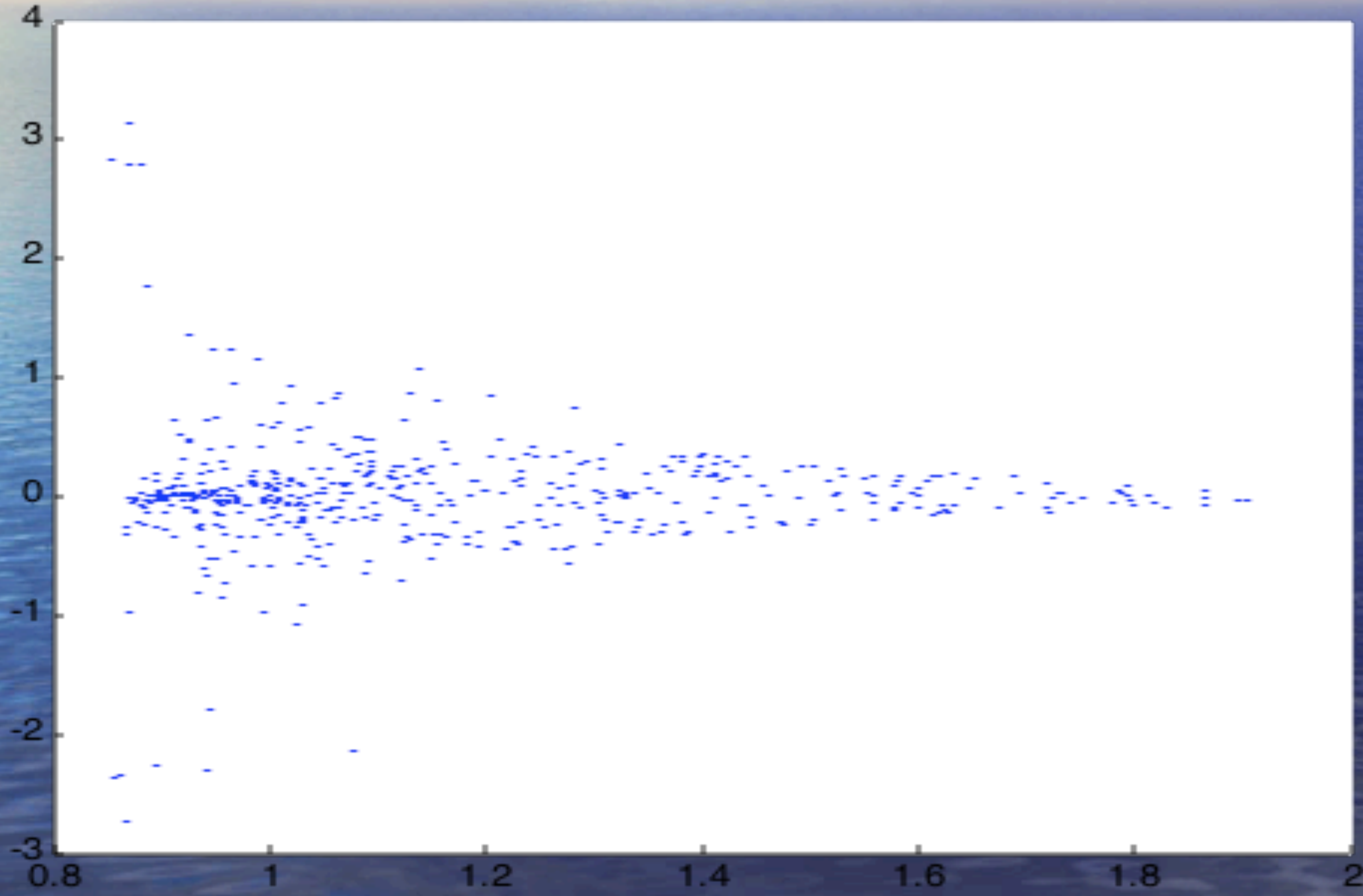


$E=0.95$
$L_z=3$
$a=0.9$
$Q=-0.5$

Apparent retention of the third integral of motion in outer region



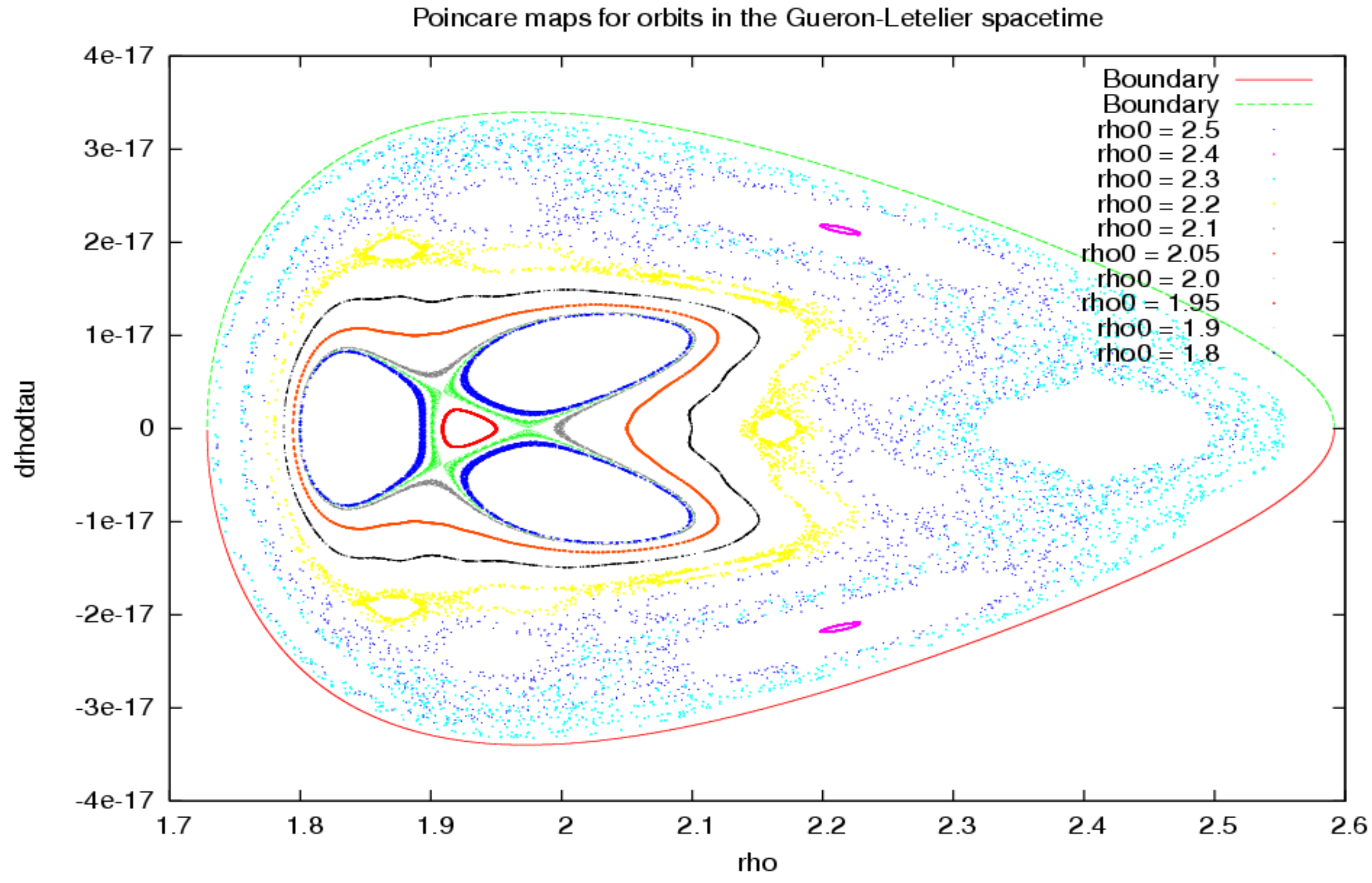
Loss of the third integral of motion in the inner region



General comments on chaos

- If motion is chaotic for any initial conditions, it is chaotic for all initial conditions, but an approximate invariant may exist in some cases (invariant tori)
- KAM theorem [Колмогоров, Арнольд, Moser]
- Clear separation between regions is unusual
  - Possible effect of qualitative difference of naked singularity from a Kerr black hole

Comparison with Guéron-Letelier

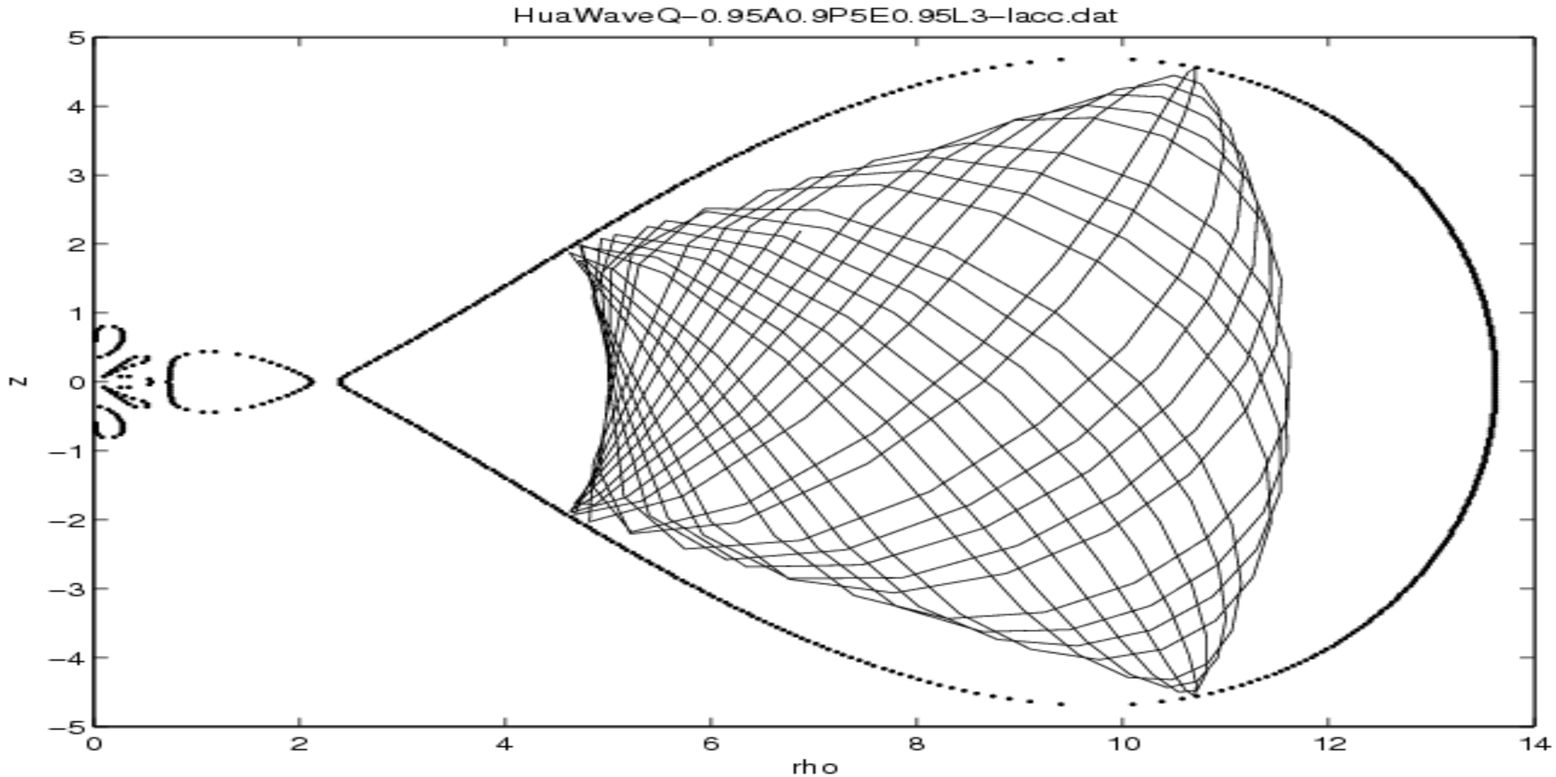


- Prolate Q leads to chaos

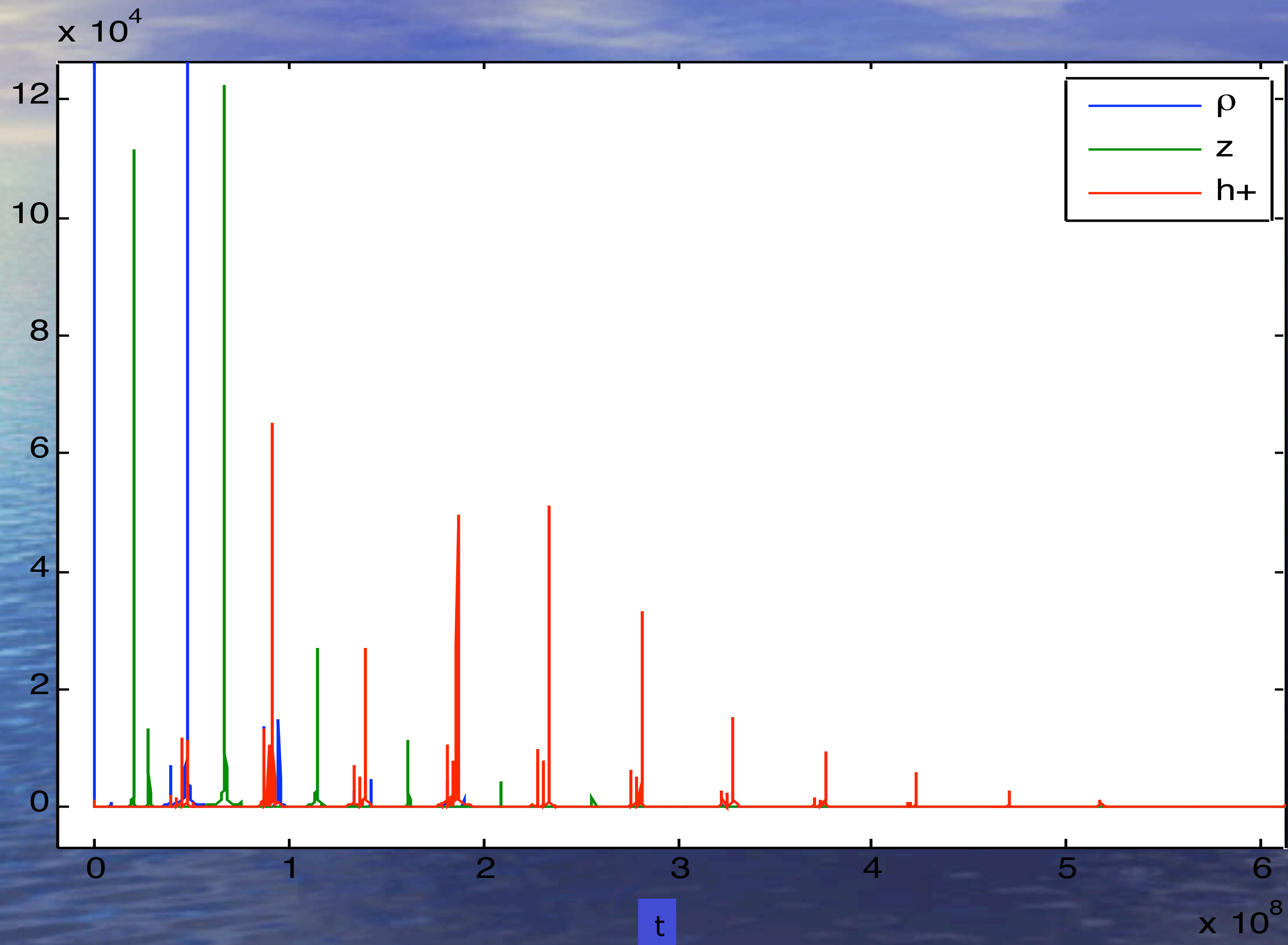
Accessibility of inner region

- Not accessible in the course of a typical inspiral
  - Decreasing energy can lead merged region to split off into inner and outer regions
  - Decreasing angular momentum leads inner and outer regions to merge
  - The latter effect appears to dominate (but we don't have a proof that this is always true)

Outer region: near invariant



Outer region: near invariant II



Both ρ and z motion consist of harmonics of two fundamental frequencies to 10^{-5}

Characterizing geodesics: precession rates

- Compare periapsis precession for equatorial orbits in spacetimes with and without an anomalous quadrupole moment Q
  - identify nearly circular equatorial orbits by azimuthal frequency
  - $\Delta\phi/(Q/M^3) \sim -0.5$
- Orbital precession for inclined orbits due to the presence of an anomalous quadrupole moment
  - identify (circular) nearly equatorial orbits by azimuthal frequency

Kludged waveforms

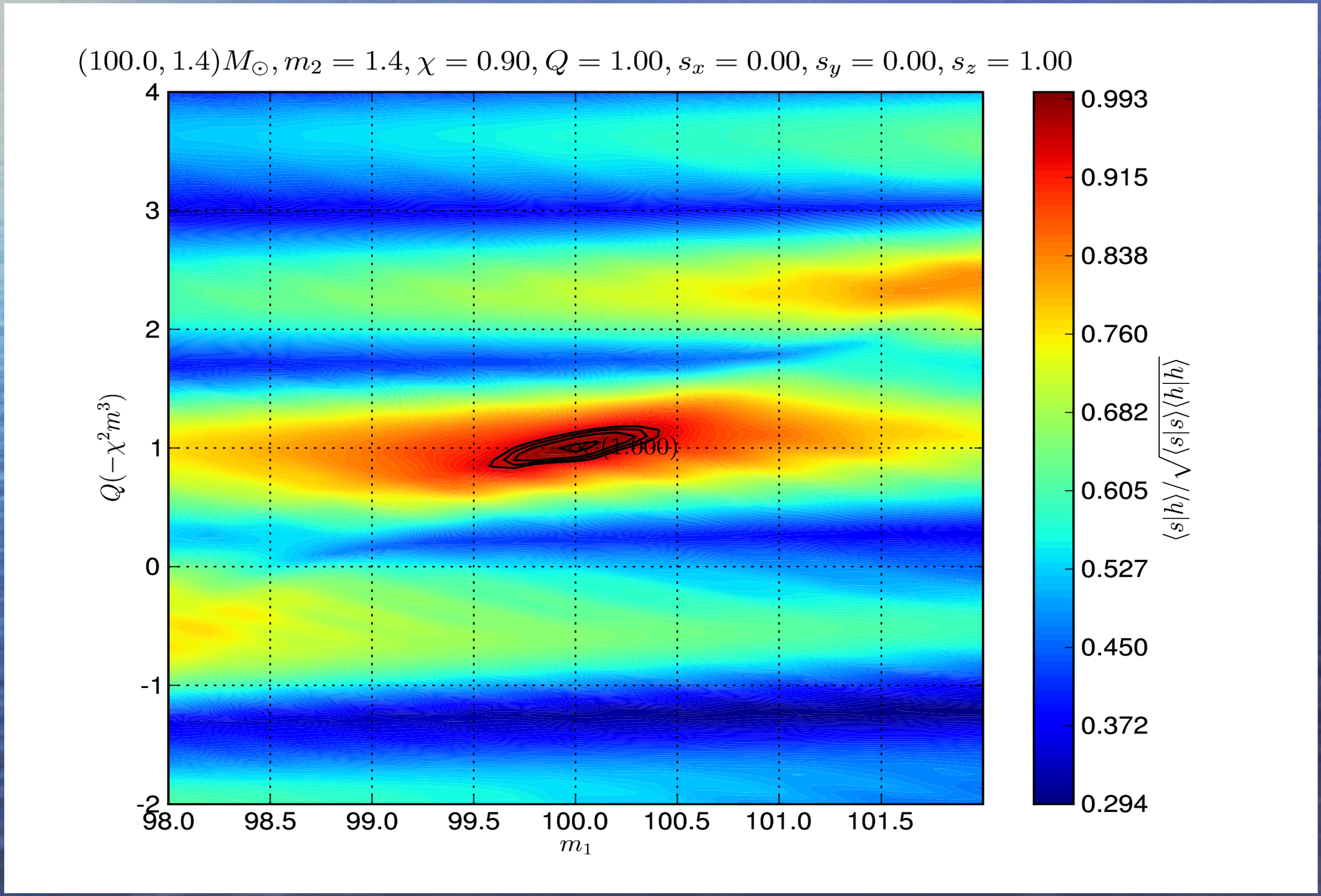
- Obtain kludged waveforms by using the quadrupolar approximation and ignoring spacetime curvature for wave propagation purposes
- Understand orbits' imprint on waves
- Frequency-domain analysis, characteristic frequencies

Kludged inspirals

- Quantitative limits on the anomalous quadrupole moment
  - Monte Carlo simulations
- What is the self force?
  - Radiation reaction; assume constant inclination?

Detection and analysis of IMRIs

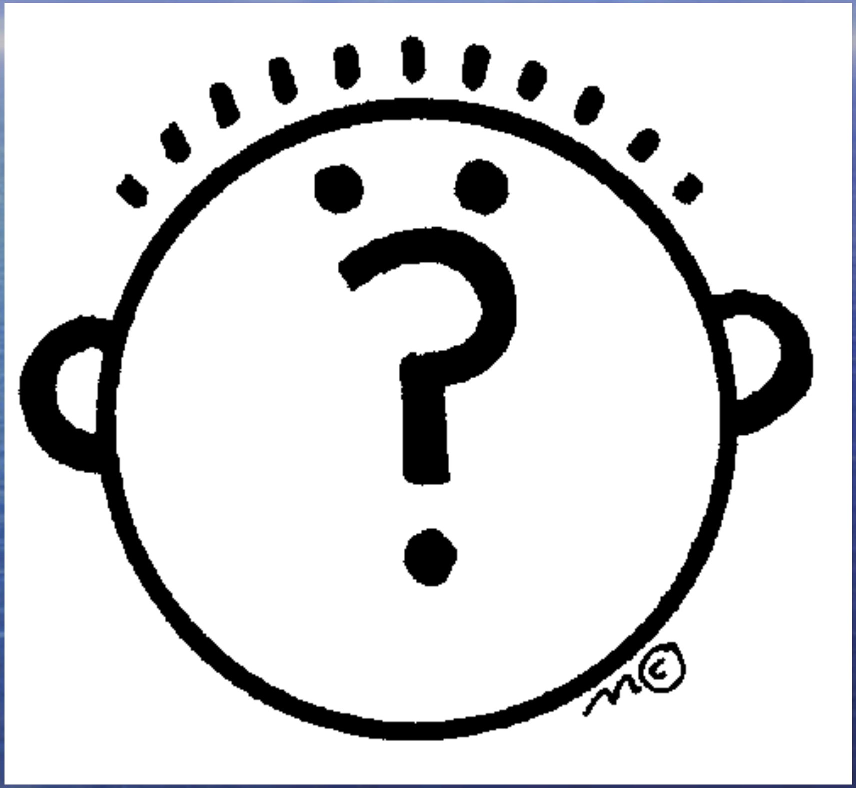
- Advanced LIGO: Brown et al, in preparation



Summary

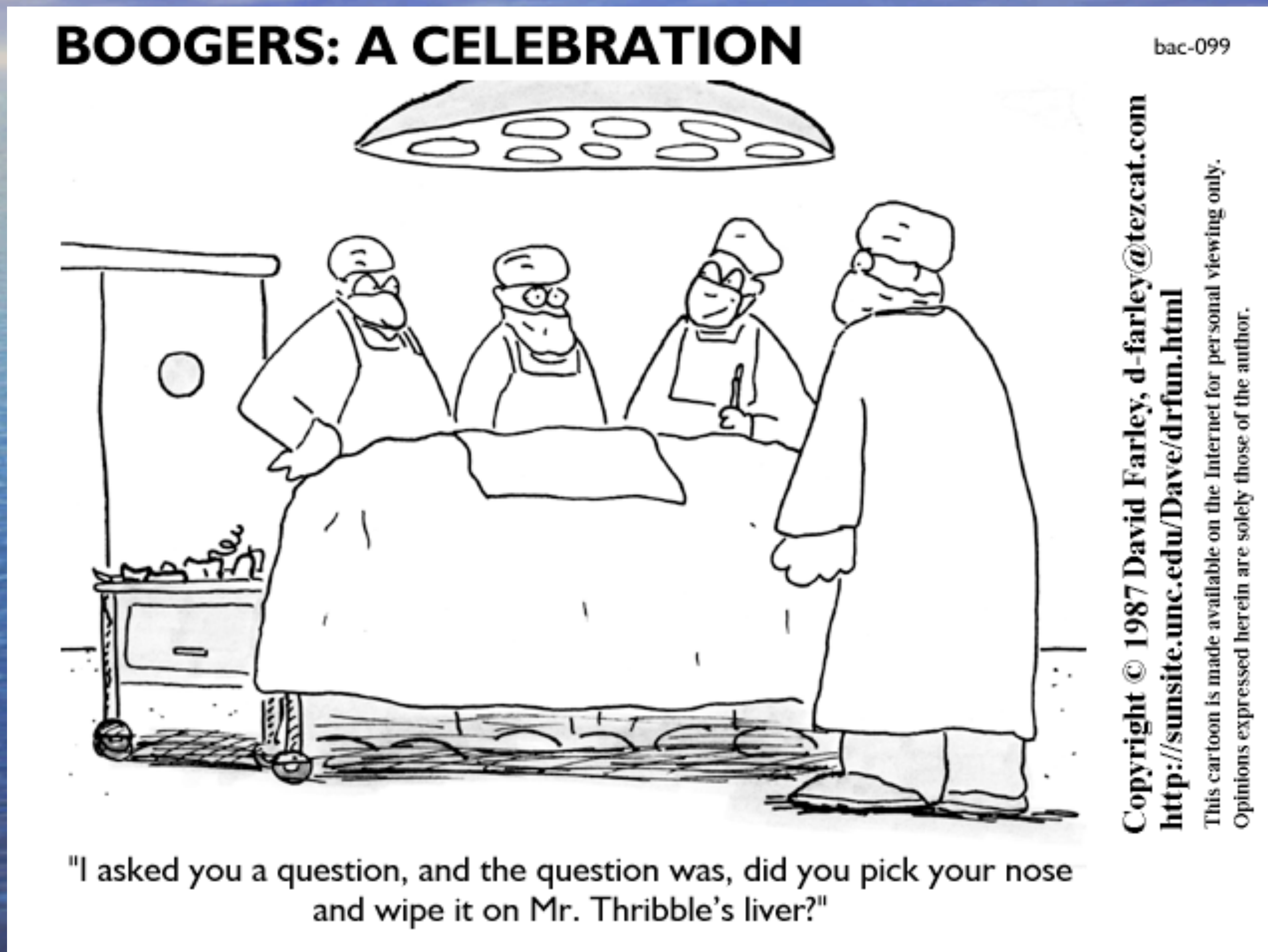
- LISA EMRIs are high quality probes of strong-gravity environment near MBH horizons; accurate extraction of (anomalous) quadrupole moments should be possible
- Q affects periastron precession, orbital plane precession due to MBH spin, and inspiral rate
- Chaotic motion may be mathematically possible in non-Kerr stationary axisymmetric spacetimes, but not astrophysically relevant
- Further study is needed of the precision with which Q can be extracted, as well as practical methods for detecting and analyzing bumpy

Questions?



Thank you!

Questions?


BOOGERS: A CELEBRATION
bac-099
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http://sunsite.unc.edu/Dave/drfun.html
This cartoon is made available on the Internet for personal viewing only.
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"I asked you a question, and the question was, did you pick your nose and wipe it on Mr. Thribble's liver?"

Needed:

Orbital precession